This paper uses a multi-stage game-theoretical model to examine the strategic interplay between research and development (R&D), licensing, and product strategies in competitive markets. Firms compete in the downstream product market as well as in upstream R&D activities related to new technologies used for developing new consumer products, and there are cooperation opportunities that derive from the interfirm licensing of those new technologies. We investigate how varying consumer preferences for uniqueness (the “snob effect”), different R&D efficiencies among firms, and the presence of older alternative technologies all influence strategic decisions concerning R&D investment, product pricing, and market entry. We identify diverse strategies that firms can adopt in licensing agreements and then explore their effects on competitive dynamics and firm profitability. Our findings suggest that licensing between competing firms can have a paradoxical effect on the market leader, potentially reducing its profitability despite enhancing a competitive advantage in R&D. This study highlights the nuanced trade-offs that firms face in multi-stage competition, and it offers insights into the optimal strategic paths for firms characterized by different product qualities and technological capabilities.

Key words: R&D competition, technology licensing, snob effect

1. Introduction

Developing and launching new products is a crucial driver of firms’ competitive advantage and profitability—especially in high-tech industries such as consumer electronics. That market exemplifies this trend with its frequent product launches. For instance, Apple unveils a new generation of iPhones on a yearly basis, while other smartphone makers might introduce new models multiple times within a year. Each generation typically features advancements in technology that include innovations in chips, display, camera, and more.

A successful product launch requires that firms either invest in R&D to develop new technologies or acquire such technologies from other firms, including competitors (i.e., through licensing). A notable example is the relationship between Apple and Samsung, two major players in the high-end segment of the smartphone market. Despite their end-product competition, Samsung is a key supplier of the displays used in Apple’s products (Tibken 2018). Thus the two companies,
while competing in consumer products, turn out to be business partners in product development. Moreover, Apple also invests in developing its own display technology. In 2023, it was reported that Apple planned to start manufacturing its own displays that could replace those from Samsung (Gurman 2023).

This example illustrates a notable structural aspect of firm competition: firms engage in horizontal competition while simultaneously exploring possibilities for cooperation. First, they compete by selling substitutable products in the same consumer market. Second, both invest independently in R&D to develop technologies for new products. Third, a cooperation opportunity may arise in which one firm becomes the technology supplier to its competitor if the former has exclusive ownership of a new technology. This coexistence of competition and cooperation is hardly unique to the smartphone industry; it is prevalent in other sectors as well. For example, Toyota provides its gas–electric hybrid technology to Mazda and Ford even as it competes with them in the hybrid vehicle market (Shepard 2010, Soble 2010). In the electric vehicle sector, Tesla has reportedly begun using the “blade battery” technology of BYD, its biggest competitor, in its own cars (He 2022).

In these competitive situations, firms face a series of interconnected decisions that range from R&D investment in technology development to product and marketing decisions, such as pricing. In addition, firms must consider the options of licensing new technologies to— and obtaining licenses from— competitors. These decisions are challenging because of the uncertainties associated with R&D investment and the implications for preceding and subsequent actions.

Furthermore, the complexity of firms’ challenges increases when the behavior of end consumers is taken into account. The way that consumers perceive new products is pivotal because it affects their purchasing decisions, which in turn shapes product demand and thereby the firms’ product and pricing strategies. A primary factor considered in this paper is the consumer’s preference for uniqueness and the exclusivity of using the most advanced technology. Consider, for instance, Samsung’s foldable smartphone: the Galaxy Z Fold series incorporates a novel folding display technology. This series of phones is priced significantly higher than competing products, making it less affordable as well as rare and exclusive in the market (Todd 2022). The introduction of new technology not only enhances the functionality of a product; its uniqueness also adds to the product’s perceived value among consumers (Kastanakis and Balabanis 2014, Cui and Im 2021).

The extant research has examined firms’ licensing policies mostly in the context of vertical competition (Crama et al. 2017). Our study focuses on R&D and licensing strategies in situations characterized by two forms of horizontal competition: R&D investment, and pricing in the consumer market. Although others have studied the co-opetition (i.e., cooperative competition) facilitated by licensing agreements (see e.g. Chen et al. 2019, Yang et al. 2023 ), our research goes further by incorporating R&D investment as an essential strategic decision. Thus we investigate how
competing firms should make strategic choices about R&D investment and licensing options. The focus is on optimizing a firm’s product entry and pricing decisions while accounting for the “snob effect” of consumers. Our study therefore sheds lights on some intricate decisions faced by firms in competitive environments.

In this paper, we conduct a formal analysis of firms’ decisions in a multi-stage, game-theoretical framework that incorporates all the critical aspects mentioned previously. Although it is possible to examine firms’ decisions at each stage—from R&D to pricing—in isolation, a comprehensive understanding of their optimal and equilibrium behavior can be achieved only by considering these interlinked decisions together. Product competition serves as an instructive example. Analyzed solely within the consumer market, it may appear mainly as a competitive pricing game. Yet once a firm’s licensing decisions are considered, the firm must also address the option of forgoing market entry in favor of potentially more profitable licensing fees. That possibility depends, in turn, on strategic decisions regarding R&D investment to develop the new technologies that make licensing possible.

Our aim is to investigate a firm’s R&D, licensing, and product decisions—as well as their interactions—in a competitive environment. The analysis begins at the consumer end of the market: examining how consumers make purchasing decisions when presented with two substitutable products. Such decisions are based on the traditional transaction utility defined by consumer surplus and also on the additional satisfaction derived from the snob effect. Products are differentiated in terms of their quality, and this is the stage at which firms set retail prices for their respective products. From this pricing game, we can derive the potential demand and profits for each firm.

While anticipating future demand and profits, a firm that owns a new technology faces these two decisions: whether to license this technology to a competitor, and whether to enter the consumer market itself. Licensing enables the rival firm to develop a new product that competes with the licensor’s product, thus introducing potential competition in the consumer market. At this stage, the firm must evaluate the prospects of competition in the consumer market and of cooperation through technology licensing. Each firm must also, decide about its own R&D investment—the success or failure of which will affect its future role as either a licensor or licensee.

The problem becomes more involved when there is an old technology that can be used in new product development. The availability of such old technology makes licensing less desirable, since firms with R&D failures will still have a backup option. For a firm with access to both new and old technologies, a strategic possibility is to license the new technology to a competitor while using the old one in its own products. This scenario is exemplified by a case reported in Forbes (Kelly 2022), where all iPhone 14 models incorporated Samsung’s latest display technology, whereas Samsung’s
flagship phone, the Galaxy S22 Ultra, used an earlier version of the display. Our research delves into firms’ R&D and licensing strategies in the presence of an existing technology. Our analysis reveals distinct licensing strategies that reflect product quality and the extent of snob effect in a competitive market. A firm with superior product quality benefits from not licensing its technology to competitors, thereby maintaining its monopolistic position in the consumer market. Conversely, licensing is a profitable strategy for the firm with lower product quality. Even if that firm successfully develops a new technology, it might find licensing to be more profitable than entering the consumer market: the profit gains from licensing may outweigh the potential losses from not enter the consumer market.

However, the licensing strategy changes when an old technology is available for product development. In this situation, a firm marketing high-quality products may find it beneficial to license its technology—especially if its competitor would otherwise use the old technology and continue to sell in the consumer market. These equilibrium strategies are a function of product quality and the relative performance of new versus old technologies.

We find that licensing plays an critical role in decisions about R&D investment and market entry both. More specifically, the firm whose product quality is superior will tend to invest less in R&D when the option of future licensing is available than when it is not. This difference showcases how a firm can benefit from its competitor’s R&D investment. If the focal firm’s own R&D fails, then it may nonetheless gain access to the new technology through licensing. The firm whose product quality is inferior will, conversely, invest more in R&D under the same conditions; doing so increases the likelihood of developing a new technology that could enhance the competitiveness of its own product or become a valuable asset for licensing.

Our study yields a surprising insight into the effects of licensing on firm profitability. We find, contrary to what one might expect, that a firm with a more competitive product may find itself at a disadvantage when licensing becomes an option. This adverse effect stems chiefly from the behavior of the firm with the less competitive product, which is motivated—by the licensing opportunity—to increase its R&D investment. In response, the firm with the superior product strategically reduces its own R&D investment. That strategic adjustment eventually has a negative effect on the quality-leading firm’s profitability.

The rest of this paper is structured as follows. Section 2 reviews the related literature. In Section 3, we introduce a model that incorporates consumer behavior, licensing, and R&D competition; Section 4 presents our analysis based on this model. In Section 5, we extend the model to include existing older technologies. We conclude in Section 6 with a brief summary of our findings as well as suggestions for future research.
2. Related Literature

Our study is related to two areas of research: R&D and technology licensing, and the snob effect. We shall review each of these areas in turn.

2.1. R&D and Technology Licensing

The strategy of licensing technology is prevalent in many industries. One research stream focuses on licensing between incumbent and entrant firms. Previous work has emphasized that firms with innovative technologies often have strategic reasons to license those technologies to their competitors. For incumbent firms, licensing can serve to deter potential entrants because it discourages them from investing in R&D to develop superior technologies and so preserves the incumbent’s market share (Gallini 1984). The form and effectiveness of licensing contracts in such situations are affected by the focal firm’s financial condition and the market demand for its product (Kulatilaka and Lin 2006). In the case of multiple weak entrants, incumbents can use licensing strategically to favor a particular competing entrant and thereby protect their market position (Rockett 1990).

Another stream of research examines the vertical structure of markets—in particular, how technology providers strategize their licensing to downstream competing firms. Erat and Kavadias (2006) explore the dynamics of how a technology provider can induce partial or full adoption of its technology among competing industrial customers. de Bettignies et al. (2023) compare the performances, with regard to the intensity of market competition, of targeted licensing to a single firm versus marketwide licensing to all firms; they identify the conditions under which each licensing strategy is optimal.

Technology licensing enables cooperation among diverse parties so they can combine their expertise and resources. Studies have examined contract designs in R&D collaboration as well as product launches by parties with differing areas and levels of expertise (Crama et al. 2017, Taneri and Crama 2021). Beyond the scope of R&D collaboration, research on supply chains and operations management explores the dynamics of co-opetition, a setting in which licensing and product competition coexist between a licensor and a licensee (Venkatesh et al. 2006, Xu et al. 2010, Yang et al. 2023).

These studies typically assume that the ownership of innovation is exogenously predetermined, with the owner offering licensing agreements to competitors. In contrast, our study investigates settings in which either competing firm may become the licensor—contingent on the success of their R&D investments and subsequent acquisition of innovation ownership. The type of competition that we study is an R&D race between two rival firms. Such competition has been modeled before (e.g., Iyer and Soberman 2016, Lin 2023), but not the possibility of licensing between the competitors. Our study underscores the large role played by this factor in the competition between firms, not only in the R&D race but also in the resulting product market.

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2.2. The Snob Effect

Our study is related also to research that models consumer behavior in the end-product market. Whereas previous work on licensing and product strategies usually accounts for positive network externalities among consumers (see e.g. Conner 1995, Sun et al. 2004, Niculescu et al. 2018), we focus in particular on the snob effect—a type of negative network externality characterized by the consumer’s preference for (oftentimes conspicuously) consuming unique and exclusive products (Leibenstein 1950).

The snob effect has been examined in the context of firms’ pricing decisions. Amaldoss and Jain (2005a) incorporate the snob effect into consumer utility models and investigate its effects on a monopolistic firm’s demand and pricing strategies, an analysis that the authors also extend to the duopoly setting (Amaldoss and Jain 2005b). Arifoğlu et al. (2020) study a firm’s dynamic pricing problem in the presence of forward-looking customers who exhibit the snob effect.

Such consumer behavior can, moreover, have an economically meaningful effect on firms’ joint pricing and operational decisions. In the context of product development, Agrawal et al. (2015) demonstrate that offering a highly durable product at a high price is a viable strategy for firms whose customers are snobbish. Tereyağoğlu and Veeraraghavan (2012) identify conditions under which adopting a “scarcity” strategy in production is optimal for firms.

We build on this work in our modeling of the snob effect in a rational expectations framework. We solve the firm’s competitive pricing problem; more importantly, our study addresses firms’ R&D competition and subsequent licensing strategies. The analysis developed here reveals the impact of the snob effect in these contexts, which has not been explored in previous studies.

3. Model

3.1. Firm Decisions

We consider a duopoly market consisting of two firms, denoted as firm A and firm B, where each firm (indexed by i or j; i, j = A, B, i ≠ j) independently makes a series of strategic decisions. First, each firm must decide whether to invest in R&D activities for a new technology. Successful R&D enables a firm to incorporate this technology into a new consumer product. In cases where one firm successfully develops the new technology while the other firm does not (because of R&D failure or of choosing not to invest), the successful firm may license its technology to the other firm at a unit licensing fee of w. The firm that did not develop the new technology can then choose to purchase the license for use in its own product. Finally, both firms decide whether to manufacture and sell their consumer products in the same end market. A firm that chooses to enter the market must determine the retail price for its product. If both firms are market entrants then the result is price competition, which will be explored in our model. The sequence of events is presented in Figure 1.
There is an inherent uncertainty in the outcome of a firm’s R&D investment, which can result in either success or failure. A larger investment generally corresponds to a greater likelihood of success. We capture this relation by modeling a firm’s R&D investment level as the probability of achieving success, denoted by $e_i \in [0,1]$. The investment incurs a quadratic cost, $c_i e_i^2/2$, where $c_i$ and $c_j$ are the respective R&D “efficiency coefficients” of firms $i$ and $j$. We assume that the probability of one firm’s success is independent of the other firm’s R&D investment; in other words, the two firms’ R&D activities are completely separate and independent of each other.

### 3.2. Market and Consumer Behavior

The end market consists of consumers with heterogeneous valuations of the new products. We assume that the valuation $v$ follows a uniform distribution over $[0,1]$. A consumer with a valuation of $v$ derives the following utility from purchasing firm $i$’s product:

$$u_i = \delta_i [v + \lambda (1 - z)] - p_i, \quad i = A, B;$$

here $p_i$ is the price of firm $i$’s product. In this utility model, the consumer’s total perceived value of a product reflects two components: the valuation of the product itself, represented by $v$; and the snob effect, represented by $\lambda (1 - z)$. Snobbish customers are those who seek uniqueness and exclusivity in the products they purchase, especially those featuring new technology. Hence this additional utility is diminished when the expected number $z$ of owners increases (a similar modeling approach is adopted by Tereyağoglu and Veeraraghavan 2012; Arifoğlu et al. 2020). The magnitude of the snob effect is captured by the parameter $\lambda$, $0 \leq \lambda \leq 1$; in the absence of a snob effect ($\lambda = 0$), the consumer behaves as a rational decision maker.$^1$

The customer’s preference for firm $i$’s product is denoted by parameter $\delta_i$, which represents the quality difference between the two firms’ products. Unlike the individual product valuation $v$, $\delta_i$ is

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$^1$ In a more general market setting, there could be a mix of rational and snobbish consumers. When we extend our main model to include both types, the results remain qualitatively the same.
a common factor that affects all customers. We assume (without loss of generality) that firm A’s product is of higher quality ($\delta_A \geq \delta_B$), which is normalized to 1; hence $\delta_A = 1$ and $\delta_B = \delta \in (0,1)$. The two products are substitutes, and a customer purchases at most one unit from either firm. A typical customer with valuation $v$ chooses product $i$ if $u_i \geq \max\{u_j, 0\}$ but chooses not to purchase either firm’s product if $u_i, u_j < 0$ for $i, j = A, B$.

Our model’s parameters, $\lambda$ and $\delta$, capture the market’s characteristics in terms of consumer behavior—as influenced by (respectively) the snob effect and the quality differences between the competing firms’ products. The analysis will be concerned mostly with how these two parameters jointly determine the two firms’ R&D and licensing decisions.

4. Analysis

We analyze the game by way of backward induction, starting with the pricing competition in the consumer market; this is followed by an analysis of the licensing game, which takes into consideration all possible outcomes of the R&D stage. If a firm’s R&D efforts are successful, it must then decide whether (or not) to manufacture and sell its consumer products in the market. This firm has the option to license its new technology to a competitor that has failed in its R&D efforts. By licensing the successful firm’s technology, the unsuccessful rival gains the capability of producing and selling its own consumer product in the end market.

4.1. Pricing in the Consumer Market

The consumer market can be either a duopoly—under which both firms $A$ and $B$ sell to consumers—or a monopoly in which only one firm does so. We shall analyze the firms’ pricing decisions in each of these scenarios.

4.1.1. Duopoly

There are two cases in which the market constitutes a duopoly: when both firms successfully develop the new technology or when only one firm succeeds but decides to license the technology to its rival. In choosing between the two firms’ products, a consumer purchases from firm $A$ if $v + \lambda(1-z) - p_A \geq \delta[v + \lambda(1-z)] - p_B$ or purchases from firm $B$ if $\delta[v + \lambda(1-z)] - p_B \geq v + \lambda(1-z) - p_A$. If neither option yields the customer a positive surplus, then no purchase is made. Given this consumer purchase behavior, we can write the demand for each firm’s product as follows:

$$
(d_A, d_B) = \begin{cases} 
(1 - p_A + \lambda(1-z), 0) & \text{if } p_A < p_B/\delta, \\
(1 - \frac{p_A - p_B}{1-\delta} + \lambda(1-z), \frac{\delta p_A - p_B}{1-\delta}) & \text{if } p_B/\delta \leq p_A < p_B + [1 + \lambda(1-z)](1-\delta), \\
(0, 1 - \frac{p_B}{\delta} + \lambda(1-z)) & \text{if } p_A \geq p_B + [1 + \lambda(1-z)](1-\delta).
\end{cases}
$$

We assume that consumers have rational expectations with respect to the purchase decisions of other consumers. In other words: the expected total number of owners is equal to the sum of
indivisuals’ demand for the products of firms A and B (i.e., \( z = d_A + d_B \)). We can then express total demand as

\[
    z = \begin{cases} 
        1 - \frac{p_A}{1+\lambda} & \text{if } p_A < p_B/\delta, \\
        1 - \frac{p_B}{\delta(1+\lambda)} & \text{if } p_A \geq p_B/\delta. 
    \end{cases}
\]

Substituting the total demand \( z \) into the demand functions of firms A and B yields the final demand functions of the two products:

\[
    (d_A, d_B) = \begin{cases} 
        (1 - \frac{p_A}{1+\lambda}, 0) & \text{if } p_A < p_B/\delta, \\
        \left(1 - \frac{\delta(p_A-p_B)+\lambda(\delta p_A-p_B)}{\delta(1-\delta)(1+\lambda)}, \frac{\delta p_A-p_B}{\delta(1-\delta)} \right) & \text{if } p_B/\delta \leq p_A < 1 - \delta + \frac{(\delta+\lambda)p_B}{\delta(1+\lambda)}, \\
        (0, 1 - \frac{p_B}{\delta(1+\lambda)}) & \text{if } p_A \geq 1 - \delta + \frac{(\delta+\lambda)p_B}{\delta(1+\lambda)}. 
    \end{cases}
\]

The demand for each firm’s product depends on the relative prices set by both firms. If those prices fall within a certain range, as in the second line of the demand function just given, then there is positive demand for the each firm’s product. But if there is a large gap between the prices set by the two firms (first or third line of the demand function), then the demand changes drastically: there is an demand for the firm’s product whose price is lower, yet there is no demand for the other firm’s product because its price is higher.

The profit functions of firms A and B are (respectively) \( \pi_A = p_A d_A \) and \( \pi_B = p_B d_B \). Lemma 1 gives the resulting equilibrium prices and profits.

**Lemma 1.** In a duopoly market, the equilibrium retail prices are \( p_A^* = \frac{2(1-\delta)(1+\lambda)}{4+3\lambda-\delta} \) and \( p_B^* = \frac{\delta(1-\delta)(1+\lambda)}{4+3\lambda-\delta} \); the equilibrium demands are \( d_A^* = \frac{2(1+\lambda)}{4+3\lambda-\delta} \) and \( d_B^* = \frac{1+\lambda}{4+3\lambda-\delta} \). The corresponding equilibrium profits of firms A and B are \( \pi_A^* = \frac{4(1-\delta)(1+\lambda)^2}{(4+3\lambda-\delta)^2} \) and \( \pi_B^* = \frac{\delta(1-\delta)(1+\lambda)^2}{(4+3\lambda-\delta)^2} \).

Both firms’ equilibrium profits increase with the strength of the snob effect \( \lambda \). The reason is that customers are willing to pay higher prices when they have a stronger preference for the uniqueness associated with a new technology. This tendency allows firms to raise their retail prices accordingly. And in equilibrium, the demand for each firm’s product increases with the snob effect \( \lambda \), thus boosting the firms’ profits.

**4.1.2. Monopoly** When only one firm succeeds in developing the new technology, the successful firm may choose to pursue its monopolistic power by not licensing that technology to the other firm. Either firm may establish itself as a monopoly, and we address the two possible cases separately.

*Firm A Monopoly.* A customer with valuation \( v \) purchases firm A’s product if \( v + \lambda(1-z) - p_A \geq 0 \), which gives the demand for that product as \( d_A = 1 - p_A + \lambda(1-z) \). By rational expectations, \( z = d_A \) and so \( z = 1 - \frac{p_A}{1+\lambda} \). Hence firm A’s profit, \( \pi_A = p_A d_A = p_A \left(1 - \frac{p_A}{1+\lambda}\right) \), is maximized at \( p_A^* = \frac{1+\lambda}{2} \) with a maximum profit of \( \pi_A^* = \frac{1+\lambda}{4} \).
**Firm B Monopoly.** A customer with valuation \( v \) purchases firm B’s product if \( \delta[v + \lambda(1 - z)] - p_B \geq 0 \). Therefore, demand for that product is given by \( d_B = 1 - p_B/\delta + \lambda(1 - z) \). Given rational expectations, \( z = 1 - \frac{p_B}{\delta(1 + \lambda)} \) and so firm B’s profit function, \( \pi_B = p_B d_B = p_B \left(1 - \frac{p_B}{\delta(1 + \lambda)}\right) \), is maximized at \( p_B^* = \frac{\delta(1 + \lambda)}{2} \) with a maximum profit of \( \pi_B^* = \frac{\delta(1 + \lambda)}{4} \).

### 4.2. Licensing Decision of the Successful Firm

When only one firm succeeds in its R&D investment, it faces the strategic choice of whether to license its new technology to the other firm. We analyze two scenarios: firm A licensing the new technology to firm B; and firm B licensing to firm A. Recall that firm A is the one with the higher-quality product.

#### 4.2.1. Firm A Licenses the New Technology to Firm B

When deciding to license its new technology to firm B, firm A must ensure a positive demand for firm B’s product in order to realize a profit from the licensing agreement. Suppose that firm A also uses this new technology in its own product; then the respective profit functions of the two firms under this licensing arrangement are

\[
\pi_A = p_A \left(1 - \frac{\delta(p_A - p_B) + \lambda(\delta p_A - p_B)}{\delta(1 - \delta)(1 + \lambda)}\right) + w \left(\frac{\delta p_A - p_B}{\delta(1 - \delta)}\right),
\]

\[
\pi_B = (p_B - w) \left(\frac{\delta p_A - p_B}{\delta(1 - \delta)}\right).
\]

The equilibrium retail prices under the unit licensing fee \( w \) are

\[
p_A^* = \frac{2\delta(1 - \delta)(1 + \lambda) + [3\delta + (1 + 2\delta)\lambda]w}{\delta(4 + 3\lambda - \delta)}, \quad p_B^* = \frac{(1 + \lambda)[\delta(1 - \delta) + (2 + \delta)w]}{4 + 3\lambda - \delta}.
\]

To ensure positive demand for firm B’s product, the licensing fee must satisfy the inequality \( w < \frac{\delta(1 + \lambda)}{2 + \lambda} \). Under this condition, firm A’s optimization problem with respect to \( w \) is given by

\[
\max_w \pi_A = \frac{(2\delta(1 - \delta)(1 + \lambda) + [3\delta + (1 + 2\delta)\lambda]w)(2\delta(1 + \lambda) - (\delta - \lambda)w)}{\delta(4 + 3\lambda - \delta)} + \frac{w(\delta(1 + \lambda) - (2 + \lambda)w)}{\delta(4 + 3\lambda - \delta)} \quad \text{s.t.} \quad w < \frac{\delta(1 + \lambda)}{2 + \lambda}.
\]

Note that firm A’s profit function is concave in \( w \) for \( \delta > \frac{(2 + \lambda)\sqrt{\lambda^2 + 16\lambda + 16 - (\lambda^2 + 8\lambda + 8)}}{2(1 + \lambda)} \) and is convex otherwise. Since

\[
\frac{\partial \pi_A}{\partial w} \bigg|_{w = \frac{\delta(1 + \lambda)}{2 + \lambda}} > 0,
\]

it follows that the optimal licensing fee set by firm A is \( w^* = \frac{\delta(1 + \lambda)}{2 + \lambda} \). This fee results in zero demand for firm B’s product in the consumer market. Hence it is optimal for firm A not to license the new technology to firm B and instead to use the new technology exclusively for its own product.
Yet if firm A decides to license the new technology to firm B and chooses not to enter the consumer market itself, then firm B effectively becomes a monopoly. In this case, the demand for firm B’s product is \( d_B = 1 - \frac{p_B}{\delta(1+\lambda)} \) and its profit function is \( \pi_B = (p_B - w)(1 - \frac{p_B}{\delta(1+\lambda)}) \), which is maximized at \( p_B^* = \frac{\delta(1+\lambda) + w}{2} \). Substituting this optimal price into firm B’s demand function now yields \( d_B = \left(1 - \frac{w}{\delta(1+\lambda)}\right)/2 \). Firm A’s profit function is then \( \pi_A = w d_B = w \left(1 - \frac{w}{\delta(1+\lambda)}\right)/2 \), which is maximized at \( w^* = \frac{\delta(1+\lambda)}{2} \). The optimal profits of the two firms are therefore \( \pi_A^* = \frac{\delta(1+\lambda)}{8} \) and \( \pi_B^* = \frac{\delta(1+\lambda)}{16} \). In this case, firm A earns less than it would if it were a monopoly.

These two cases indicate that the firm with a superior quality product should not license its technology to a competitor with an inferior quality product, since doing so would result in the former firm achieving suboptimal profits.

4.2.2. Firm B Licenses the New Technology to Firm A

When firm B licenses its new technology to firm A and also sells its own new product, the profit functions of the two firms are given by

\[
\begin{align*}
\pi_A &= (p_A - w) \left(1 - \frac{\delta(p_A - p_B) + \lambda \delta p_A - p_B}{\delta(1-\delta)(1+\lambda)}\right), \\
\pi_B &= p_B \left(\frac{\delta p_A - p_B}{\delta(1-\delta)}\right) + w \left(1 - \frac{\delta(p_A - p_B) + \lambda \delta p_A - p_B}{\delta(1-\delta)(1+\lambda)}\right).
\end{align*}
\]

Given the licensing fee \( w \), the equilibrium retail prices for firms A and B are

\[
p_A^* = \frac{2(1-\delta)(1+\lambda)}{4 - \delta + 3\lambda} + \frac{(\delta^2 + 2[1+\lambda(3+\lambda)] + \lambda^2)w}{\delta(1+\lambda)(4 - \delta + 3\lambda)}, \quad p_B^* = \frac{\delta(1-\delta)(1+\lambda) + [3\delta + \lambda(2+\delta)]w}{4 - \delta + 3\lambda}.
\]

The boundary condition that ensures positive demand for each firm’s product in this equilibrium is \( w < \frac{\delta(1+\lambda)^2}{\delta + \lambda(2+\lambda)} \). Substituting these equilibrium prices into firm B’s profit function yields that firm’s profit maximization problem with respect to the licensing fee \( w \):

\[
\max_w \pi_B = \left(\frac{\delta(1-\delta)(1+\lambda) + [3\delta + \lambda(2+\delta)]w}{\delta(1+\lambda)(4 - \delta + 3\lambda)}\right)^2 + \frac{w(2\delta(1+\lambda)^2 - [2\delta + \lambda(\delta - \lambda)]w)}{\delta(1+\lambda)(4 - \delta + 3\lambda)} \quad \text{s.t.} \quad w < \frac{\delta(1+\lambda)^2}{\delta + \lambda(2+\lambda)}.
\]

The optimal solution for this profit maximization problem is characterized in our next lemma.

**Lemma 2.** There exists a \( \bar{\delta} = \frac{(1+\lambda)\sqrt{9\lambda^2 + 36\lambda + 4(2+\lambda)(1+3\lambda)} - (2+\lambda)(1+3\lambda)}{2} \) such that firm B’s optimal licensing fee \( w^* \) is given by

\[
w^* = \begin{cases} 
\frac{\delta(1+\lambda)^2}{\delta + \lambda(2+\lambda)} & \text{if } \delta \leq \bar{\delta}, \\
\frac{\delta(1+\lambda)[8 + 2\delta + \lambda(\lambda + 2)(2\delta + 7)]}{2(\delta^2 + (2+\lambda)(\lambda^2 + 7\lambda + 4\delta - \lambda^2))} & \text{if } \delta > \bar{\delta}.
\end{cases}
\]

Moreover, the optimal licensing fee \( w^* \) decreases with \( \lambda \) when \( \delta \leq \bar{\delta} \) but increases with \( \lambda \) when \( \delta \geq \bar{\delta} \).
Lemma 2 reveals an important structural change in firm B’s licensing fee, which is affected by firm B’s product quality $\delta$. In particular, if firm B’s product quality is low (as indicated by a lower value of $\delta$) then it charges a lower licensing fee in the presence of a stronger snob effect. That lower licensing fee induces firm A to refrain from setting a retail price so high that the result would be low volume and hence low revenue for firm B.

If firm B licenses the new technology to firm A but chooses not to enter the consumer market, then firm A becomes the monopoly. In this case, the demand for firm A’s product is $d_A = 1 - \frac{p_A}{1+\lambda}$ and its profit function is $\pi_A = (p_A - w)(1 - \frac{p_A}{1+\lambda})$, which is maximized at $p^*_A = \frac{1+\lambda+w}{2}$. Substituting this optimal price into the demand function yields firm A’s demand: $d_A = (1 - \frac{w}{1+\lambda})/2$. Firm B can then generate income only through licensing; its profit function thus becomes $\pi_B = wd_A = w(1 - \frac{w}{1+\lambda})/2$, which is maximized at $w^* = \frac{1+\lambda}{2}$. As a result, the optimal profits of the two firms are $\pi^*_B = \frac{1+\lambda}{8}$ and $\pi^*_A = \frac{1+\lambda}{16}$.

By comparing firm B’s profits across different scenarios, we can derive its optimal licensing and product policy. The following proposition summarizes that policy as well as firm A’s licensing policy.

**Proposition 1.** When licensing is a viable option, the optimal strategies are as follows.

1. It is never optimal for firm A to license its technology to firm B.
2. It is always optimal for firm B to license its technology to firm A, and firm B’s decision about entering the consumer market can be characterized by $\hat{\delta}$ and $\bar{\delta}$:
   (a) if $\delta < \hat{\delta}$, then firm B does not enter the consumer market;
   (b) if $\hat{\delta} \leq \delta < \bar{\delta}$, then firm B enters the consumer market but does not have positive demand;
   (c) if $\delta \geq \bar{\delta}$, then firm B enters the consumer market with positive demand.

Note that, in this proposition, for any $\lambda$ we have $\hat{\delta} = [4\lambda^2 + 7\lambda + 2 - 2(1 + \lambda)(3 + 2\lambda)]\lambda$ and $\hat{\delta} < \bar{\delta}$. The regions for the three entry policies are illustrated in Figure 2.

It is always optimal for firm B to license its new technology to firm A, but firm B adopts one of three distinct strategies for entering the consumer market depending on the strength of the snob effect ($\lambda$) and the extent of product differentiation ($\delta$). In region $R_3$, firm B enters the end market, actively competes with firm A, and achieves positive sales of its own product. In this case, firm B earns profits on the licensing fee from firm A and also on its own product sales. In the middle region, $R_2$, firm B’s product is technically available in the market but its retail price is such that it attracts no demand: all customers purchase firm A’s product. Despite this lack of sales, the strategic presence of firm B’s product in the market plays a crucial role. In region $R_1$, firm B opts out of the consumer market, licensing its technology to firm A and letting that firm be the monopoly. In this case, no products from firm B are present in the market.
The difference between regions $R_2$ and $R_1$ lies in the strategic presence of firm $B$’s product. In $R_2$, firm $B$’s product does not sell yet its availability in the market acts as a strategic deterrent. That presence prevents firm $A$ from setting an excessively high retail price, which could diminish firm $B$’s income from the licensing fee if firm $A$’s sales volume is reduced too drastically. So in this region, firm $B$ uses its product as a strategic instrument to influence firm $A$’s pricing behavior.

For any fixed level of $\lambda$, firm $B$’s strategy changes from completely abandoning the consumer market, to strategic deterrence, and finally to competing with firm $A$ in the end market. As its product quality $\delta$ increases, firm $B$’s inclination to enter and compete in the consumer market grows and thereby leads to increased profitability.

4.3. R&D Competition

For the R&D competition stage, we focus on how the two firms make investment decisions simultaneously and independently. Our analysis addresses two distinct scenarios. First, when the licensing option is not available and the two firms compete throughout the R&D and market stages; second, when licensing is possible and so the successful firm can license the new technology to its competitor. In comparing these two cases, we aim to isolate the effects of licensing and thus examine how it affects each firm’s R&D investment decisions.

Both firms seek to maximize their expected profits, which depend on the outcomes of their R&D decisions. These outcomes include four possible cases: both firms succeed in their R&D investment (SS), both firms fail (FF), or one of the two firms (either $A$ or $B$) succeeds while the other fails (SF or FS). Considering these cases and the subsequent pricing equilibrium, we may write the the firms’ expected profits as follows:

$$E\pi_A = e_A e_B \pi_{A}^{SS} + e_A (1 - e_B) \pi_{A}^{SF} + (1 - e_A) e_B \pi_{A}^{FS} + (1 - e_A) (1 - e_B) \pi_{A}^{FF} - \frac{c_A e_A^2}{2}; \quad (2)$$

$$E\pi_B = e_A e_B \pi_{B}^{SS} + e_A (1 - e_B) \pi_{B}^{SF} + (1 - e_A) e_B \pi_{B}^{FS} + (1 - e_A) (1 - e_B) \pi_{B}^{FF} - \frac{c_B e_B^2}{2}. \quad (3)$$

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Electronic copy available at: https://ssrn.com/abstract=4815893
In these expressions, the first and second superscripts indicate the respective R&D outcomes of firms \( A \) and \( B \) while \( \pi \), represents firm \( i \)'s equilibrium profit in those scenarios; as mentioned previously, we use S and F to denote (respectively) “success” and “failure”. The equilibrium profits were derived in our previous analysis. Note that if both firms fail in their R&D efforts (FF), then neither firm can enter the market and each firm’s profit is zero: \( \pi_A^{FF} = \pi_B^{FF} = 0 \).

We now analyze the equilibrium of the R&D competition for these two firms under two conditions—one with licensing and the other without licensing—while assuming that \( c_A = c_B = 1 \). The cost structure is discussed in Section 4.5.

### 4.3.1. R&D Competition without Licensing

Suppose that licensing is not an option. Then a firm that fails in its R&D efforts is effectively barred from entering the market, which renders the successful firm a monopoly. But if both firms succeed in their R&D, they constitute a duopoly in the market and engage in price competition. The equilibrium investment levels of the two firms are

\[
e_A^* = \frac{[(1 + \lambda)\delta^3 + 2(5\lambda + 9\lambda + 2)\delta^2 + (9\lambda^3 + 17\lambda^2 + 32\lambda + 32)\delta - 4(4 + 3\lambda)^2]t}{T},
\]

\[
e_B^* = \frac{\delta[(3 - \lambda)\delta^2 + 2(\lambda - 9\lambda - 14)\delta - 9\lambda^3 + 7\lambda^2 + 64\lambda + 52]t}{T};
\]

the expressions for \( t \) and \( T \) are given in Appendix D. Our next proposition states the analytical results from our comparing the R&D investment and profits of firms \( A \) and \( B \) when licensing is not an option.

**Proposition 2.** If licensing is not feasible, then firm \( A \) always invests more in R&D and also earns a higher profit than does firm \( B \); that is, \( e_A > e_B \) and \( \pi_A > \pi_B \).

This proposition highlights the impact of firm-specific R&D investment on profitability. Although both firms must invest in R&D in order to enter the end market with a consumer product, the firm with the higher-quality product—in our setting, firm \( A \)—always invests more. This higher investment level improves firm \( A \)'s likelihood of R&D success, which could allow it to monopolize the market if firm \( B \) fails in its R&D effort. Next we examine the impact of product differentiation (\( \delta \)) and the snob effect (\( \lambda \)) on these firms’ respective R&D investment and profits.

**Proposition 3.** When licensing is not an option, the following statements hold:

1. firm \( A \)'s (resp., firm \( B \)'s) R&D investment and profit are decreasing (resp., increasing) in \( \delta \);
2. both firms’ R&D investment and profits increase with \( \lambda \).

Proposition 3 describes the effect of market forces (viz., product differentiation and consumer behavior) on R&D investment and profits in a pure competitive environment. As the competitiveness of firm \( B \)'s product increases (indicated by a higher value of \( \delta \)), that firm increases its R&D
investment and consequently earns a higher profit; in contrast, firm A reduces its R&D investment and earns less. At the same time, a greater snob effect (increased $\lambda$) benefits both firms because it reflects stronger consumer preferences for uniqueness and exclusivity, promotes increased R&D investment by both firms, and leads to higher profits for each.

4.3.2. R&D Competition under Licensing When licensing between the two firms is introduced as a strategic option, it significantly changes firm B’s product decisions. In this scenario, there exists a unique equilibrium in the R&D competition and we have the following comparisons of R&D investment and profits between the two firms.

**Proposition 4.** When licensing is a viable option, firm A invests more in R&D—and also earns higher profit—than does firm B; that is, $e_A > e_B$ and $\pi_A > \pi_B$.

This proposition shows that the relative investment levels and profits of the two firms are not affected by licensing. Yet unlike in the case without licensing, where firm A’s profit is always decreasing in $\delta$, here firm A can actually benefit when firm B’s product is of high quality.

**Proposition 5.** If licensing is a viable option, then firm A’s profit is higher when $\delta \in (\hat{\delta}(\lambda), \check{\delta}_1(\lambda))$ than when $\delta \in (0, \hat{\delta}(\lambda))$; also, firm B’s profit always increases with $\delta$.

In a competitive market without licensing options, it follows from part 1 of Proposition 3 that firm A’s profit is always negatively affected by firm B’s product quality $\delta$. Yet when licensing becomes a feasible option, our analysis uncovers a potential positive impact of firm B’s product quality on firm A’s profit. It is noteworthy that this benefit arises solely from the existence of the licensing opportunity—that is, given that other market factors (i.e., market size and the snob effect) remain constant.

Figure 3 illustrates the properties stated in Proposition 5. Panel (a) of the figure shows that firm A’s investment decreases region-wise as $\delta$ increases from region $R_1$ to $R_3$ whereas firm B’s investment increases region-wise. Figure 3(b) shows that firm A’s profit in $R_2$ is higher than in $R_1$, correlating with a higher value of $\delta$ in $R_2$ than in $R_1$. This comparison across these two ranges of $\delta$ indicates that firm A can achieve a higher profit when firm B’s quality improves in the shift from $R_1$ to $R_2$. Panel (a) reveals also the mechanism that drives this result: firm B significantly increases its R&D investment in $R_2$ while firm A reduces its investment, thereby saving a significant cost. Furthermore, if only firm B succeeds in R&D then it licenses its technology but chooses not to effectively enter the market; this outcome leads to firm A facing less competition in the consumer market—an obvious benefit to that firm.
4.4. Impact of Licensing on R&D Investment and Profitability

We now turn to a comparative analysis of the equilibrium levels of firms’ R&D investment, with and without licensing options, based on our previous results in these settings. Our objective is to understand not only how licensing influences firms’ R&D decisions but also the subsequent implications for their profitability.

**Proposition 6.** In the presence of a licensing option, firm A reduces its investment in R&D as compared with the no-licensing scenario—whereas, under the same conditions, firm B increases its R&D investment.

The reasons for these systematic shifts in R&D investment are as follows. For firm A, the possibility of licensing serves as a backup that improves its chances of obtaining the new technology with less investment, thereby saving cost. In contrast, firm B takes licensing as an opportunity to augment its revenue stream through licensing fees—that is, in addition to selling its own consumer product. This potential for increased revenue incentivizes firm B to invest more heavily in R&D. Thus there are two forces, cost reduction for firm A and revenue enhancement for firm B, that drive the decrease and increase in their respective levels of R&D investment.

The introduction of a licensing option leads to considerable changes in the R&D investment levels of competing firms, which in turn has diverse implications for their profitability. A key aspect of our analysis involves determining whether the presence of licensing options improves the firms’ performance. Toward that end, we compare the firms’ profits with and without licensing; our findings are summarized in the following proposition.

**Proposition 7.** From comparing each firm’s profit with and without licensing, the following results are obtained.
1. There exists a $\tilde{\lambda}(\delta)$ such that firm A’s profit under licensing is smaller than under no-licensing when $\lambda \geq \tilde{\lambda}(\delta)$; otherwise, firm A’s profit is greater.
2. Firm B’s profit is always higher under licensing than under no-licensing.

According to this proposition, licensing invariably enhances firm B’s profitability. For a firm whose product is inferior, the benefit of higher investment levels under licensing outweighs the associated costs. In contrast, the impact of licensing is not always positive for firm A (whose product quality is superior). When the snob effect exceeds a certain threshold, firm A may find itself adversely affected by the presence of licensing options. This negative outcome reflects the reduction of firm A’s R&D investment in response to firm B’s increased investment. Although that reduction saves costs, it also reduces the likelihood of achieving a monopoly in the consumer market (should firm B’s R&D efforts fail). Our analysis shows that the potential profit losses may outweigh firm A’s cost savings from lower R&D investment.

4.5. Differential R&D Costs

In our main model, the firms’ R&D costs were assumed to be equal (and were normalized to 1, $c_A = c_B = 1$). In this section, we investigate how the firms’ performance is affected by different cost structures. Our analysis of the pricing and licensing stages remains qualitatively consistent with our main model; the focus here is on the R&D stage and performance implications of the associated cost. Because the model’s complexity increases sharply when variable R&D costs are introduced, analyzing the firms’ R&D competition becomes intractable when the parameters $c_A$ and $c_B$ are not fixed. Hence this stage of the game is examined by means of numerical analysis.

We first consider a specific setting in which the marginal cost is (a) the same for both firms but (b) other than 1; that is, $c_A = c_B \neq 1$. More specifically, the values of $c_A$ and $c_B$ are varied but are always less than 1. The effect of licensing on the firms’ respective R&D investment levels remains consistent with our previous findings. We find that firm A’s (resp., firm B’s) R&D investment decreases (resp., increases) under licensing, as stated in Proposition 6. Furthermore, our analysis establishes that firm B always benefits from licensing—that is, regardless of the cost structure. A notable change occurs, however, when the common cost parameter becomes smaller. In that event, we find that firm A’s profit is more likely to be negatively affected when its cost $c_A$ decreases. Figure 4 plots the areas in which firm A earns less under licensing for three different cost levels; we can see that the area expands as $c_A$ decreases from 0.8 to 0.5.

We also study the cases in which R&D costs diverge between the two firms (i.e., when $c_A \neq c_B$). Once again, we find that the effect of licensing on their respective R&D investment is consistent with our previous findings. The impact of R&D cost on firm A’s profit is qualitatively the same as in the first setting, where firms A and B share a common cost; this result is illustrated in Figure 5.
Our finding that R&D cost has a negative effect on firm A’s profit in the two cases just described is indicative of a counterintuitive phenomenon: firm A finds the licensing option to be less favorable than the no-licensing option as its R&D becomes more cost efficient—contrary to what would be expected in a traditional competitive situation. Our analysis, which juxtaposes scenarios with and without licensing, attributes this adverse effect on firm A’s performance to the presence of a licensing option. The effect becomes even more salient as firm A’s R&D cost efficiency improves. We discussed previously how licensing gives firm B additional revenue opportunities; for firm A, in contrast, licensing complicates the preservation of its market share and profitability.

5. Extension: Presence of an Old Technology

Our analysis so far has assumed that the two firms can enter the consumer market if one or the other can either successfully develop a new technology or license from the successful firm. That is to say, we have been presuming that no alternative technology exists for either firm. In practice, however, firms often continue to use an existing (older) technology when new technology is not
available; doing so allows them to enter the consumer market with a product of comparatively lesser quality. In this section, we extend the base model by incorporating the existence of an alternative older technology. This old technology enables a firm that fails in developing a new technology, or that does not invest in R&D, to manufacture its consumer products for market entry. If an old technology exists, then neither firm can monopolize the consumer market. Of course, the advantage of developing the new technology is profiting from its licensing and/or achieving greater product differentiation. Yet our model becomes analytically intractable with the addition of a new parameter for the technical performance of an existing technology. We therefore assume that \( \lambda = 0 \) (i.e., the market consists only of rational consumers) so that we may focus on the effects of the existence of an old technology.\(^2\)

We introduce a new parameter, \( \theta \in [0, 1] \), to represent the discounted quality of products that are based on existing technology. So when using this old technology, firm A’s product quality is discounted to \( \theta \) — in contrast to the full quality of 1 that is achievable with new technology. Firm B’s product quality, when using the old technology, is similarly adjusted to \( \theta \delta \); this adjustment reflects both the quality discount and the relatively inferior quality of firm B’s product.

The old technology enriches the competitive landscape by offering firms an alternative pathway to market entry in case their R&D efforts fail. This availability of an old technology ensures that neither firm is excluded from market participation owing only to unsuccessful R&D, thereby preventing a monopoly scenario regardless of firms’ R&D outcomes. It also gives rise to a broader range of competitive strategies in R&D and in the consumer market, since firms must decide between leveraging the old technology for market entry or developing a new technology for purposes of greater product differentiation and higher profits. The detailed equilibrium pricing strategies under different combinations of using old or new technologies by firms are described in Appendix A.

5.1. Licensing

The existence of an old technology broadens the strategic options for firms—in particular, for the firm whose R&D efforts are successful. That firm can choose among three distinct licensing policies in conjunction with its product strategy: (i) licensing the new technology and also using it for its own product; (ii) licensing the new technology but using the old technology for its own product; and (iii) licensing the new technology without entering the consumer market. These policy options require the successful firm to decide not only about whether to license the new technology but also about which technology to use for its own product. The existing technology serves as a backup option for either firm; it follows that, in order to license successfully, the firm’s licensing fee must be low enough that the licensee’s profit from purchasing the new technology exceeds what it

\(^2\) The analysis and results for \( \lambda > 0 \) are presented in Appendix B.
can achieve with the old technology. For each firm, the optimal licensing fees associated with the licensing policies described here are reported in Appendix A.

5.1.1. Firm A Licenses the New Technology to Firm B Given the optimal licensing fees and possible product performance in the consumer market, we may characterize conditions for each licensing policy to be optimal as follows.³

**Proposition 8.** If firm A succeeds and firm B fails, then it is optimal for firm A to:
1. not license the new technology to firm B when \((\delta, \theta) \in H_1\);
2. license the new technology to firm B and use the old technology for its own product when \((\delta, \theta) \in H_2\);
3. license the new technology to firm B without entering the market when \((\delta, \theta) \in H_3\).

![Figure 6 Optimal Licensing Strategies of Firm A and Firm B](https://ssrn.com/abstract=4815893)

Figure 6(a) illustrates the areas corresponding to firm A’s licensing strategies as outlined in Proposition 8. In contrast to the case where no old technology exists (when firm A should never license its technology), this graph indicates scenarios where licensing is an optimal policy for firm A. However, licensing is optimal only if firm A chooses either not to enter the end market or to enter but with the old technology. In other words, firm A should not allow firm B to compete with its products while using the same new technology.

The first area of interest, \(H_3\), emerges when firm B’s product and the old technology are both competitive. If firm A enters the end market, regardless of whether it uses the new or the old technology, then excessive competition between the two firms will reduce their respective profit

³ The detailed expressions for \(H_1\), \(H_2\), and \(H_3\) are given in Appendix A.
margins. Therefore, licensing the new technology to firm B and avoiding product competition will yield firm A a higher profit than otherwise.

In area $H_2$, licensing is still optimal but firm A also enters the consumer market with its own product that uses the old technology. Firm A’s product is still of high quality because the old technology is still competitive in this region. If firm A licenses the new technology to firm B, then the latter’s product quality will improve and thus attract more demand; hence licensing is a profitable policy for firm A in this case. The result is a co-opetition scenario in which firm A benefits both from licensing fees and from the revenue generated by its own product sales; here firm A has the dual role of licensor and competitor.

In the area labeled by $H_1$, it is not optimal for firm A to license the new technology to firm B. Firm A’s profit margin from selling its new-technology product is greater than its profit margin from licensing, an outcome that is not altered by the competition from firm B’s old-technology product.

5.1.2. Firm B Licenses the New Technology to Firm A

Given how our model is complicated by the introduction of an old technology—especially as regards determining the boundary conditions for optimal strategies—we again employ numerical methods to solve for firm B’s optimal licensing policy. Our findings from this analysis are illustrated in panel (b) of Figure 6, which shows four distinct strategic policy areas.

In area $V_1$, it is optimal for firm B to license the new technology to firm A while also using that technology for its own end product. In the two areas labeled by $V_2$, firm B’s optimal strategy is to license the new technology while using the old technology for its own product. This policy allows firm B to capitalize on licensing fees while avoiding strong competition from firm A. Because there are two $V_2$ areas, we discuss them separately relative to their neighboring areas, $V_3$ and $V_4$.

In $V_3$, it is optimal for firm B to license the new technology to firm A and not enter the end market; under this strategy, firm B generates revenue solely from licensing fees. For low and moderate values of $\delta$, firm B’s licensing policy changes from $V_1$ to $V_2$ and finally to $V_3$ as the old technology’s performance becomes stronger relative to that of the new technology. At the same time, firm B’s profit margin from licensing increases while its margin from its own product—if it chooses to enter the market—decreases as a result of competition from firm A. Thus firm B’s product strategy changes from using new to old technology and finally to not entering the end market.

Finally, firm B’s best policy in $V_4$ is not to license the new technology to firm A. In our base model, where the old technology is absent, firm B always benefits from licensing its new technology to firm A. However, the introduction of an old technology gives rise to a new policy. As Figure 6(b)
shows, not licensing is optimal only under certain conditions: when (i) the old technology’s performance is relatively low (indicated by small to moderate values of \( \theta \)) and (ii) the quality of firm B’s product is high (indicated by high values of \( \delta \)). As \( \theta \) becomes larger in the area \( V_2 \) that is above \( V_4 \), firm B finds it profitable to use the old technology and gains additional revenue from licensing its new technology to firm A.

5.2. R&D Competition and the Impact of Licensing

In line with our base model’s analysis, we compare the R&D investment levels and overall performance of each firm under scenarios with and without licensing options. In light of the intricate dynamics due to the availability of an old technology, the model at this stage is analytically intractable; hence we must again resort to numerical analysis.

Our findings, which are illustrated in Figure 7, focus on the implications of licensing for firm A’s R&D investment level and performance. The outcomes under firm A’s various levels of R&D investment can be categorized into three distinct scenarios that may arise depending on the performance of the old technology and on the quality of firm B’s product.

![Figure 7](https://ssrn.com/abstract=4815893)

**Figure 7** Comparisons of Firm A’s R&D Investment and Profit with and without a Licensing Option in the Presence of an Old Technology

First, firm A invests more in R&D under licensing (light gray area) when the old technology’s performance is comparatively high. Second, firm A’s investment levels are not affected by the availability of the licensing option (dark gray area). This occurs when firm B chooses not to license (i.e., \( V_4 \) in panel (b) of Figure 6). Except in these areas, firm A invests more when no licensing option is available.

The discussion here centers on firm A because we have consistently found that, under licensing, firm B always invests more and achieves higher profit than in the setting without licensing.
These findings explicate firm A’s responses to varying technological and market conditions. Observe that firm A’s profitability under licensing does not follow changes in R&D investment levels in an straightforward manner. In particular, Figure 7(b) shows that firm A’s profit is higher under licensing when (i) the old technology’s performance ($\theta$) is low and (ii) the difference in product quality ranges from low to high. Firm A’s profit can also be higher under licensing when (i) the extent of quality difference is low (i.e., high values of $\delta$) and (ii) the old technology’s performance is either medium or high (light gray region in the graph’s upper right area). Firm A’s profit is unaffected by the availability of the licensing option in $V_4$, where firm B’s optimal choice is not to license. In all the other areas, firm A’s profit is reduced by the licensing option.

6. Conclusion

We study the R&D competition between two firms manufacturing and selling differentiated consumer products, and we emphasize the preference of consumers for the uniqueness and exclusivity of the most advanced technology. The outcome of a firm’s R&D investment is uncertain, since the firm may fail to develop a new technology. A firm that succeeds in that endeavor must then decide whether to license its technology to a rival, thereby enabling the latter to manufacture its own version of the consumer product. Because these firms’ products are viewed as substitutes, the decision to license is not simply a matter of generating additional revenue from licensing fees; the licensing firm must also account for the resulting competition in the consumer market. It follows that firms should strategically align their licensing and product strategies in order to optimize their respective overall profits.

We develop a multi-stage, game-theoretical model that incorporates several relevant aspects: firms’ R&D competition, the option of licensing, product entry decisions, and pricing policies in the consumer market. We derive the optimal licensing and product policies for each firm and investigate their equilibrium R&D strategies.

Licensing plays a pivotal role in firms’ R&D decisions and also in their product and pricing policies. From a revenue perspective, licensing enables the focal firm to increase its revenue even as doing so intensifies product competition in the consumer market. In terms of investment, licensing serves as a backup option in case a firm’s own R&D investment fails to produce a new technology. In a real-world context featuring the interlinked nature of decisions ranging from R&D investment to product pricing, the multi-stage model presented here offers an in-depth understanding of how these strategic decisions should be optimally chosen. This comprehensive analysis yields new insights into the complex dynamics of competitive and cooperative strategies.

Although licensing always benefits the firm whose product is of lower quality, it may reduce the profits of a rival whose product is of higher quality. We identify three factors—the snob effect,
the firm’s R&D cost efficiency, and the existence of an old technology—that can render licensing unprofitable for the leading firm.

Our findings have a number of managerial implications. First, firms’ licensing and product policies should be considered holistically. A novel strategy involves one firm licensing new technology to its competitor while keeping its own consumer product in the market as a form of strategic deterrence—that is, even it generates no sales. This approach can safeguard the licensor’s revenue by affecting its rival’s pricing behavior.

Second, the optimal licensing decision depends on product quality. Whereas firms that market a product of superior quality typically decide against licensing, it is usually optimal for the firm whose product is of inferior quality to license, leading to increased R&D investment despite the outcome uncertainties. This dynamic underscores the strategic importance of R&D for these firms.

Third, a firm’s licensing strategy must be recalibrated when an older technology is present. For the firm with a high-quality product, licensing its new technology to a competitor is beneficial if that technology appreciably outperforms the old technology. In contrast, the firm with a marginally less competitive product may prefer to opt out of licensing to its rival.

These implications point to the nuanced considerations that a firm should take into account when determining its R&D, licensing, and product strategies. Each strategic decision—whether (or not) to license, how much to invest in R&D, and how to position products in the consumer market—requires careful analysis of the firm’s own product competitiveness and R&D efficiency as well as the potential impact of existing technologies.

We have sought to develop a comprehensive model of R&D competition, but there remain several related questions for future research. One area involves the impact of new technologies on market demand. Future work could investigate how these changes affect firms’ licensing and R&D strategies. Also, our model presupposes complete information in the R&D competition even though each firm’s R&D marginal cost may actually be private information. Future studies could relax this assumption and study the resulting dynamics under incomplete information. The temporal aspect of R&D investment is another key factor whose aspects include the pressure of deadlines and the uncertainty of just when an R&D breakthrough will occur. We leave these topics for future investigation.

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Appendix A: Analysis for Section 5

In this section, we present the analysis of the optimal licensing fees described in Section 5.

A.1. Product and Pricing Policy

Since both firms can develop their own product using either the new or the old technology, there are four possible combinations of policies: (i) both firms use the new technology, denoted by (n, n); (ii) firm A uses the new technology and firm B uses the old one, (n, o); (iii) firm A uses the old one and firm B uses the new one, (o, n); and (iv) both firms use the old technology, (o, o). We first derive the demand function of the two firms and then solve for the equilibrium prices.

A.1.1. (n, n): This case is the same as the one described in Section 4.1.1.

A.1.2. (n, o): In this case, the utility maximization problem of a consumer with product valuation \(v\) is

\[
\max \{v - p_A, \delta v - p_B, 0\}.
\]

The demand functions of the two firms are given by

\[
(d_A, d_B) = \begin{cases} 
(1 - p_A/\theta, 0) & \text{if } p_A < p_B - (\delta - \theta), \\
(1 - \frac{p_A - p_B}{1 - \theta}, \frac{p_A - p_B}{1 - \theta}) & \text{if } p_B - (\delta - \theta) \leq p_A < \frac{p_B}{\delta}, \\
(0, 1 - p_B/\delta) & \text{if } p_A \geq \frac{p_B}{\delta}.
\end{cases}
\]

Under these demand functions, the equilibrium prices are \(p_A^* = \frac{2(1 - \delta \theta)}{4 - 9\theta}\) and \(p_B^* = \frac{4(1 - \delta \theta)}{4 - 9\theta}\).

A.1.3. (o, n): The utility maximization problem of a consumer with valuation \(v\) is

\[
\max \{\theta v - p_A, \delta v - p_B, 0\}.
\]

There are two subcases, as follow.

1. When \(\theta \leq \delta\), the demand functions of the two firms are given by

\[
(d_A, d_B) = \begin{cases} 
(1 - p_A/\theta, 0) & \text{if } p_A < p_B - (\delta - \theta), \\
(1 - \frac{p_A - p_B}{\delta - \theta}, \frac{p_A - p_B}{\delta - \theta}) & \text{if } p_B - (\delta - \theta) \leq p_A < \frac{p_B}{\delta}, \\
(0, 1 - p_B/\delta) & \text{if } p_A \geq \frac{p_B}{\delta}.
\end{cases}
\]

2. When \(\theta > \delta\), the demand functions of the two firms are given by

\[
(d_A, d_B) = \begin{cases} 
(1 - p_A/\theta, 0) & \text{if } p_A < \frac{p_B}{\delta}, \\
(1 - \frac{p_A - p_B}{\theta - \delta}, \frac{p_A - p_B}{\theta - \delta}) & \text{if } \frac{p_B}{\delta} \leq p_A < p_B + (\theta - \delta), \\
(0, 1 - p_B/\delta) & \text{if } p_A \geq p_B + (\theta - \delta).
\end{cases}
\]

The equilibrium prices of these two subcases are:

\[
(p_A^*, p_B^*) = \begin{cases} 
\left(\frac{\theta - \delta}{49 - \theta}, \frac{2\theta - \delta}{49 - \theta} \right) & \text{if } \theta < \delta; \\
\left(\frac{\theta - \delta - \delta}{49 - \theta}, \frac{3\theta - \delta}{49 - \theta} \right) & \text{if } \theta > \delta.
\end{cases}
\]

A.1.4. (o, o): The utility maximization problem of a consumer with valuation \(v\) is

\[
\max \{\theta v - p_A, \delta v - p_B, 0\}.
\]

So in this case, the demand functions of two firms are

\[
(d_A, d_B) = \begin{cases} 
(1 - p_A/\theta, 0) & \text{if } p_A < p_B/\delta, \\
(1 - \frac{p_A - p_B}{1 - \theta}, \frac{p_A - p_B}{1 - \theta}) & \text{if } p_B/\delta \leq p_A < p_B + \theta(1 - \delta), \\
(0, 1 - p_B/\delta) & \text{if } p_A \geq p_B + \theta(1 - \delta).
\end{cases}
\]

The equilibrium prices are then \(p_A^* = \frac{2(1 - \delta \theta)}{4 - 3\theta}\) and \(p_B^* = \frac{5\theta(1 - \delta)}{4 - 3\theta}\).

Table A.1 summarizes the two firms’ equilibrium prices and profits under the policies just described.

A.2. Licensing

We now derive the optimal licensing fees when a firm with the new technology decides to license it to a firm without that technology.
The equilibrium prices are $p^\text{optimal licensing policy}$ is given in our next lemma.

As before, the constraint ensures that it is optimal for firm $B$ here the constraint ensures that it is optimal for firm $A$

Table A.1  Equilibrium Results for Competition with Rational Customers

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\theta \leq \delta$</td>
<td>$\frac{2(1-\delta)}{4-\delta}$</td>
<td>$\frac{4(1-\delta)}{2(1-\delta)}$</td>
<td>$\frac{3\delta(\delta-\theta)}{4(4-\delta)}$</td>
<td>$\frac{3\delta(\delta-\theta)}{2(1-\delta)}$</td>
</tr>
<tr>
<td>$\theta &gt; \delta$</td>
<td>$\frac{\delta(\delta-\theta)}{4-\delta}$</td>
<td>$\frac{\delta(\delta-\theta)}{2(1-\delta)}$</td>
<td>$\frac{\delta(\delta-\theta)}{2(1-\delta)}$</td>
<td>$\frac{\delta(\delta-\theta)}{2(1-\delta)}$</td>
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</table>

A.2.1. Firm A Licenses the New Technology to Firm B

If firm $A$ licenses the new technology to firm $B$ and also uses it for its own product, then the two firms' respective profit functions are given by

$$
\pi_A = p_A \left( 1 - \frac{p_A - p_B}{1 - \delta} \right) + w \left( \frac{p_A - p_B}{1 - \delta} \right),
$$

$$
\pi_B = (p_B - w) \left( 1 - \frac{p_A - p_B}{1 - \delta} - \frac{p_B}{\delta} \right).
$$

The equilibrium prices are $p_A^* = \frac{2(1-\delta)+3w}{4-\delta}$ and $p_B^* = \frac{\delta(1-\delta)+2w}{4-\delta}$.

When $w < \delta/2$, there is positive demand for firm $B$'s product. In this case, firm $A$'s optimization problem with respect to (w.r.t.) $w$ becomes

$$
\max_w \pi_A = \left( \frac{2-w}{4-\delta} \right) + w \left( \frac{\delta-2w}{\delta(4-\delta)} \right)
$$

s.t. \( \frac{(1-\delta)(\delta-2w)^2}{\delta(4-\delta)^2} \geq \frac{\delta\theta(1-\delta)}{(4-\delta)^2} \);

here the constraint ensures that it is optimal for firm $B$ to purchase the new technology (instead of using its own old technology). Firm $A$'s optimal licensing fee is given in the following lemma.

**Lemma A.1.** Firm A’s profit maximization problem has a unique solution when $\delta < \frac{4(1+\theta)}{\theta} - \sqrt{\frac{4(1+\theta)}{\theta} + 1} = \delta_1^{AB}$ and $w^* = \frac{\delta}{2} - \frac{\delta(1-\delta)}{2(1-\delta)(4-\delta)}$; otherwise, the problem does not have a feasible solution.

Next we consider the scenario where firm $A$ licenses to firm $B$ but continues to use the old technology for its own end product. As in Section A.1.3, there are two subcases.

1. When $\theta \leq \delta$, customers prefer firm $B$’s product and so the two firms’ profit functions are given by

$$
\pi_A = p_A \left( \frac{p_B - p_A}{\delta - \theta} - \frac{p_A}{\theta} \right) + w \left( 1 - \frac{p_B - p_A}{\delta - \theta} \right),
$$

$$
\pi_B = (p_B - w) \left( 1 - \frac{p_B - p_A}{\delta - \theta} - \frac{p_A}{\theta} \right).
$$

The equilibrium prices are $p_A^* = \frac{\theta(3w+\delta-\theta)}{4\delta-\theta}$ and $p_B^* = \frac{2\delta(\delta-\theta)+2w}{4\delta-\theta}$. Thus, firm $A$’s optimization problem w.r.t. $w$ is

$$
\max_w \pi_A = \frac{\theta(\delta - w) (\delta - \theta + 3w)}{4(\delta - \theta)^2} + w \left( \frac{2(\delta - w)}{4\delta - \theta} \right)
$$

s.t. \( \frac{4(\delta - \theta)(\delta - w)^2}{(4\delta - \theta)^2} \geq \frac{\delta\theta(1-\delta)}{(4-\delta)^2} \).

As before, the constraint ensures that it is optimal for firm $B$ to purchase the new technology. Firm $A$’s optimal licensing policy is given in our next lemma.
Lemma A.2. There exist $\delta_2^{AB}$ and $\delta_3^{AB}$ such that firm A’s profit maximization problem w.r.t. $w$ does not have a feasible solution when $\theta < \delta < \delta_3^{AB}$. It has a unique optimal solution when $\delta_3^{AB} \leq \delta < 1$, in which case the optimal licensing fee is

$$w^* = \begin{cases} \delta & \text{if } \delta_3^{AB} \leq \delta < \delta_2^{AB}, \\ \frac{\delta - \frac{\theta}{2}}{\frac{4(\theta - \delta)(1 - \theta)}{4\theta - \delta}} & \text{if } \frac{\delta_2^{AB}}{2} \leq \delta < 1. \end{cases}$$

2. When $\theta > \delta$, customers still prefer firm A’s product to that of firm B even if firm A uses the old technology and firm B uses the new technology. In this case, the profit functions of the two firms are:

$$\pi_A = p_A \left(1 - \frac{p_A - p_B}{\theta - \delta}\right) + w\left(\frac{p_A - p_B}{\theta - \delta} - \frac{p_B}{\delta}\right);$$

$$\pi_B = (p_B - w)\left(\frac{p_A - p_B}{\theta - \delta} - \frac{p_B}{\delta}\right).$$

The equilibrium prices are $p_A^* = \frac{\theta[3w + 2(\theta - \delta)]}{4\theta - \delta}$ and $p_B^* = \frac{\delta(\theta - \delta) + \theta + 2(\delta - \delta/2)w}{4\theta - \delta}$. There is positive demand for firm B’s product if $w < \delta/2$, so firm A’s optimization problem w.r.t. $w$ is

$$\max_w \pi_A = \frac{\theta[3w + 2(\theta - \delta)](2\theta - w)}{(4\theta - \delta)^2} + w\left(\frac{\theta(\delta - 2w)}{\delta(4\theta - \delta)}\right)$$

s.t. $\frac{\theta(\delta - 2w)^2}{\delta(4\theta - \delta)^2} \geq \frac{\delta\theta(1 - \delta\theta)}{(4 - \delta\theta)^2}$.

Lemma A.3 describes firm A’s optimal licensing fee.

Lemma A.3. There exists a $\delta_4^{AB}$ such that firm A’s profit maximization problem has the unique optimal solution $w^* = \frac{\delta}{2} - \frac{\delta(4\theta - \delta)\sqrt{\theta(\theta - \delta)(1 - \theta)}}{2(\theta - \delta)(4\theta - \delta)}$ if $\delta < \delta_4^{AB}$; otherwise, the problem does not have a feasible solution.

Finally, we address the scenario in which firm A licenses the new technology to firm B without producing its own end product. In this case, firm B becomes a monopoly. Then firm B’s profit function is $\pi_B = (p_B - w)(1 - p_B/\delta)$, which is maximized at $p_B^* = \frac{\delta + w}{2}$. Hence firm A’s optimization problem w.r.t. $w$ is given by

$$\max_w \pi_A = w\left(\frac{\delta - w}{2\delta}\right)$$

s.t. $\frac{(\delta - w)^2}{4\delta} \geq \frac{\delta\theta(1 - \delta\theta)}{(4 - \delta\theta)^2}$. \hspace{1cm} (A.1)

The optimal licensing fee is $w^* = \delta/2$, under which the two firms’ optimal profits are $\pi_A^* = \frac{\delta}{8}$ and $\pi_B^* = \frac{\delta}{16}$.

A.2.2. Firm B Licenses the New Technology to Firm A The optimal licensing fees are derived following the same procedure as described in Section A.2.1, so we omit the details.

Table A.2 summarizes the optimal licensing fees derived in Section A.2.

Appendix B: Snobs and Old Technology

In the presence of an old technology, snobs may not have the same sensitivity to the new and old technologies. Let $\lambda^o, \lambda^o \in [0, 1]$ denote the parameters of the snob effects regarding the new and old technologies, respectively, and suppose that snobs are more sensitive to the new technology: $\lambda^o > \lambda^o$. We assume further that $\lambda^o = \lambda$ and $\lambda^o = 0$; this will allow us to focus on the snob effect associated with the new technology. Thus a snob derives utility $u_A^o = v + \lambda(1 - z) - p_A$ or $u_B^o = \delta[v + \lambda(1 - z)] - p_B$ from purchasing a new-technology product from firm A or B, respectively; we analogously write $u_A^o = \theta v - p_A$ and $u_B^o = \theta\delta v - p_B$ for purchasing a product with the old technology.
Table A.2  Optimal Licensing Fees

| Policy (i) | \( w = \begin{cases} \frac{\delta}{2} - \frac{\delta(4-\delta)\sqrt{\theta(1-\delta)(1-\delta\theta)}}{2(1-\delta)(4-\delta^2)} & \text{if } \delta < \tilde{\delta}_{1}^{1AB}, \\ \frac{n/a}{\delta} & \text{if } \delta \geq \tilde{\delta}_{1}^{1AB}; \end{cases} \) |
| A Licenses to B |
| Policy (ii) | \( w = \begin{cases} \frac{\delta}{2} - \frac{\delta(4\theta-\delta)\sqrt{\theta(1-\delta)(1-\theta\delta)}}{2(\theta-\delta)(4-\delta^2)} & \text{if } \delta < \tilde{\delta}_{4}^{1AB}, \\ \frac{n/a}{\delta} & \text{if } \tilde{\delta}_{4}^{1AB} \leq \delta < \tilde{\delta}_{5}^{1AB}, \\ \frac{\delta}{2} - \frac{(4\delta-\theta)\sqrt{\delta(1-\delta)(1-\theta\delta)}}{2(\delta-\theta)(4-\delta^2)} & \text{if } \tilde{\delta}_{3}^{1AB} \leq \delta < \tilde{\delta}_{2}^{1AB}, \\ \frac{n/a}{\delta} & \text{if } \tilde{\delta}_{2}^{1AB} \leq \delta < 1; \end{cases} \) |
| Policy (iii) | \( w = \delta/2. \) |

| Policy (i) | \( w = \begin{cases} 1 - \frac{\theta(4-\delta)\sqrt{(1-\delta)(\delta-\theta)}}{(1-\delta)(4\theta-\delta)} & \text{if } \theta \geq 1/4 \text{ and } \delta < \tilde{\delta}_{3}^{BA}, \\ \frac{n/a}{\delta} & \text{if } (\delta, \theta) \in \Omega_1, \end{cases} \) |
| B Licenses to A |
| Policy (ii) | \( w = \begin{cases} 1 - \frac{(4-\delta)\sqrt{\delta(1-\delta)(1-\theta\delta)}}{2(1-\delta)|4\theta-\delta|} & \text{if } \delta \geq 1/4 \text{ and } \delta < \tilde{\delta}_{4}^{BA}, \\ \frac{n/a}{\delta} & \text{if } (\delta, \theta) \in \Omega_2; \end{cases} \) |
| Policy (iii) | \( w = \begin{cases} 1 - \frac{|4\delta\sqrt{(\theta-\delta)}|}{4\theta-\delta} & \text{if } \theta \geq 1/4 \text{ and } \delta < \tilde{\delta}_{5}^{BA}, \\ \frac{n/a}{\delta} & \text{if } (\delta, \theta) \in \Omega_3. \end{cases} \) |

Notes: \( \Omega_1 = \{(\delta, \theta) \mid \theta < 1/4 \text{ and } \delta < \theta, \text{ or } \tilde{\delta}_{3}^{BA} \leq \delta < \theta \text{ and } \theta \geq 1/4, \text{ or } \theta \leq \delta < \tilde{\delta}_{4}^{BA} \}; \) \( \Omega_2 = \{(\delta, \theta) \mid \theta < 1/4, \text{ or } \tilde{\delta}_{3}^{BA} \leq \delta < \theta \text{ and } \theta \geq 1/4, \text{ or } \delta \geq \theta \}; \) \( \Omega_3 = \{(\delta, \theta) \mid \theta < 1/4, \text{ or } \tilde{\delta}_{5}^{BA} \leq \delta < \theta \text{ and } \theta \geq 1/4, \text{ or } \delta \geq \theta \}. \) \( n/a \) = no feasible solution under this condition (i.e., no positive licensing fee can satisfy the other firm’s individual rationality constraint).

The next two figures illustrate our numerical analysis of the impact of licensing on firm A’s R&D investment level and profits for different values of \( \lambda. \) Figure B.1 shows that firm A invests more in R&D under the no-licensing option as the snob effect become stronger; that is, the area of \( e^{N}_{A} > e^{L}_{A} \) increases with \( \lambda. \) In Figure B.2 we can see the areas in which firm A’s profit is smaller under the licensing option than under the no-licensing option.

Appendix C: Proofs

C.1. Proof of Lemma 1

The profit functions of firms A and B are given by

\[
\pi_{A} = \left\{ \begin{array}{ll}
p_{A} \left( 1 - \frac{p_{A} - p_{B}}{1 + \lambda} \right) & \text{if } p_{A} < p_{B} / \delta, \\
p_{A} \left( \frac{\delta(p_{A} - p_{B}) + \lambda(\delta p_{A} - p_{B})}{\delta(1-\delta)(1+\lambda)} \right) & \text{if } \frac{p_{B}}{\delta} \leq p_{A} < 1 - \delta + \frac{\lambda}{\delta(1+\lambda)} p_{B}, \\
0 & \text{if } p_{A} \geq 1 - \delta + \frac{\lambda}{\delta(1+\lambda)} p_{B}; \end{array} \right.
\]
\[ \pi_B = \begin{cases} 
 p_B \left( 1 - \frac{p_B}{\delta (1+\lambda)} \right) & \text{if } p_B < \frac{\delta (1+\lambda) [p_A - (1-\delta)]}{\delta + \lambda}, \\
 p_B \left( \frac{\delta [p_A - (1-\delta)]}{\delta + \lambda} \right) & \text{if } \frac{\delta (1+\lambda) [p_A - (1-\delta)]}{\delta + \lambda} \leq p_B < \frac{\delta (1+\lambda)}{\delta + \lambda}, \\
 0 & \text{if } p_B \geq \frac{\delta (1+\lambda)}{\delta + \lambda}. 
 \end{cases} \]

The best response functions, which can be found by solving the first-order optimality conditions of each firm, are given by

\[ p_A^*(p_B) = \begin{cases} 
 \frac{\delta (1-\delta) (1+\lambda) + (\delta + \lambda) p_B}{2 \delta (1+\lambda)} & \text{if } p_B < \frac{\delta (1-\delta) (1+\lambda)}{2 \delta + \lambda + \lambda}, \\
 p_B / \delta & \text{if } \frac{\delta (1-\delta) (1+\lambda)}{2 \delta + \lambda + \lambda} \leq p_B < \frac{\delta (1+\lambda)}{2 \delta + \lambda}, \\
 \frac{1+\lambda}{2} & \text{if } p_B \geq \frac{\delta (1+\lambda)}{2 \delta + \lambda}. 
 \end{cases} \] \hspace{1cm} (C.1)

\[ p_B^*(p_A) = \begin{cases} 
 \frac{\delta p_A}{2} & \text{if } p_A < \frac{2 (1-\delta) (1+\lambda)}{2 + \lambda - \delta}, \\
 \frac{\delta [p_A - (1-\delta)]}{\delta + \lambda} & \text{if } \frac{2 (1-\delta) (1+\lambda)}{2 + \lambda - \delta} \leq p_A < \frac{2 + \lambda - \delta}{2}, \\
 \delta (1+\lambda) / 2 & \text{if } p_A \geq \frac{2 + \lambda - \delta}{2}. 
 \end{cases} \] \hspace{1cm} (C.2)

Both functions are monotonic in the other firm’s pricing decision. Hence there exists a unique equilibrium, consisting of \( p_A^* = \frac{2 (1-\delta) (1+\lambda)}{4 + 3 \lambda - \delta} \) and \( p_B^* = \frac{\delta (1-\delta) (1+\lambda)}{4 + 3 \lambda - \delta} \), under which the profits of firms A and B are \( \pi_A^* = \frac{4 (1-\delta) (1+\lambda)^2}{(4 + 3 \lambda - \delta)^2} \) and \( \pi_B^* = \frac{\delta (1-\delta) (1+\lambda)^2}{(4 + 3 \lambda - \delta)^2} \).
C.2. Proof of Lemma 2

Differentiating $\pi_B$ with respect to $w$ yields

$$\frac{\partial \pi_B}{\partial w} = \frac{\delta [\delta^2 + 2(2 + \lambda)\delta + 7\lambda(2 + \lambda) + 8] - 2[\delta^2 + (2 + \lambda)\lambda^2 + 7\lambda + 4]\delta - \lambda^3]}{\delta(1 + \lambda)(4 + 3\lambda - \delta)^2},$$

$$\frac{\partial^2 \pi_B}{\partial w^2} = \frac{-2[\delta^3 - 2(2 + \lambda)(\lambda^2 + 7\lambda + 4)\delta]}{\delta(1 + \lambda)(4 + 3\lambda - \delta)^2}.$$

We have $\frac{\partial^2 \pi_B}{\partial w^2} = 0$ for $\delta = \hat{\delta}_0 := \frac{(1 + \lambda)(4 + \lambda)\sqrt{\lambda^2 + 8\lambda + 4 - (2 + \lambda)(\lambda^2 + 7\lambda + 4)}}{2[\delta^2 + (2 + \lambda)(\lambda^2 + 7\lambda + 4)\delta - \lambda^3]}$. It follows that $\pi_B$ is concave in $w$ if $\delta \geq \hat{\delta}_0$ and is convex in $w$ if $\delta < \hat{\delta}_0$. Solving the first-order condition $\frac{\partial \pi_B}{\partial w} = 0$ now gives $\hat{w} = \frac{\delta(1 + \lambda)[8 + \delta^2 + \lambda(\lambda + 2)(2\delta + 7)]}{2[\delta^2 + (2 + \lambda)(\lambda^2 + 7\lambda + 4)\delta - \lambda^3]}$.

Comparing $\hat{w}$ with the boundary condition $w < \frac{\delta(1 + \lambda)^2}{\delta + \lambda(2 + \lambda)}$, we obtain $\hat{w} < \frac{\delta(1 + \lambda)}{\delta + \lambda(2 + \lambda)}$ if $\delta > \hat{\delta}$ and $\hat{w} \geq \frac{\delta(1 + \lambda)^2}{\delta + \lambda(2 + \lambda)}$ if $\delta \leq \hat{\delta}$; here $\hat{\delta} = \frac{(1 + \lambda)\sqrt{9\lambda^2 + 36\lambda + 4 - (2 + \lambda)(1 + 3\lambda)}}{2(\lambda^2 + 2\lambda)[\lambda^2 + 7\lambda + 4])} > \hat{\delta}_0$. We can also determine the optimal solution when $\pi_B$ is convex in $w$. If $\delta < \hat{\delta}_0$, then $\pi_B$ is increasing in $w$. Therefore, the optimal solution is on the boundary; that is, $w^* = \frac{\delta(1 + \lambda)^2}{\delta + \lambda(2 + \lambda)}$. As a consequence, firm $B$’s optimal licensing fee is

$$w^* = \begin{cases} \frac{\delta(1 + \lambda)^2}{\delta + \lambda(2 + \lambda)} & \text{if } \delta < \hat{\delta}, \\ \frac{\delta(1 + \lambda)[8 + \delta^2 + \lambda(\lambda + 2)(2\delta + 7)]}{2[\delta^2 + (2 + \lambda)(\lambda^2 + 7\lambda + 4)\delta - \lambda^3]} & \text{if } \delta \leq \delta < 1. \end{cases}$$

The properties of $w^*$ with respect to $\lambda$ can be verified using the first-order derivatives as follows:

1. if $\delta < \hat{\delta}$, then
$$\frac{\partial w^*}{\partial \lambda} = \frac{-2\delta(1 - \delta)(1 + \lambda)}{[\delta + \lambda(2 + \lambda)]^2} < 0;$$

2. if $\delta \geq \hat{\delta}$, then
$$\frac{\partial w^*}{\partial \lambda} = \frac{\delta m}{2[\delta^2 + (2 + \lambda)(\lambda^2 + 7\lambda + 4)\delta - \lambda^3]^2}.$$

The closed-form expressions for $m$ are given in Appendix D. Since $m > 0$ for $\delta \geq \hat{\delta}$, it follows that $\frac{\partial w^*}{\partial \lambda} > 0$.

C.3. Proof of Proposition 1

We compare firm $B$’s profits across all possible licensing policies to find the optimal strategy as well as its boundaries $\check{\delta}$ and $\hat{\delta}$. Let $\pi_B^{Ln}$, $\pi_B^{Lno}$, and $\pi_B^{NL}$ denote firm $B$’s profit when (respectively) it licenses the new technology to firm $A$ and also uses the new technology for its own end product, it licenses to firm $A$ without entering the consumer market, and it declines to license the new technology.

(i) If $\delta \geq \hat{\delta}$ then

$$\pi_B^{Ln} - \pi_B^{NL} = \frac{\delta(1 - \delta)(1 + \lambda)^2(2 + \lambda)^2}{4[\delta^2 + (2 + \lambda)(\lambda^2 + 7\lambda + 4)\delta - \lambda^3]} > 0,$$

$$\pi_B^{Ln} - \pi_B^{Lno} = \frac{(1 + \lambda)[2\delta^3 + (8\lambda^2 + 20\lambda + 7)\delta^2 + (-\lambda^3 + \lambda^2 - 2\lambda)\delta + \lambda^3]}{8[\delta^2 + (2 + \lambda)(\lambda^2 + 7\lambda + 4)\delta - \lambda^3]} > 0.$$

In this case, it is optimal for firm $B$ to license the new technology to firm $A$ and also use that technology for its own product.

(ii) If $\delta < \hat{\delta}$ then

$$\pi_B^{Ln} - \pi_B^{NL} = \frac{\delta \lambda(1 + \lambda)^3}{\delta + \lambda(2 + \lambda)^2} - \frac{\delta(1 + \lambda)}{4} = \frac{\delta(1 + \lambda)(4\lambda(1 + \lambda - \delta) - (\delta + \lambda)^2)}{4[\delta + \lambda(2 + \lambda)^2]} > 0,$$

$$\pi_B^{Ln} - \pi_B^{Lno} = \frac{\delta \lambda(1 + \lambda)^3}{\delta + \lambda(2 + \lambda)^2} - \frac{1 + \lambda}{8} = \frac{(1 + \lambda)[-\delta^2 + 2\lambda(4\lambda^2 + 7\lambda + 2)]\delta - \lambda^2(2 + \lambda)^2}{8[\delta + \lambda(2 + \lambda)^2]}.$$
The first inequalities in both (i) and (ii) show that no-licensing is never optimal for firm $B$. Let $\hat{\delta} = [4\lambda + 7\lambda + 2 - 2(1+\lambda)\sqrt{2\delta(3+2\lambda)}] / \lambda$ (i.e., the solution to $\pi^{L_1}_B - \pi^{L_0}_B = 0$); then $\pi^{L_1}_B < \pi^{L_0}_B$ if $\delta < \hat{\delta}$ and $\pi^{L_1}_B \geq \pi^{L_0}_B$ if $\delta \leq \hat{\delta}$. Furthermore, if $\hat{\delta} \leq \delta < \bar{\delta}$ then the equilibrium demand for firm $B$’s product falls to zero.

C.4. Proof of Proposition 2

If there is no licensing option then the equilibrium investment levels of the two firms can be found by solving their best response functions, as derived in Section 4.3.1. Also, we have $e^*_A - e^*_B = \frac{4N_1}{T^2} > 0$ and $E \pi_A - E \pi_B = \frac{4e^*_A}{T^2} > 0$, where the closed-form expressions for $S_1$ and $S_2$ are given in Appendix D.

C.5. Proof of Proposition 3

We can easily verify the following properties of $e^*_A$:

\[
\frac{\partial e^*_A}{\partial \delta} = -\frac{4(1+\lambda)^2(4+3\lambda - \delta)M_1}{T^2},
\]

\[
\frac{\partial e^*_A}{\partial \lambda} = \frac{4(4+3\lambda - \delta)M_2}{T^2}.
\]

The closed-form expressions for $M_1$ and $M_2$ can be found in Appendix D. Note also that the inequalities $M_1 > 0$ and $M_2 > 0$ hold for $\delta \in [0,1]$, from which it follows that $\frac{\partial e^*_A}{\partial \delta} < 0$ and $\frac{\partial e^*_A}{\partial \lambda} > 0$.

The proof of other properties is similar and is therefore omitted here.

C.6. Proof of Proposition 4

We compare the two firms’ equilibrium R&D investment levels and profits across three regions of firm $B$’s optimal licensing strategy.

(i) For $(\delta, \lambda) \in R_1$, we have $e_A - e_B = \frac{\delta N_1}{T_1}$ and $E \pi_A - E \pi_B = \frac{(1+\lambda)N_2N_1}{T^2}$, where the closed-form expressions for $N_1$, $N_2$, $N_3$, and $T_1$ are given in Appendix D. In addition, $N_1$, $N_2$, $T_1 > 0$, and $N_3 > 0$ all hold for $\delta < \hat{\delta}$. Therefore, $e_A > e_B$ and $E \pi_A > E \pi_B$.

(ii) For $(\delta, \lambda) \in R_2$ and $(\delta, \lambda) \in R_3$, the proof follows the same procedure.

C.7. Proof of Proposition 5

For $(\delta, \lambda) \in R_1$, we have

\[
\frac{\partial E \pi_A}{\partial \delta} = -\frac{16(1+\lambda)^4(4+3\lambda - \delta)n_1}{T_1^3}.
\]

The closed-form expression for $n_1$ can be found in Appendix D, and $n_1 > 0$ holds for $\delta < \hat{\delta}$. Therefore, we have $\frac{\partial E \pi_A}{\partial \delta} < 0$. For any $\lambda \in (0,1)$, the maximum value of $E \pi_A$ in the region $R_1$ is $E \pi_A^{R_1}|_{\delta=0}$. So for $(\delta, \lambda) \in R_2$, we compare $E \pi_A^{R_2}|_{\delta=\delta_+}$ with $E \pi_A^{R_1}|_{\delta=0}$ and obtain $E \pi_A^{R_2}|_{\delta=\delta_+} > E \pi_A^{R_1}|_{\delta=0}$. By continuity, there exists a $\hat{\delta}_1(\lambda)$ such that $E \pi_A^{R_2}$ for $\delta \in (\hat{\delta}_1, \bar{\delta})$ is greater than $E \pi_A^{R_1}$ for $\delta \in (0, \hat{\delta})$.

Next, for $(\delta, \lambda) \in R_1$ we can also write

\[
\frac{\partial E \pi_B}{\partial \delta} = \frac{512(1+\lambda)^4(4+3\lambda - \delta)^3n_2n_3}{T_1^3}.
\]

The closed-form expressions for $n_2$ and $n_3$ are given in Appendix D, and $n_2, n_3 > 0$ hold for $\delta < \hat{\delta}$. Thus we have $\frac{\partial E \pi_B}{\partial \delta} > 0$. We can use the same procedure to establish that $\frac{\partial E \pi_B}{\partial \delta} > 0$ for $(\delta, \lambda) \in R_2$ and $(\delta, \lambda) \in R_3$. Moreover, we can show that $E \pi_B^{R_1}|_{\delta=\delta_-} < E \pi_B^{R_2}|_{\delta=\delta_+}$. It follows that, across all three regions, $E \pi_B$ increases with $\delta$ for any value of $\lambda$. 

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C.8. Proof of Proposition 6

As before, we let the superscripts \( L \) and \( N \) denote investment levels under (respectively) licensing and no-licensing. Then, for \((\delta, \lambda) \in R_1\) we may write

\[
e_A^N - e_A^L = \frac{4t(1 + \lambda)k_1}{TT_1},
\]

\[
e_B^L - e_B^N = \frac{t(4 + 3\lambda - \delta)^2k_2}{TT_1}.
\]

The closed-form expressions for \( T, k_1, \) and \( k_2 \) are given in Appendix D; also, the inequalities \( T, k_1, k_2 > 0 \) hold for \( \delta < \delta \). Therefore, \( e_A^N > e_A^L \) and \( e_B^L > e_B^N \). This derivation proves our statement about the investment levels in \( R_1 \). We omit the analogous proof for regions \( R_2 \) and \( R_3 \).

C.9. Proof of Proposition 7

We compare each firm’s profit with and without the licensing options (as indicated by superscripts \( L \) and \( N \), respectively) in all three regions: \( R_1, R_2, \) and \( R_3 \).

1. Region \( R_1 \) can be expressed either as \( R_1 = \{ \delta \mid \delta < \delta(\lambda), 0 \leq \lambda \leq 1 \} \) or as \( \{ \lambda \mid \lambda > \lambda(\delta), 0 \leq \delta \leq 13 - 4\sqrt{10} \} \). As a result,

\[
\pi_A^N - \pi_A^L = \frac{z_1}{4(TT_1)^2} \quad \text{and} \quad \pi_B^L - \pi_B^N = \frac{\lambda^2(4 + 3\lambda - \delta)^2z_2}{2(TT_1)^2}.
\]

See Appendix D for the closed-form expressions for \( z_1 \) and \( z_2 \). Let \( f = \pi_A^N - \pi_A^L \); then \( \frac{\partial^2 f}{\partial \lambda^2} < 0 \), so \( f \) is convex in \( \lambda \). When \( \lambda = 1 \), there exists a \( \delta < 13 - 4\sqrt{10} \) such that \( f > 0 \) if \( \delta < \delta \) and \( f \leq 0 \) otherwise. Note that \( f|_{\lambda=\hat{\lambda}(\delta)} < 0 \) holds for \( \delta < 13 - 4\sqrt{10} \). When \( \delta < \delta \), there exists a \( \lambda(\delta) \) such that \( f < 0 \) if \( \lambda < \lambda(\delta) \), and \( f > 0 \) for \( \lambda(\delta) < \lambda < 1 \). When \( \delta > \delta \), we always have \( f < 0 \). Observe that \( z_2 \) is increasing in \( \delta \). Since \( z_2|_{\lambda=\delta} > 0 \), it follows that \( z_2 > 0 \); that is, \( \pi_B^L > \pi_B^N \).

2. For \((\delta, \lambda) \in R_2 \) and \((\delta, \lambda) \in R_3 \), we can use the same procedure to prove that \( \pi_A^L > \pi_A^N \) and \( \pi_B^L > \pi_B^N \).

C.10. Proof of Proposition 8

Firm \( A \)'s licensing strategies were analyzed in Section A.2.1. The optimal licensing strategy can be found by comparing the profits under each licensing policy—and also under no-licensing—based on Lemmas A.1, A.2, and A.3. Let \( \pi_A^{L_n}, \pi_A^{L_o}, \) and \( \pi_A^{L_no} \) denote firm \( A \)'s optimal profits for the cases of (respectively) licensing the new technology to firm \( B \) while also using that technology for its own end product, licensing to firm \( B \) while using the old technology for its own end product, and licensing to firm \( B \) without entering the end market. We shall use \( \pi_A^{NL} \) to denote firm \( A \)'s profit if it does not license to firm \( B \).

1. We start by comparing \( \pi_A^{L_n} \) with \( \pi_A^{NL} \). For \( \delta < \delta_A^{LB} \), we have

\[
\pi_A^{L_n} - \pi_A^{NL} = \frac{\theta(\theta + 7)\delta^2 - 16(1 + \theta)\delta + 16}{4(1 - \delta)(4 - \delta)^2} - \frac{4(1 - \delta)\theta(1 - \theta)}{4(1 - \delta)(4 - \delta)^2} < 0.
\]

Hence this licensing policy is suboptimal. For \( \delta \geq \delta_A^{LB} \), licensing is not feasible because firm \( B \)'s profit constraint cannot be met. Therefore, licensing and using the new technology for its own product is never optimal for firm \( A \) and so can be eliminated from further consideration.

2. We then compare \( \pi_A^{L_o} \) with \( \pi_A^{NL} \) under two scenarios, as described next.
(i) When $\theta \leq \delta$, firm $A$’s licensing problem has a feasible solution if and only if $\delta_3^{AB} \leq \delta \leq 1$. Let $\Phi = \pi_A^{Lo} - \pi_A^{NL}$, where $\Phi$ increases in $\delta$ and $\Phi|_{\delta=1} = 0$.\(^5\) We also have $\Phi|_{\delta=1} = \frac{\theta(4+\theta)(\theta^2+8\theta+36)+64}{2(8+\theta)(4-\theta)^2}$. Note that (a) $\Phi|_{\delta=1}$ is increasing in $\theta$ and (b) there exists a $\hat{\theta}_1 \in (\delta_3^{AB}, 1)$ such that $\Phi = |_{\delta=1} < 0$ if $\theta < \hat{\theta}_1$ and $\Phi|_{\delta=1} \geq 0$ otherwise. Therefore, if $\theta < \hat{\theta}_1$ then we always have $\Phi < 0$; if $\theta \geq \hat{\theta}_1$ then there exists a $\delta_1^{AB}$ such that $\Phi < 0$ (or $\pi_A^{Lo} < \pi_A^{NL}$) for $\delta_3^{AB} \leq \delta < \delta_1^{AB}$ and $\Phi > 0$ (or $\pi_A^{Lo} > \pi_A^{NL}$) for $\delta_1^{AB} \leq \delta < 1$.

(ii) When $\theta > \delta$, if $\delta < \delta_2^{AB}$ then

$$\pi_A^{Lo} - \pi_A^{NL} = \frac{1-(\delta \theta^2 - \delta (1-\delta)(8-\delta)\theta^2 + (17\delta^2 + 16) \theta - 16\delta]}{4(\theta - \delta)(4-\delta\theta)^2} < 0.$$  

Also, if $\delta_2^{AB} \leq \delta < \theta$ then, for firm $A$, licensing the new technology to firm $B$ while using the old technology for its own product is not feasible (and so is never optimal) because firm $B$’s constraint cannot be met.

3. Finally, we compare $\pi_A^{Lo}$ and $\pi_A^{NL}$:

$$\pi_A^{Lo} - \pi_A^{NL} = \frac{\delta}{8} - \frac{4(1-\delta\theta)}{(4-\delta\theta)^2} = \frac{\phi_1}{8(4-\delta\theta)^2},$$

where the closed-form expression for $\phi_1$ is given in Appendix D. Observe that $\phi_1$ increases with $\delta$ and also with $\phi_1|_{\delta=1} = 0$. Moreover, $\phi_1|_{\delta=1} = \theta^2 + 24 \theta - 16$ and so, if $\theta = \hat{\theta}_2 = 4(\sqrt{10} - 3)$, then $\phi_1|_{\delta=1} = 0$. As a consequence, $\phi_1|_{\delta=1} < 0$ if $\theta < \hat{\theta}_2$ and $\phi_1|_{\delta=1} \geq 0$ otherwise. Therefore, if $\theta < \hat{\theta}_2$ then $\phi_1 < 0$ (or $\pi_A^{Lo} < \pi_A^{NL}$) always holds; if $\theta \geq \hat{\theta}_2$, then there exists a $\delta_2^{AB}$ such that $\phi_1 < 0$ (or $\pi_A^{Lo} < \pi_A^{NL}$) for $\delta < \delta_2^{AB}$ and $\phi_1 \geq 0$ (or $\pi_A^{Lo} \geq \pi_A^{NL}$) for $\delta \geq \delta_2^{AB}$.

The preceding analysis suggests that no-licensing can be out-performed both by (a) licensing the new technology while using the old technology and (b) licensing without entering the market. Hence we now compare $\pi_A^{Lo}$ with $\pi_A^{Lno}$ when both strategies outperform the no-licensing strategy—that is, when $\delta \geq \max\{\delta_1^{AB}, \delta_2^{AB}\}$—in two distinct circumstances: (i) $\delta_2^{AB} < \delta_1^{AB}$; and (ii) $\delta_2^{AB} \geq \delta_1^{AB}$.

(i) If $\delta_2^{AB} < \delta_1^{AB}$, then

$$\pi_A^{Lo} - \pi_A^{Lno} = \frac{\theta(2\theta + 7\delta)}{8(\theta + 6\delta)} > 0.$$  

Thus, we always have $\pi_A^{Lo} > \pi_A^{Lno}$.

(ii) If $\delta_2^{AB} \geq \delta_1^{AB}$, then the analysis can be further divided into the following two cases.

1. If $\delta_1^{AB} \leq \delta < \delta_2^{AB}$, then

$$\pi_A^{Lo} - \pi_A^{Lno} = \frac{\phi_2}{8(\delta - \theta)(4-\delta\theta)^2},$$

where the closed-form expression for $\phi_2$ is given in Appendix D and $\phi_2$ is concave in $\delta$. The sign of $\phi_2$ is determined as follows. (i) If $\delta = \delta_1^{AB}$, then $\pi_A^{Lo} = \pi_A^{NL}$. Since $\pi_A^{Lo} > \pi_A^{NL}$ for $\delta \geq \max\{\delta_1^{AB}, \delta_2^{AB}\}$, it follows that $\pi_A^{Lo} - \pi_A^{Lno} < 0$; that is, $\phi_2 < 0$. (ii) If $\delta = \delta_2^{AB}$, then $\pi_A^{Lo} - \pi_A^{Lno}|_{\delta=\delta_2^{AB}} = \frac{6(2\theta + 7\delta)}{\pi_A^{Lo}} > 0$; that is, $\phi_2|_{\delta=\delta_2^{AB}} > 0$. Combining (i) and (ii), we can see that there exists a $\delta_3^{AB}$ such that $\phi_2 < 0$ if $\delta_1^{AB} \leq \delta < \delta_3^{AB}$ and $\phi_2 \geq 0$ if $\delta_3^{AB} \leq \delta < \delta_2^{AB}$. That is: $\pi_A^{Lo} < \pi_A^{Lno}$ if $\delta_1^{AB} \leq \delta < \delta_3^{AB}$ and $\delta \geq \delta_2^{AB}$; and $\pi_A^{Lo} \geq \pi_A^{Lno}$ if $\delta_3^{AB} \leq \delta < \delta_2^{AB}$ and $\delta \geq \delta_2^{AB}$.

If $\delta = \delta_3^{AB}$, then the optimal licensing fee is $w^* = 0$ and so $\hat{\pi}_A = \frac{4(1-\delta)}{14-32\delta} < \pi_A^{NL}$.  

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2. If \( \delta_2^{AB} \leq \delta < 1 \), then
\[
\frac{\pi^L_A - \pi_{L0}^A}{\delta(\theta + 8\delta)} > 0.
\]
Thus, we always have \( \pi^L_A > \pi_{L0}^A \) for \( \delta \geq \delta_2^{AB} \).
The preceding analysis yields the following optimal licensing strategies.

**Case 1.** For \( \delta_2^{AB} < \tilde{\delta}_1^{AB} \), the optimal licensing strategy is as follows:
1. if \( (\delta, \theta) \in H_1 = \{ (\delta, \theta) \mid \delta < \delta_1^{AB} \text{ and } \delta < \delta_2^{AB} \} \), then it is optimal for firm A not to license the new technology to firm B;
2. if \( (\delta, \theta) \in H_2 = \{ (\delta, \theta) \mid \delta > \tilde{\delta}_2^{AB} \} \), then it is optimal for firm A to license the new technology to firm B and use the old technology in its own product;
3. if \( (\delta, \theta) \in H_3 = \{ (\delta, \theta) \mid \delta < \tilde{\delta}_1^{AB} \text{ and } \delta > \delta_2^{AB} \} \), then it is optimal for firm A to license the new technology to firm B without entering the market.

**Case 2.** For \( \delta_2^{AB} \geq \tilde{\delta}_1^{AB} \), there exists a \( \tilde{\delta}_3^{AB} \in (\tilde{\delta}_1^{AB}, \tilde{\delta}_2^{AB}) \) such that the optimal licensing strategy is as follows:
1. if \( (\delta, \theta) \in H_1 = \{ (\delta, \theta) \mid \delta < \delta_1^{AB} \text{ and } \delta < \delta_2^{AB} \} \), then it is optimal for firm A not to license the new technology to firm B;
2. if \( (\delta, \theta) \in H_2 = \{ (\delta, \theta) \mid \delta > \tilde{\delta}_3^{AB} \text{ and } \delta > \tilde{\delta}_2^{AB}, \text{ or } \delta > \tilde{\delta}_1^{AB} \text{ and } \delta < \tilde{\delta}_2^{AB} \} \), then it is optimal for firm A to license the new technology to firm B and use the old technology to produce its own product;
3. if \( (\delta, \theta) \in H_3 = \{ (\delta, \theta) \mid \delta < \tilde{\delta}_3^{AB} \text{ and } \delta > \tilde{\delta}_2^{AB} \} \), then it is optimal for firm A to license the new technology to firm B without entering the market.

**C.11. Proof of Lemma A.1**
Differentiating \( \pi_A \) with respect to \( w \) yields \( \frac{\partial \pi_A}{\partial w} = \frac{(8+\theta)(\delta-2w)}{\delta(4-\delta)^2} > 0 \) for \( w < \delta/2 \). It follows that \( \pi_A \) is increasing in \( w \) and so there is no interior solution to firm A’s profit maximization problem. Hence the optimal solution is a boundary solution that satisfies the binding constraint \( \frac{(1-\delta)(\delta-2w)^2}{\delta(4-\delta)^2} - \frac{\theta(1-\delta)}{(4-\delta)^2} = 0 \). The solution is given by \( w^* = \frac{\delta}{2} \frac{(-\delta)\sqrt{\theta(1-\delta)(1-\delta)}}{2(1-\delta)(4-\delta)} \), which satisfies the constraint \( w < \delta/2 \). If \( \delta < \frac{4(2(1+\theta) - \sqrt{4(\theta^2+1)+\theta})}{7\theta} \), then \( w^* > 0 \) holds. Therefore, firm A’s problem has a optimal solution if \( \delta < \frac{4(2(1+\theta) - \sqrt{4(\theta^2+1)+\theta})}{7\theta} \) but is not feasible otherwise.

**C.12. Proof of Lemma A.2**
Let \( g(w) = \frac{4(\theta-\delta)(\delta-\delta w)^2}{(4-\delta)^2} - \frac{\theta(1-\delta)}{(4-\delta)^2} \geq 0 \). Then the Karush–Kuhn–Tucker conditions for firm A’s profit maximization problem are given by
\[
\frac{\partial \pi_A}{\partial w} - \mu g(w) = 0, \quad g(w) \geq 0, \quad \mu g(w) = 0;
\]
and \( \frac{\partial g}{\partial w} = \frac{8\delta^2 + \theta^2 - 2(\delta + \theta)w}{(4-\delta)^2} \) and \( \frac{\partial g}{\partial w} = \frac{8\theta(\theta - 1)(\theta - w)}{(4-\delta)^2} \). We can now check the solutions as follows.

**Case 1:** \( \mu = 0 \). In this case, \( w^* = \frac{8\delta^2 + \theta^2}{2(\delta + \theta)} \). Substituting \( w^* \) into \( g \) yields
\[
g(w) = \frac{4\theta^2 \delta^5 + 32\theta(2\theta - 1)\delta^4 + (64 - 3\theta^4 + 16\theta^3 - 64\theta)\delta^3 - \theta^2(\theta^3 - \theta^2 - 24\theta + 16)\delta^2 - \delta^2(48 + \theta - 8\theta^2) - 16\theta^3}{(4 - \delta)^2(\theta + 8\delta)^2}.
\]
Note that $g$ is decreasing in $\delta$. Because $g|_{\delta=0} < 0$ and $g|_{\delta=1} > 0$, there must exist a $\tilde{\delta}_2^{AB}$ such that $g \geq 0$ if $\delta \geq \tilde{\delta}_2^{AB}$ and $g < 0$ otherwise.

**Case 2:** $\mu > 0$. Here $g(w) = 0$ and $w^*_2 = \delta - \frac{\theta\delta(1-\delta)(1-\theta)}{2(4-\delta\theta)\sqrt{\theta(1-\delta\theta)(\delta-\theta)}}$. We also can find

$$\mu = \frac{8\delta + \theta}{4(\delta - \theta)} - \frac{(\theta + 2\delta)(4 - \theta\delta)\sqrt{\delta\theta(1 - \delta)(\delta - \theta)}}{4\delta\theta(\delta - \theta)(1 - \delta\theta)}.$$

If $\delta < \tilde{\delta}_2^{AB}$, then $\mu > 0$. The condition for $w^*_2 > 0$ can be characterized as $\delta > \tilde{\delta}_3^{AB}$, and it can be shown that $\tilde{\delta}_3^{AB} < \tilde{\delta}_2^{AB}$. In short, firm $A$’s maximization problem has an optimal solution if $\delta \geq \tilde{\delta}_3^{AB}$ but has no feasible solution if $\theta < \delta < \tilde{\delta}_3^{AB}$. The optimal solution is

$$w^* = \begin{cases} w_2^* & \text{if } \tilde{\delta}_3^{AB} \leq \delta < \tilde{\delta}_2^{AB}, \\ w_1^* & \text{if } \tilde{\delta}_2^{AB} \leq \delta < 1. \end{cases}$$

**C.13. Proof of Lemma A.3**

The analysis for the case $\theta > \delta$ follows the same procedure as in Section C.11; we omit the details.

**Appendix D: Notations**

Here we present the closed-form expressions for the notation used in Appendix C. We use $(Y)_i'$ and $(Y)_i^*$ to denote the partial derivatives of the function $Y$ with respect to $\delta$ and $\lambda$, respectively.

1. $m = \delta^4 - 2(\lambda^3 + 3\lambda^2 + 3\lambda + 3)\delta^3 + 6(2\lambda^2 + 11\lambda^3 + 24\lambda^2 + 23\lambda + 9)\delta^2 + 12(4\lambda^3 + 18\lambda^2 + 27\lambda + 16)\delta + 21\lambda^4 + 44\lambda^3 + 24\lambda^2 + 32 > 0$
2. $t = (1 + \lambda)(4 + 3\lambda - \delta)^2 > 0$
3. $t_A = (1 + \lambda)\delta^3 + 2(5\lambda + 9\lambda + 2)\delta^2 + (9\lambda^3 + 17\lambda^2 + 32\lambda + 32)\delta - 4(4 + 3\lambda)^2 > 0$
4. $t_B = \delta[(3 - \lambda)\delta^2 + 2(\lambda - 9\lambda - 14)\delta - 9\lambda^3 + 7\lambda^2 + 64\lambda + 52] > 0$
5. $T = -(\lambda + 1)^2\delta^5 - 4(2\lambda^3 + 5\lambda^2 + 4\lambda - 3)\delta^4 + 2(\lambda^4 + 16\lambda^3 + 39\lambda^2 - 62\lambda - 118)\delta^3 - 4(18\lambda^5 + 91\lambda^4 + 190\lambda^3 - 13\lambda^2 - 466\lambda - 360)\delta^2 - (81\lambda^6 + 414\lambda^5 + 853\lambda^4 + 2612\lambda^3 + 7372\lambda^2 + 9312\lambda + 4096)\delta + 16(3\lambda + 4)^4 > 0$
6. $S_1 = (1 - \delta)[\delta^2 + (3\lambda^2 - 5)\delta + (3\lambda + 4)^2] > 0$
7. $S_2 = (1 - \delta)\delta^5 - 2(2\lambda^2 + 9\lambda + 8)\delta^2 + (10 + 16\lambda - 9\lambda^3 - 5\lambda^2)\delta + 2(4 + 3\lambda)^2 > 2(4 + 3\lambda)^2 - 2(\lambda^2 + 9\lambda + 8)\delta^2 > 2(4 + 3\lambda)^2 - 2(\lambda^2 + 9\lambda + 8) = 2(1 + \lambda)(8 + 7\lambda) > 0$
8. $M_1 = \frac{(t_A)_{\lambda}^2 - (t_A)_{\lambda}^{-1}T}{4(1 + \lambda\lambda)^2(1 + 3\lambda - \delta)}$; $M_2 = \frac{(t_A)_{\lambda}^{-1}T - (t_A)_{\lambda}T^{-1}}{4(1 + 3\lambda - \delta)}$
9. $t_{A1} = (27 - 5\lambda)\delta - 2(17\lambda^2 + 125\lambda + 140)\delta - 45\lambda^3 + 187\lambda^2 + 696\lambda + 496 > 0$
10. $t_{B1} = -8(\lambda^2 + 17\lambda + 5)\delta^2 + 2(7\lambda^2 + 3\lambda - 8)\delta + (3 - \lambda)(4 + 3\lambda)^2 > 0$
11. $T_1 = -(40\lambda^3 + 125\lambda^2 + 130\lambda - 83)\delta^4 - 4(68\lambda^4 + 243\lambda^3 + 316\lambda^2 + 559\lambda + 546)\delta^3 - 2(180\lambda^5 + 571\lambda^4 + 538\lambda^3 - 3453\lambda^2 - 9440\lambda - 6224)\delta^2 + 4(81\lambda^6 + 280\lambda^5 - 3139\lambda^4 - 13738\lambda^3 - 18496\lambda - 8224)\delta - (45\lambda^4 + 146\lambda^3 - 979\lambda^2 - 2984\lambda - 2032)(3\lambda + 4)^2 > 0$
12. $k_1 = \frac{t_A t_{A1}}{4(1 + \lambda\lambda)} > 0$; $k_2 = \frac{t_B t_{B1}}{4(3\lambda + 4)^2} > 0$
13. $t_{A2} = \delta^6 - (\lambda^3 + 3\lambda^2 - \lambda^2 - \lambda + 8)\delta^5 + (4\lambda^7 + 14\lambda^6 + 4\lambda^5 - 34\lambda^4 - 65\lambda^3 - 85\lambda^2 - 48\lambda + 16)\delta^4 - \lambda(25\lambda^7 + 159\lambda^6 + 428\lambda^5 + 644\lambda^4 + 627\lambda^3 - 24\lambda - 128)\delta^3 + \lambda^2(2 + \lambda)(26\lambda^6 + 117\lambda^5 + 190\lambda^4 + 126\lambda^3 + 122\lambda^2 + 272\lambda + 192)\delta^2 - \lambda^3(9\lambda^7 + 113\lambda^6 + 487\lambda^5 + 937\lambda^4 + 632\lambda^3 - 504\lambda^2 - 1024\lambda - 448)\delta + \lambda^4(4 + 3\lambda)^2(2 + \lambda)^4 > 0$

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14. $t_{B2} = \delta [\delta^2 + (2 + \lambda)(\lambda^2 + 7\lambda + 4)\delta - \lambda^3][(3 - \lambda)\delta^4 - 4(\lambda^3 - \lambda^2 - 2\lambda + 4)\delta^3 - 2(2\lambda^5 + 10\lambda^4 + 71\lambda^3 + 185\lambda^2 + 166\lambda + 32)\delta^2 - 4(7\lambda^5 - \lambda^4 - 92\lambda^3 - 200\lambda^2 - 152\lambda - 32)\delta - 49\lambda^5 - 65\lambda^4 + 26\lambda^3 + 700\lambda^2 + 608\lambda + 192] > 0$

15. $T_2 = \delta^6 - (\lambda^4 + 3\lambda^3 - \lambda^2 - \lambda + 8)\delta^5 + (4\lambda^7 + 14\lambda^6 + 4\lambda^5 - 34\lambda^4 - 64\lambda^3 - 85\lambda^2 - 48\lambda + 16)\delta^4 - \lambda(25\lambda^7 + 159\lambda^6 + 428\lambda^5 + 644\lambda^4 + 627\lambda^3 + 361\lambda^2 - 24\lambda - 128)\delta^3 + (2 + \lambda)(26\lambda^6 + 117\lambda^5 + 190\lambda^4 + 126\lambda^3 + 122\lambda^2 + 272\lambda + 192)\delta^2 - \lambda^3(9\lambda^7 + 113\lambda^6 + 487\lambda^5 + 937\lambda^4 + 632\lambda^3 - 504\lambda^2 - 1024\lambda - 448)\delta + \lambda^4(4 + 3\lambda)^2(2 + \lambda)^2 > 0$

16. $N_1 = (32\lambda^2 + 63\lambda + 47)\delta^2 - 2(45\lambda^2 + 137\lambda + 108)\delta - 9\lambda^3 + 175\lambda^2 + 472\lambda + 304 > 166\lambda^2 + 472\lambda + 304 - 2(45\lambda^2 + 137\lambda + 108)\delta > 166\lambda^2 + 472\lambda + 304 - 2(45\lambda^2 + 137\lambda + 108) > 76\lambda^2 + 198\lambda + 88 > 0$

17. $N_2 = (27 - 5\lambda)\delta^2 - 2(17\lambda^2 + 125\lambda + 140)\delta - 45\lambda^3 + 187\lambda^2 + 696\lambda + 496 > 142\lambda^2 + 696\lambda + 496 - 2(17\lambda^2 + 125\lambda + 140)\delta > 142\lambda^2 + 696\lambda + 496 - 2(17\lambda^2 + 125\lambda + 140) = 108\lambda^2 + 446\lambda + 216 > 0$

18. $N_3 = (9 - 64\lambda^4 - 272\lambda^3 - 401\lambda^2 - 248\lambda)\delta^4 + 4(56\lambda^4 + 191\lambda^3 + 200\lambda^2 - 135\lambda - 264)\delta^3 + 2(2187\lambda^2 + 5560\lambda + 3472 - 72\lambda^3 - 371\lambda^2 - 392\lambda^3)\delta^2 + (4(3\lambda + 4)(21\lambda^4 + 28\lambda^3 - 685\lambda^2 - 1748\lambda - 1120)\delta + (4 + 3\lambda)^2 + (433\lambda^2 + 1496\lambda + 10409 - 9\lambda^4 - 96\lambda^3) > 0$

19. $y_A = -(\lambda + 1)\delta^3 - 2(5\lambda^2 + 9\lambda + 2)\delta^2 - (9\lambda^3 + 17\lambda^2 + 32\lambda + 32)\delta + 4(4 + 3\lambda)^2 > 0$

20. $y_B = \delta[(3 - \lambda)\delta^2 + 2(\lambda^2 - 9\lambda - 14)\delta - 9\lambda^3 + 7\lambda^2 + 64\lambda + 52] > 0$

21. $y_{A1} = (320\lambda^5 + 1680\lambda^4 + 3365\lambda^3 + 2221\lambda^2 - 1301\lambda + 1043)\delta^6 + 2(1088\lambda^6 + 5920\lambda^5 + 13003\lambda^4 + 21343\lambda^3 + 34177\lambda^2 + 12529\lambda - 16156)\delta^5 + (2880\lambda^7 + 11808\lambda^6 + 15587\lambda^5 - 52285\lambda^4 - 288123\lambda^3 - 203779\lambda^2 + 439304\lambda + 452304)\delta^4 - 4(1296\lambda^7 + 6171\lambda^6 - 12457\lambda^5 - 143639\lambda^4 + 51719\lambda^3 + 845868\lambda^2 + 1473936\lambda + 715456)\delta^3 + (6480\lambda^8 + 39771\lambda^7 + 5987\lambda^6 - 665339\lambda^5 + 550237\lambda^4 + 12005200\lambda^3 + 28637344\lambda^2 + 27045376\lambda + 9263360)\delta^2 - 2(4 - 3\lambda)(1431\lambda^7 + 471\lambda^6 - 114527\lambda^5 + 92569\lambda^4 + 2308920\lambda^3 + 5656192\lambda^2 + 5411200\lambda + 1865472)\delta + (4 + 3\lambda)^2(405\lambda^7 + 5229\lambda^6 - 45429\lambda^5 - 89597\lambda^4 + 469808\lambda^3 + 1560032\lambda^2 + 1637888\lambda + 589568) > 0$

22. $y_{B1} = -(8\lambda^2 + 17\lambda + 5)\delta^2 + 2(7\lambda^2 + 3\lambda - 8)\delta + (3 - \lambda)(4 + 3\lambda)^2 > 0$

23. $n_1 = \frac{2(y_{A1})T_1}{(4 + \lambda^2)(4 + 3\lambda^2)}; n_2 = -(8\lambda^2 + 17\lambda + 5)\delta^2 + 2(7\lambda^2 + 3\lambda - 8)\delta + (3 - \lambda)(4 + 3\lambda)^2$

24. $n_3 = -(128\lambda^4 + 544\lambda^3 + 772\lambda^2 + 603\lambda + 279)\delta^5 - (384\lambda^5 + 1440\lambda^4 + 1942\lambda^3 - 829\lambda^2 - 4827\lambda - 3272)\delta^4 + 2(528\lambda^5 + 1656\lambda^4 - 3711\lambda^3 - 20247\lambda^2 - 26848\lambda - 11600)\delta^3 - 2(432\lambda^6 + 2154\lambda^5 - 8005\lambda^4 - 59447\lambda^3 - 122600\lambda^2 - 108816\lambda - 36096\delta^2 + (420\lambda^4 - 671\lambda^3 - 7099\lambda^2 - 11592\lambda - 5744)(3\lambda + 4)^2\delta - (18\lambda^4 + 45\lambda^3 - 297\lambda^2 - 836\lambda - 544)(3\lambda + 4)^3$

25. $z_1 = \frac{t_2y_{A1}^{T_2} - (1 + \lambda)y_{A1}^{T_2}}{4(4 + 3\lambda - 8)\delta}; z_2 = \frac{16(y_{B1}T_1)^2 - (y_{B1}T_1)}{(4 + 3\lambda - 8)\delta^2}$

26. $\phi_1 = \theta^2\delta^3 - 8\theta^2\delta^2 + 16(2\theta + 1)\delta - 32$

27. $\phi_2 = 4(4 - \theta^2)(2\delta + \theta)(\sqrt{\theta/(1 - \theta)}(\delta - \theta) - \delta[\theta^2\delta^3 - \theta(\theta^2 + 16\theta + 8)\delta^2 + 2(-\theta^3 + 4\theta^2 + 8\theta + 8)\delta - 2\theta(8 - \theta)]$