Voluntary Technology Sharing to Rivals

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Abstract

Technology sharing to rivals and new-product introductions enabled by those technologies have often been observed across different industries. We develop a game-theoretic model to examine why a firm would voluntarily share its technology and help its rival develop a new product. We find that the cannibalization consideration in the rival’s multiproduct pricing imposes externality to the focal firm, which largely gives rise to its incentive to share technology, in addition to the potential benefit from the change in demand elasticity. Surprisingly, the rival does not always embrace the shared technology. In equilibrium, the new product would be introduced only when the existing product valuation is low but the technology transfer rate is not too low, when the existing product valuation is fairly high but the technology transfer rate is not too high, or when the existing product valuation is high but the technology transfer rate is neither too high nor too low. We show that social welfare increases with the new product introduction to a large extent except when the existing product’s valuation is high but the technology transfer rate is low. The new product introduction increases consumer surplus only when the existing product valuation is low. Compared to technology sharing to an independent third party, the focal firm is more likely to share its technology to the rival.

Keywords: technology sharing, new product development, cannibalization externality, spoke model
1 Introduction

Rapid technology development and innovation have become the hallmark of the new century. One such an example is the great advancement of smart wearable technology as exemplified by the wide popularity of smart watches. Along with the technology development, one interesting phenomenon is new-technology firms’ voluntary technology sharing to their rivals. For instance, Zepp Health Corporation, as a biometric and activity data-driven company, is globally recognized in the smart-watch market. Timex Group, a leader in watchmaking, primarily produces and sells traditional watches. While these two firms compete in the watch market, interestingly, Zepp shared its core technology with Timex and helped Timex develop its own smart watches. As a result, Timex launched its Metropolitan smartwatch collection in 2020.\footnote{https://www.timex.com/browse/smartwatches/timex-smart/metropolitan/ (accessed July 2021).} Technology sharing to rivals in such examples seems to be against traditional business wisdom—keeping a technology proprietary can equip a firm with competitive advantage against its rivals—and challenge our understanding of the technology world. Why would a firm share its proprietary technology with its rival?

Prior studies have identified various factors as firms’ incentive to share their technologies, such as expected gains from market expansion (Conner, 1995), network effects (e.g., Niculescu et al., 2018), and information disclosure which deters costly innovation competition (Huang et al., 2020). These studies focus on either technology sharing to new market entries or technology sharing between competing firms to improve their existing products. Our focus, in contrast, is on a firm’s technology sharing with its rival that enables the rival to introduce a new product competing against the firm. We examine how the new product introduced by the rival affects the competition between the two firms and thus their incentives to share and to adopt the technology (if shared), in the absence of the aforementioned factors.

The phenomenon of new-product introduction enabled by technology shared by rivals has been often observed across different industries. In a similar vein to Zepp’s technology sharing to Timex in the watchmaking industry, in the automobile industry, BYD Company Ltd., as a pioneer in battery technology, primarily produces and sells battery electric vehicles, and Toyota Motor Corporation, as a traditional automobile maker, produces and sells gasoline-fueled vehicles. While these two firms compete in the Chinese automobile market, BYD initiated a project to share its core technology of
battery electric vehicles to Toyota which enabled Toyota to also produce battery electric vehicles attractive to Chinese consumers.\textsuperscript{2} In the camera industry, Panasonic Corporation was a pioneer in digital camera technology, and Leica Camera was a manufacturer of traditional analog cameras. Although the two firms competed against each other in the camera market decades ago when digital cameras appeared to be a viable substitute to traditional cameras, Panasonic shared some of its digital technologies (e.g., image processing) with Leica, which helped Leica produce its own line of digital cameras.\textsuperscript{3} 

Evidently, technology sharing with rivals and the associated new-product introductions have profound impact to the involved firms, consumers, and society, especially given the sheer market sizes in some industries. For example, global sales of smart watches reached 91.4 million in 2020, and the global market is forecast to maintain double-digit growth through 2024.\textsuperscript{4} In this paper, we aim to answer the following questions: Why would a firm share its proprietary technology with its rival? Does the rival always embrace and benefit from the shared technology? Under what conditions is one willing to share and the other willing to adopt the technology such that a new product would be introduced to the market? How does technology sharing affect social welfare and consumer surplus?

To answer these questions, we develop a game-theoretic model with two firms competing in a market. One firm owns a new proprietary technology and determines whether to share its technology with its rival. Without technology sharing, the firms serve the market with one product each using their own technologies. When the new technology is shared by the firm, the rival may choose to adopt the technology to introduce a new product into the market. If it does, the rival serves the market with both its existing product and the new product. We assume that the valuations of the two existing products are comparable, and the valuation of the new product depends on the technology transfer rate. We consider these products are horizontally differentiated, and use a spoke model to characterize the firms’ competition before and after the new-product introduction. We compare the firms’ profits with and without the introduction of the new product. When their profits are greater with the new-product entry than without, the firm has incentive to share its technology


and the rival has incentive to adopt the technology (if shared).

We find that the focal firm has an incentive to share its technology only if its existing product valuation is low but the technology transfer rate is high (i.e., the new product’s valuation is not too low) or if its existing product valuation is high but the technology transfer rate is low (i.e., the new product’s valuation is not too high). An introduction of a new product enabled by the shared technology, on one hand, imposes additional competition against the firm. On the other hand, it turns the rival into a multiproduct firm, which must consider cannibalization between its two products, softening the interfirm competition in the existing product market. The focal firm’s incentive to share its technology is dictated by these two effects—the benefit from cannibalization externality entices the firm to share, whereas the additional competition might discourage the firm from sharing. When the existing product’s valuation is low, the firm has an incentive to share its technology only if the technology transfer rate is high. In this case, when the new product is introduced, the cannibalization between its two products is salient, which induces the rival to increase its existing product’s price, softening the interfirm competition in the existing product’s market. As a result, the focal firm can raise its product price despite the additional competition from the rival’s new product. The benefit for the firm from such a price boosting effect is more than compensating for its demand loss to the rival’s new product, giving rise to the focal firm’s incentive to share its technology. When the existing products’ valuation is high, the focal firm has incentive to share its technology if the technology transfer rate is not too high. The condition on technology transfer rate ensures that the valuation of the new product is not too high and thus the additional competition against the focal firm is not too severe, while the focal firm continues to benefit from the cannibalization externality. Interestingly, in this case, the focal firm might reduce its product price, and thus its demand can even be enhanced despite the additional competition from the rival’s new product. Such a demand enhancement effect might dominate the price factor, giving rise to the focal firm’s incentive to share its technology.

While conventional wisdom would suggest that the rival should always embrace the shared technology, our analyses reveal that the reality is more complicated. When the existing product’s valuation is high but the new product’s valuation is low (i.e., the technology transfer rate is low), we find that the option to use the additional product hurts the rival. In this case, anticipating the new-product entry and the additional competition brought in by it, the focal firm would lower
its product price, intensifying the competition in the existing product market. Meanwhile, the relatively low-value new product is significantly disadvantaged in competing with the focal firm’s product, and does not generate adequate profit. Consequently, the rival’s profit is reduced if the new product is introduced, so it has no incentive to adopt the technology.

Only when the two firms’ incentives are aligned would the new product be introduced to the market. In equilibrium, we find that the new product would be introduced only when the existing product valuation is low but the technology transfer rate is not too low, when the existing product valuation is fairly high but the technology transfer rate is not too high, or when the existing product valuation is high but the technology transfer rate is neither too high nor too low.

The implications of the new-product introduction for social welfare and consumer surplus are also nuanced. We show that social welfare increases with the new-product introduction to a large extent because of the benefits from the additional consumer coverage and the further consumer segmentation by the new product. However, these two benefits are dwarfed by the loss from product misallocation when the existing product’s valuation is high but the technology transfer rate is low. In that case, social welfare decreases. The new-product introduction increases consumer surplus only when the existing product valuation is low, under which additional consumer coverage is pronounced.

To further elaborate on a firm’s incentive to share technology with a rival, we compare the main model with a case in which the firm might share its technology with a third-party firm instead of to a rival. First, we find that, different from the main model, the third-party firm always has incentive to adopt the shared technology in this case. In equilibrium, the new product would be introduced to market only when the existing product valuation is low but the new product’s valuation is not too low or when the existing product valuation is fairly high but the new product’s valuation is not too high. The driving force lies in that under these conditions the new product decreases the demand elasticity for the existing products, which softens the competition in that market, giving rise to the firm’s incentive to share. Second, compared to technology sharing with a third-party firm (in which cannibalization is absent), the firm under our main model is more likely to have incentive to share its technology—the technology could be shared even when the existing product’s valuation is higher or lower. A greater likelihood of introduction of the new product in equilibrium highlights the importance of the cannibalization externality. Finally, we also find that social welfare is more
likely to increase in the technology sharing with the rival rather than with a third-party firm.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces our model. In Section 4, we provide an equilibrium analysis for the subgames with and without the new product in the market. In Section 5, we examine the conditions under which the focal firm has an incentive to share its technology and under which its rival has an incentive to adopt the technology, and we investigate how the new-product entry affects social welfare and consumer surplus in equilibrium. Section 6 compares the cases of technology sharing with rivals and with third parties. Section 7 discusses managerial implications and concludes.

2 Literature Review

This study relates mainly to three streams of literature: the studies on innovation cooperation, on technology openness, and on product-line extension.

The first related stream concerns innovation cooperation (or R&D cooperation) between competing firms. The aim of this type of cooperation is either to reduce production cost by joint effort in process innovation (e.g., D’Aspremont and Jacquemin, 1988; Kamien et al., 1992) or to develop new products by joint effort in product innovation (e.g., Bourreau et al., 2016). Three different cooperation modes have been assessed: cartelization in which firms form cartels to coordinate their innovation investments (D’Aspremont and Jacquemin, 1988), research joint ventures in which firms share their innovation efforts (Bourreau et al., 2016), and a combination of cartelization and research joint venture (Kamien et al., 1992). This stream of literature focuses on how firms trade off between product competition and innovation cooperation (Bourreau and Doğan, 2010; Bourreau et al., 2016), choosing the best mode in terms of firms’ profitability (Kamien et al., 1992), and how the firms’ coopetition affects technological improvement (D’Aspremont and Jacquemin, 1988; Kamien et al., 1992) and social welfare (D’Aspremont and Jacquemin, 1988; Bourreau et al., 2016).

Other relevant aspects on competing firms’ innovation cooperation have been also explored. For instance, Goyal and Joshi (2003) examine the formation of innovation-cooperation networks in an oligopoly market and suggest that strategically stable networks are often asymmetric, with some firms having many links and others having few or none. Suetens (2008) and Levy (2012) examine the relationship between price collusion and innovation cooperation. In contrast to this stream of
literature, we focus on a firm’s voluntary technology sharing with a rival, in which the firm has complete ownership of the technology, which might be the outcome of its sole innovation effort. While the literature generally suggests the benefits of innovation-expenditure reduction (Bourreau and Doğan, 2010; Bourreau et al., 2016), technological improvement (Kamien et al., 1992) or R&D-productivity enhancement (Branstetter and Sakakibara, 2002) as the competing firms’ incentives to cooperate, our study demonstrates a different factor that might induce a firm to voluntarily share its proprietary technology. Specifically, we demonstrate that the benefit from the positive externality of cannibalization in the rival’s multiproduct pricing can motivate a firm to voluntarily share its technology, enabling its rival to develop a new product.

The second related stream of literature concerns firms’ technology (or intellectual-property) openness: openness to complementors or openness to same-side competitors. Boudreau (2010) compares two distinct approaches to opening a technology platform for complementary innovation—giving access to independent developers of complementary components or giving control over the platform itself—and shows that platforms can gain more accelerated development with the access-granting opening approach. Parker and Alstyne (2018) characterize the optimal level of openness for a platform by addressing the trade-off between the platform’s ability to charge for access and the developers’ ability to innovate upon it. Technology openness to same-side competitors is more relevant to our study. Conner (1995) shows that enabling new entrants via technology openness can be more profitable for a firm than dominating the market alone because of the benefit from user-base (market) expansion. Sun et al. (2004) show that accommodation with new competitors can be more profitable than a firm’s own product-line extension in the presence of strong network effects. Niculescu et al. (2018) further suggest that a firm can calibrate the amount of technology sharing to the entrant’s absorptive capacity, leading to a mutually beneficial outcome even under intermediate network effects. They further show that the firm’s ability to engineer the strength of network effects can promote its technology sharing. Huang et al. (2020) demonstrate that a firm’s technology openness can serve as a shrewd information-disclosure strategy to signal its technology’s value, driving away unnecessary innovation competition that could otherwise dwarf innovation benefits.

While the market-expansion, network, and information-disclosure effects might encourage firms’ technology sharing with their competitors, the above studies mainly consider either new products developed by new entrants (Conner, 1995; Sun et al., 2004; Niculescu et al., 2018) or existing rivals’
products improved via the technology (Huang et al., 2020). In contrast to these studies, we focus on a firm’s technology sharing with its rival and the technology enabling the rival to introduce a new product to compete with the firm. We show that, even in the absence of the aforementioned effects, the rival’s introduction of the additional product can benefit the focal firm because of the change in the competition structure and, particularly, the externality imposed by the rival’s cannibalization concern in its multiproduct pricing.

The third related stream of literature is on firms’ product-line extension (e.g., Wilson and Norton, 1989; Desai, 2001). Most of these studies are concerned with the profitability of the introduction of the new products from the viewpoint of the parties who own the new products. Only a few papers have examined changes in the profits for other parties in the same market, which are more related to our work. With consumers’ status preferences in which consumers appreciate the status value granted by the products (Rao and Schaefer, 2013; Gao et al., 2017), Li (2019) shows that a firm’s vertical product-line extension might increase the rival’s profit because the status preferences can reduce consumers’ overall price sensitivity, softening the interfirm competition. Joshi et al. (2016) show that a firm’s horizontal line expansion can benefit its rival if consumers have relatively high valuation but low heterogeneity in their horizontal preferences for the extension product (compared with the existing products). In that case, a broader scope of products permits the firm to effectively price discriminate by raising prices for its core consumers, while the rival can optimally respond by lowering prices to gain a larger market share, benefiting both firms. Thomadsen (2012) demonstrates that a firm’s horizontal expansion of product line can also be conducive to its rival as long as a moderate number of consumers are unserved before the expansion and the consumers’ preferences for the new product are positively correlated with the firm’s original product but negatively correlated with the rival’s product.

In our study, a focal firm has an incentive to share its technology only if its rival’s introduction of a new product, enabled by the shared technology, benefits the focal firm. In contrast to the aforementioned studies, we demonstrate a potential benefit of technology sharing from a competition-altering perspective; even in the absence of consumers’ status preferences, low heterogeneity in consumers’ horizontal preferences, unserved consumers, or the required correlation, a firm might want to share its technology and can gain from its rival’s introduction of a new product because the firm might benefit from the change in the demand elasticity caused by a new-product introduction and, more
importantly, enjoy the positive externality imposed by the rival’s cannibalization concern in pricing its products.

3 Model

We consider a market with two competing firms, 1 and 2. Firm 1 produces Product 1 and Firm 2 produces Product 2. The two products are imperfect substitutes produced under different proprietary technologies. In the example of the watchmaking industry, Firm 1 can be Zepp using smart-watch technology, and Firm 2 can be Timex using traditional watch technology. We consider that Firm 1 can share its proprietary technology with Firm 2. Under technology sharing, Firm 2 might produce a new product, Product 3. Doing so leads to three products competing in the same market. We assume that Firm 2 is unable to develop Product 3 without Firm 1’s technology. Therefore, without technology sharing, only Products 1 and 2 compete in the market.

We consider the products are horizontally differentiated in that consumers have different preferences among the products because the products, for example, have different design characteristics and features such as layout, appearance, and color. In particular, we use the spoke model (Chen and Riordan, 2007) to characterize consumer preference and product competition before and after the new-product entry. As shown in Figure 1, we consider a 3-spoke model. Each spoke, with length of $\frac{1}{2}$, terminates at a center and originates at the other end. Product $i$ is located at the origin of Spoke $i$, $i \in \{1, 2, 3\}$. Consumers are uniformly distributed on the spokes. The total mass of consumers is normalized to 1, and the mass of consumers is $\frac{1}{3}$ on each spoke. For a consumer located on Spoke $i$, Product $i$ is her first preferred product, and each of the other two products is equally likely to be her second preferred product. Each consumer chooses between her first and second preferred products, if both are available on the market. If one of these two products is unavailable, consumers only consider the available preferred product. For instance, in the presence of all of the three products, half of the consumers located on Spoke 1 choose between Products 1 and 2, and the other half choose between Products 1 and 3. In the absence of Product 3, half of the consumers located on Spoke 1 choose between Products 1 and 2, and the other half only consider buying Product 1 or not.

As in other well-known horizontal-differentiation models (e.g., the Hotelling model), the utility
that a consumer derives from a first or second preferred Product \( i \) is the maximum value \( q_i \) that the consumer derives from the product minus the misfit cost and the product price \( p_i \). In particular, the consumer at a distance of \( x_i \) from the preferred Product \( i \) derives a utility of \( u_i \) from buying product \( i \) and \( u_j \) from buying the other preferred product \( j \), where \( j \neq i, j \in \{1, 2, 3\} \), and

\[
\begin{align*}
    u_i &= q_i - x_i t - p_i \\
    u_j &= q_j - (1 - x_i) t - p_j.
\end{align*}
\]

The maximum value \( q_i \) or \( q_j \) depends on the product’s quality. The terms \( x_i t \) and \( (1 - x_i) t \) are the misfit costs, with \( t \) being the unit misfit cost. The distances \( x_i \) and \( 1 - x_i \) measure the degree of misfit—greater distances lead to greater misfit. The utility of the outside option is normalized to 0.

When Products \( i \) and \( j \) are a consumer’s two preferred products and both are available, the consumer compare \( u_i \) and \( u_j \) and buys the product that offers a greater nonnegative utility. These consumers are located on Spokes \( i \) and \( j \), and we call this consumer segment submarket \( \tilde{i} \tilde{j} \), which is of size \( \frac{1}{3} \). When only one preferred product is available, a consumer buys the product if and only if the product offers positive utility.

We are interested in the conditions under which Firm 1 has incentive or disincentive to share its technology with its rival, considering the effect of the introduction of Product 3 on the subsequent competition and resulting profits. To sharpen our focus on technology sharing, we assume that the maximum value of the two existing products is comparable and, in particular, we let \( q_1 = q_2 \equiv q \).
We let \( q_3 = \theta q \), where \( \theta, \theta \leq 1 \), measures the technology transfer rate; that is, the maximum value that Product 3 can deliver under the shared technology is not greater than that of Product 1, which is manufactured under the original technology. For ease of exposition, we consider \( \theta \) to be not too small (e.g., \( \theta \geq \frac{43}{100} \)). Allowing smaller \( \theta \) does not alter our main insights. We also assume that the maximum value of the existing products is not too low (i.e., \( q \geq \frac{74}{6} \)) nor too high (i.e., \( q \leq \frac{54}{20} t \)). The lower bound ensures the market is competitive without technology sharing; otherwise, the firms might become local monopolies. The upper bound ensures the existence of a pure-strategy equilibrium. Allowing larger \( q \) does not change our results qualitatively.

The timeline of the game is as follows. In Stage 1, Firm 1 decides whether to share its technology with Firm 2. If offered, Firm 2 decides whether to adopt the technology to introduce the new product into the market. In Stage 2, the two firms engage in price competition by simultaneously determining their respective product price(s). In Stage 3, consumers make their purchase decisions.

4 Equilibrium Analysis

We use the concept of subgame-perfect Nash equilibrium and use backward induction to derive the equilibrium. Based on the firms’ technology-sharing and technology-adoption decisions in Stage 1, we have two subgames in Stage 2: with and without new-product entry. We first derive the equilibrium for each subgame in this section and then derive the conditions under which Firm 1 has an incentive to share its technology and under which Firm 2 has an incentive to adopt the technology.

4.1 The Case without New-Product Entry

In the case without new-product entry, only Products 1 and 2 are present in the market. These two products directly compete in Submarket 12, while holding a monopoly position in Submarkets 13 and 23, respectively. In Submarket 12, by letting \( u_1 = u_2 \) in Equation (1), we can obtain the marginal consumer who is indifferent to purchasing Products 1 and 2 located at \( m_{12} \), the distance from Product 1:

\[
m_{12} = \frac{t - p_1 + p_2}{2t}.
\]
Consumers located between Product 1 and $m_{12}$ purchase Product 1, and those located between $m_{12}$ and Product 2 purchase Product 2. In Submarket $i3$, consumers buy Product $i$ when $u_i \geq 0$. Accordingly, we can formulate Product 1’s and 2’s demands as

$$
\begin{align*}
  d_1 &= \frac{1}{3}m_{12} + \frac{1}{3} \min \{ \frac{q-p_1}{t}, 1 \} \\
  d_2 &= \frac{1}{3}(1 - m_{12}) + \frac{1}{3} \min \{ \frac{q-p_2}{t}, 1 \}
\end{align*}
$$

and their profit functions as

$$
\pi_1 = p_1d_1 \quad \text{and} \quad \pi_2 = p_2d_2.
$$

Each firm chooses its price to maximize profit. By solving the first-order conditions of the profit functions in Equation (4), we can derive each firm’s best response to its rival. Based on these best responses, we can derive the equilibrium outcome as summarized in Lemma 1.

**Lemma 1.** In the case without new product entry, the equilibrium prices of Products 1 and 2 are

$$
p_1^* = p_2^* = \begin{cases} 
  q - t & \text{if } 2t < q \\
  \frac{1}{5}(2q + t) & \text{otherwise},
\end{cases}
$$

the equilibrium demands of the two products are

$$
d_1^* = d_2^* = \begin{cases} 
  \frac{1}{2} & \text{if } 2t < q \\
  \frac{1}{10t}(2q + t) & \text{otherwise},
\end{cases}
$$

and the equilibrium profits for the two firms are

$$
\pi_1^* = \pi_2^* = \begin{cases} 
  \frac{1}{2}(q - t) & \text{if } 2t < q \\
  \frac{1}{30t^2}(2q + t)^2 & \text{otherwise}.
\end{cases}
$$

Lemma 1 characterizes two types of equilibria and indicates a kink in the equilibrium outcome (at $q = 2t$). In general, in maximizing their profits, each firm considers both the competitive segment and their respective monopoly segment. When the product value is low (i.e., $q \leq 2t$), the optimal price reflects the trade-offs in both segments. When the product value is high enough (i.e., $q > 2t$),
both firms price their products as if they focus only on their monopoly consumer segments—they charge prices such that the consumers with the highest misfit cost in their monopoly segments derive zero utility and are just served. Intuitively, in this case, the firms are able to charge high prices and derive high revenue from their monopoly segments (due to high product value) such that the trade-off in the monopoly segment dominates that in the competitive segment.

4.2 The Case with New-Product Entry

If Firm 1 shares its technology with Firm 2 and Firm 2 serves the market with both Products 2 and 3, then all consumers’ two preferred products are both available in the market.

Similar to Submarket $\overline{i2}$, we have the marginal consumer in submarket $\overline{i3}$, $i \in \{1, 2\}$, who is indifferent to purchasing Products $i$ and 3. Similarly to $m_{12}$ in Equation (2), we define $m_{i3}$ as the marginal consumer’s distance from product $i$. By letting $u_i = u_3$, we can derive

$$m_{i3} = \frac{t - p_i + p_3 + (1 - \theta)q}{2t}. \quad (8)$$

Provided that the marginal consumers derive a positive utility, consumers located between Product $i$ and $m_{i3}$ purchase Product $i$, and those located between $m_{i3}$ and Product 3 purchase Product 3. Each product’s demand, which is the sum of demands from the two submarkets in which it is situated, can be derived as

$$\begin{aligned}
  d_1 &= \frac{1}{3}m_{12} + \frac{1}{3}m_{13} \\
  d_2 &= \frac{1}{3}(1 - m_{12}) + \frac{1}{3}m_{23} \\
  d_3 &= \frac{1}{3}(1 - m_{13}) + \frac{1}{3}(1 - m_{23})
\end{aligned} \quad (9)$$

The two firms’ profits can be formulated as

$$\pi_1 = p_1d_1 \text{ and } \pi_2 = p_2d_2 + p_3d_3. \quad (10)$$

Similar to the case without new-product entry, each firm chooses its price(s) to maximize profit. By solving the first-order conditions of the profit functions in Equation (10), we can derive the equilibrium outcome as summarized in Lemma 2.
Lemma 2. If Firm 2 serves the market with both Products 2 and 3, the equilibrium prices of the three products are

\[
(p_1^*, p_2^*, p_3^*) = \begin{cases} 
\left(\frac{1}{6}(8t + q - \theta q), \frac{1}{12}(20t + q - \theta q), \frac{1}{12}(4t - q + \theta q)\right) & \text{if } \frac{13t}{4+2\theta} < q \\
\left(\frac{1}{3}(t + 2q), \frac{1}{3}(-2t + 3q + \theta q), \frac{1}{3}(-2t + q + 3\theta q)\right) & \text{if } \frac{3t}{1+\theta} < q \leq \frac{13t}{4+2\theta} \\
\left(\frac{1}{2}(-2t + 3q + \theta q), \frac{1}{2}(-2t + 3q + \theta q), \frac{1}{2}(-2t + q + 3\theta q)\right) & \text{if } \frac{14t}{7+5\theta} < q \leq \frac{3t}{1+\theta} \\
\left(\frac{1}{2}(2q + t), \frac{1}{2}(2q + t), \max\left\{\frac{1}{2}(-6t + 3q + 5\theta q), \frac{\theta q}{2}\right\}\right) & \text{if } q \leq \frac{14t}{7+5\theta}, 
\end{cases}
\]  

(11)

the equilibrium demands of the products are

\[
(d_1^*, d_2^*, d_3^*) = \begin{cases} 
\left(\frac{1}{12t}(8t + q - \theta q), \frac{1}{12t}(20t + 7q - 7\theta q), \frac{1}{12t}(20t - 11q + 11\theta q)\right) & \text{if } \frac{13t}{4+2\theta} < q \\
\left(\frac{1}{12t}t + 2q), \frac{1}{12t}(11t + q - 3\theta q), \frac{1}{12t}(11t - 5q + 3\theta q)\right) & \text{if } \frac{3t}{1+\theta} < q \leq \frac{13t}{4+2\theta} \\
\left(\frac{1}{12t}(4t + q - \theta q), \frac{1}{12t}(4t + q - \theta q), \frac{1}{12t}(2t - q + \theta q)\right) & \text{if } \frac{14t}{7+5\theta} < q \leq \frac{3t}{1+\theta} \\
\left(\frac{1}{12t}(2q + t), \frac{1}{12t}(2q + t), \min\left\{\frac{1}{12t}(4t - 2q), \frac{\theta q}{2t}\right\}\right) & \text{if } q \leq \frac{14t}{7+5\theta}, 
\end{cases}
\]  

(12)

and the equilibrium profits for the two firms, \((\pi_1^*, \pi_2^*)\), are

\[
\begin{align*}
\left\{ \frac{1}{56t}(8t + q - \theta q)^2, \frac{1}{56t}[4(10t - q + \theta q)^2 + 27(1 - \theta)^2q^2]\right\} & \quad \text{if } \frac{13t}{4+2\theta} < q \\
\left\{ \frac{1}{56t}(t + 2q)^2, \frac{1}{56t}(-22t^2 + 2(13 + 11\theta)qt - (1 + 10\theta - 3\theta^2)q^2)\right\} & \quad \text{if } \frac{3t}{1+\theta} < q \leq \frac{13t}{4+2\theta} \\
\left\{ \frac{1}{56t}(4t + q - \theta q)(-2t + 3q + \theta q), \frac{1}{56t}[-16t^2 + 2(9 + 7\theta)qt + (1 - \theta)(1 - 5\theta)q^2]\right\} & \quad \text{if } \frac{14t}{7+5\theta} < q \leq \frac{3t}{1+\theta} \\
\left\{ \frac{1}{56t}(2q + t)^2, \frac{1}{56t}(2q + t)^2 + \max\{\theta q^2, \frac{\theta q}{2t}\}\right\} & \quad \text{if } q \leq \frac{14t}{7+5\theta} 
\end{align*}
\]  

(13)
Figure 2 illustrates the equilibrium outcome. In Region I, corresponding to the first case in Lemma 2, both $q$ and $\theta$ are high such that the existing products offer high value and the new product offers comparable value to consumers. In this case, Firms 1 and 2 compete aggressively with each other in Submarkets 12 and 13, which shapes the equilibrium prices. In contrast to the equilibrium without Product 3, Product 2's price is higher than Product 1's, because in this case Firm 2 has its exclusive Submarket 23 and charging a low price could cannibalize its Product 3's demand.

In Region II, corresponding to the second case in Lemma 2, in which either $q$ or $\theta$ is not high, on one hand, Firm 2 has incentive to take advantage of its exclusive Submarket 23 by charging high prices. On the other hand, it cannot charge very high prices because, otherwise, some consumers in its exclusive Submarket 23 do not purchase. As a result, Firm 2’s pricing strategy is largely shaped by its exclusive Submarket 23. Firm 2 chooses the optimal prices for its two products to cover all the consumers in this submarket and extract all the marginal consumer’s surplus; that is, the marginal consumer located at $m_{23}$ has zero surplus. Firm 1 reacts to its rival’s prices by optimally choosing its price for Product 1 to compete against both Products 2 and 3.

Moving further southwest to Region III (corresponding to the third case in Lemma 2), due to the reduction in the product valuation, the competition in Submarket 13 changes: If the same pricing strategy in Region II were followed, some consumers in Submarket 13 would not be served. In this case, because $q$ is not too small, Firm 1 finds it optimal to set the highest price to serve the entire residual demand in Submarket 13 such that Submarket 13 is just fully covered.

In Region IV (corresponding to the fourth case in Lemma 2), the value of the existing products is low, and the new product’s value is even lower due to attrition in the technology transfer. In this case, Product 1 does not compete with Product 3, and Products 1 and 2 compete directly as in the absence of Product 3. Firm 2 uses Product 3 to serve the residual demands in Submarkets 13 and 23. While its value is not too low (above the dashed line in Figure 2), Product 3 covers the entire residual demands in Submarkets 13 and 23. When Product 3’s value is relatively low, Firm 2 charges a monopoly price for Product 3 and leaves some consumers uncovered in Submarkets 13 and 23. As a result, in Region IV, or, when

$$q \leq \frac{14t}{1 + 5\theta} \equiv \Bar{q}(\theta),$$

(14)
the entry of the new product has no impact on the competition between the existing products—the prices, demands, and profits of Products 1 and 2 remain unchanged with the entry of Product 3.

5 Technology Sharing as the Equilibrium Outcome

We next compare the equilibria of the two subgames discussed in Section 4. We first compare the prices and demands before and after the introduction of the new product, then examine the conditions under which technology sharing and adoption arise as an equilibrium in Stage 1, and finally examine the effect of technology sharing and adoption on social welfare and consumer surplus.

We use regular notations (e.g., $p_1^*$) for the equilibrium outcome in the absence of the new product and the notations with hats (e.g., $\hat{p}_1^*$) for the equilibrium outcome in the presence of the new product.

5.1 Comparison of Prices and Demands

We first investigate how the equilibrium prices and demands of the existing products change with the entry of Product 3. Comparing equilibria in the subgames summarized in Lemmas 1 and 2 yields the following results.

**Proposition 1.** After Product 3 is introduced to the market,

(a) Product 2’s price becomes (weakly) higher, and its demand becomes (weakly) lower; that is, $\hat{p}_2^* \geq p_2^*$ and $\hat{d}_2^* \leq d_2^*$;

(b) Product 1’s price becomes (weakly) higher if and only if $q \leq \min\left\{\frac{5t}{2}, \frac{14t}{\theta + \theta}\right\}$, and its demand becomes (weakly) lower if and only if $q \leq \max\left\{\frac{5t}{2}, \frac{t}{1-\theta}\right\}$.

The new-product entry has two direct effects. First, in Submarket 23 some consumers who would have purchased the existing product in the absence of the new product now might purchase the new product: The new product can cannibalize Firm 2’s existing product’s demand. Second, in Submarket 13 some consumers who would have purchased the existing product in the absence of the new product now might purchase the new product: The new product creates additional competition for Firm 1’s product. To clearly show these two effects, we consider two benchmarks with only one effect in each. We call the case in which Product 3 serves only Submarket 23 (but not Submarket 13) and Submarket 23 is competitive in the presence of Product 3 the benchmark
with cannibalization only. We call the case in which Product 3 serves only Submarket $\bar{1}3$ (but not Submarket $\bar{2}3$) and Submarket $\bar{1}3$ is competitive in the presence of Product 3 the benchmark with additional competition only. Corollary 1 summarizes each direct effect on its associated firm for the benchmarks.

**Corollary 1.** After Product 3 is introduced to the market,

(a) in the benchmark with cannibalization only, Product 2’s price is increased, and Firm 2’s profit increases;

(b) in the benchmark with additional competition only, Product 1’s price is decreased except when $q \leq \frac{10t}{4t+\theta}$, and Firm 1’s profit decreases except when $q \leq \frac{309t}{147+859}$.

In the benchmark with cannibalization only, Firm 2 raises Product 2’s price to soften the cannibalization after the new-product introduction. In the benchmark with additional competition only, in general, Firm 1 should be induced to lower its price because of the extra competition. However, there is a caveat. When the existing product’s valuation is low (in the region between the two dashed lines in Figure 3), Product 1’s price can be increased. The driving force for the higher price is related to the decreased demand elasticity. In the absence of the new product, some consumers in Submarket $\bar{1}3$ are left out, and the marginal consumer derives zero surplus from the alternative (not purchasing). In the presence of the new product, the marginal consumer derives zero or positive surplus and has Product 3 as the alternative. If Firm 1 adjusts its price, the marginal effect of its demand is greater in the former than in the latter. In other words, the demand is more elastic in the former. Consequently, Product 1’s price becomes higher after Product 3 is introduced.

In our setting, except in Region IV of Figure 2 (i.e., $q \leq \bar{q}(\theta)$) in which the prices and demands of Products 1 and 2 remain unchanged, the introduction of Product 3 induces Firm 2 to charge a higher price for Product 2 driving down demand. Intuitively, concerned about the cannibalization between its own products in the exclusive Submarket $\bar{2}3$, Firm 2 raises Product 2’s price after introducing Product 3, reducing Product 2’s demand. In other words, the main driving force for these results is the direct effect of cannibalization concern. Although the additional competition also has an indirect effect on Firm 2 because the new product alters Product 1’s price, which can in turn affect Product 2’s price, the indirect effect on Firm 2 is dwarfed by the direct effect.

The effects of the introduction of Product 3 on Firm 1’s product price and demand are more
As illustrated in Figure 3, whether Firm 1 increases Product 1’s price and whether Product 1’s demand grows after the entry of Product 3 depends on product valuation and technology transfer rate. On one hand, Firm 2’s cannibalization concern imposes an externality on Firm 1. After Product 3 is introduced, Firm 2’s cannibalization concern lessens the competition pressure against Firm 1 in Submarket 12, which we call a cannibalization externality for Firm 1. The cannibalization externality tends to induce Firm 1 to raise Product 1’s price. On the other hand, the new product creates additional competition against Firm 1 because Firm 1 must compete against Firm 2 in Submarket 13. If the existing product’s valuation is low, the effect of decreased demand elasticity resulting from the additional competition can also drive up the price. As a result, Firm 1 charges a higher price for its product. If the existing product’s valuation is high, the additional competition pressure tends to induce Firm 1 to lower Product 1’s price. Whether Product 1’s price rises depends on the balance between the additional-competition and the cannibalization-externality effects. When Product 3’s valuation (i.e., $\theta q$) is markedly high, the former effect dominates the latter effect, and Firm 1 reduces Product 1’s price after the introduction of Product 3.

Product 1’s demand becomes lower to a large extent after Product 3’s introduction, primarily because the new product, as a substitute for Product 1, takes away some of the demand in Submarket 13 under competition. In fact, in Regions I and II of Figure 2, Product 1’s sales in Submarket 12 increase because either Firm 2 raises Product 2’s price more than Firm 1 raises Product 1’s price or
Firm 1 has an incentive to share Technology Transfer Rate \((\theta)\) and Existing Products’ Maximum Value \((q)\) for different values of \(\theta\) and \(q\).

Firm 2 has an incentive to adopt Technology Transfer Rate \((\theta)\) and Existing Products’ Maximum Value \((q)\) for different values of \(\theta\) and \(q\).

![Graphs showing Technology Sharing Incentive for Firm 1 and Technology Adoption Incentive for Firm 2](image)

(a) Technology Sharing Incentive for Firm 1  
(b) Technology Adoption Incentive for Firm 2

Figure 4: Technology Sharing and Adoption Incentives for the Two Firms \((t = 1)\)

Firm 2 raises Product 2’s price, but Firm 1 lowers Product 1’s price. However, that sales increase cannot be compensated by the sales loss to the new product in Submarket \(\bar{13}\) such that Product 1’s total demand decreases. The only exception is when Product 1’s valuation is high but the technology transfer rate is relatively low (the top-left corner in Figure 3). In this case, the new product’s valuation is low, and Product 1’s sales loss to the new product is mild. As a result, Firm 1’s product demand can be even higher after its rival’s introduction of the new product.

5.2 Conditions for Technology Sharing and Adoption

We next examine the conditions under which Firm 1 has incentive to share its proprietary technology and under which Firm 2 has incentive to adopt the technology if it is available. We then investigate the conditions under which the new product would be introduced in equilibrium—that is, the firms’ decisions in Stage 1 and the equilibrium outcome of the game.

By comparing each firm’s equilibrium profits in the two subgames summarized in Lemmas 1 and 2, we conclude the following results.

**Proposition 2.** (a) Firm 1 has an incentive to share its technology if and only if \(\bar{q}(\theta) \leq q \leq \frac{19+8\theta-3\sqrt{3(9+20\theta-2\theta^2)}}{(1-\theta)^2}t\), where \(\bar{q}(\theta)\) is defined in Equation (14).
(b) Firm 2 has an incentive to adopt the technology except when

\[ q \geq \min \left\{ \frac{4(37-10\theta)-6\sqrt{6(13+122\theta-81\theta^2)}}{31(1-\theta)}t, \frac{1+11\theta+\sqrt{3+42\theta+115\theta^2}}{4+10\theta-3\theta^2}t \right\} \]

Figure 4(a) illustrates the regions in which Firm 1 has and does not have incentive to share its technology. When its existing product valuation is low and the technology transfer rate is relatively low (i.e., \( q \leq \bar{q}(\theta) \)) such that the new product’s valuation would be low, Firm 1 has no incentive to share. When its existing product valuation is high and the technology transfer rate is relatively high (i.e., \( q > \frac{19+8\theta-3\sqrt{3(9+20\theta-2\theta^2)}}{(1-\theta)^2} \)) such that the new product’s valuation would be high, Firm 1 has no incentive to share either. In the former case, Firm 2 would only use Product 3 to serve some of the residual demand. Therefore, the introduction of Product 3 affects neither Firm 2’s pricing of Product 2 nor the competition between the two firms. As a result, Firm 1 would not benefit from the entry of Product 3 and so does not share. In the latter case, Product 3 would also offer high value to consumers, and would create significant competition pressure for Firm 1 in Submarket 13, which is detrimental to Firm 1.

As illustrated in Figure 4(a), only if its existing product valuation is low but the new product’s valuation is not too low or if its existing product valuation is high but the new product’s valuation is not too high, does Firm 1 have an incentive to share its technology. Firm 1’s incentive to share its technology is dictated by two effects—the benefit from the cannibalization externality entices Firm 1 to share, whereas the additional competition might discourage it from sharing. The lower bound ensures the occurrence of the cannibalization upon the new-product introduction, and the upper bound ensures that the additional competition against Firm 1 is not too severe.

Intuitively, when the new product is introduced, Firm 2 becomes a firm with multiple products, which are substitutes and simultaneously compete with Firm 1’s product. Firm 2 must consider not only the interfirm competition against Firm 1, but also the intrafirm cannibalization between its two products. As shown in Proposition 1, Firm 2 raises its existing product’s price with the introduction of the new product, which softens the competition against and benefits Firm 1. In particular, below the upper dashed line in this region of Figure 4(a), by Proposition 1 and Figure 3, Firm 1 is also able to raise its price as its optimal choice. Although its demand decreases, this price boosting effect benefits Firm 1 more than the sales loss, and thus its profit increases. Above
the upper dashed line in this region of Figure 4(a), although Firm 1 reduces its price because of the additional competition in Submarket 13, its demand increases, primarily because of the sales increase in Submarket 12 due to its relatively low price compared to the competing product’s. This demand enhancement effect dominates the price factor and benefits Firm 1 in this region.

Interestingly, even in the benchmark with additional competition only (where cannibalization is absent), Firm 1 might have incentive to share, as suggested by Corollary 1. Although Firm 1 is hurt by the extra competition in general, it comes with an exception when the existing product’s valuation is very low (in the region between the lower two dotted lines in Figure 4(a)). In this case, the new product, on one hand, takes away some demand from Firm 1. On the other hand, when the existing product’s valuation is low, as shown in Corollary 1, it decreases Product 1’s demand elasticity and drives up Product 1’s price, which in turn reduces the competition in Submarket 12. When the existing product’s valuation is very low, the demand loss in Submarket 13 is small such that the benefit from the reduced competition in Submarket 12 can dominate, increasing Firm 1’s profit and inducing Firm 1 to share its technology.

For Firm 2, the potential use of Product 3 to serve the market is generically desirable. Firm 2 might benefit from the use of Product 3 in various ways: (1) When the existing product’s valuation is low, Firm 2 can use the new product to serve some consumers who are left out in the market in its absence. (2) When the existing product’s valuation is high, Firm 2 can both use the new product to better serve some of the existing customers who have low degrees of fit with its existing product and use the new product to seize some consumers from its rival. Because of these benefits, as illustrated in Figure 4(b), Firm 2 has an incentive to adopt the technology to a large extent.

Surprisingly, as illustrated in the upper-left corner in Figure 4(b), Firm 2 might have no incentive to adopt the technology—the new product does not always increase Firm 2’s profit, and the option to use this additional product can even hurt Firm 2. One would expect that this option should bring no harm to Firm 2 because, after all, the firm can set the new product’s price extremely high such that no consumers buy the new product, which seems to be equivalent to the case without the new product. This reasoning would be logical if the rival did not react strategically. However, Firm 1 strategically chooses its price, anticipating that, as long as its product price is not too low, Firm 2 would use the new product to compete with it in Submarket 13 and to serve those consumers who have better fit with the new product. In particular, having the option of Product 3 can hurt
Firm 2 when the existing products’ valuation is high but the new product’s valuation is low (i.e., \( q \) is high but \( \theta \) is low, the upper-left corner in Figure 4(b)). In this case, anticipating the entry of Product 3 and the additional competition brought in by it, Firm 1 would lower its product price, which worsens the competition in Submarket 12. On the other hand, the relatively low-value new product is at a significantly disadvantaged position in competing with Product 1 in Submarket 13, and does not generate adequate profit. As a result, although it helps win over some demand from its rival in Submarket 13, the new product hurts Firm 2 in the existing product’s market, which is of higher value to Firm 2. Consequently, Firm 2’s profit shrinks if the new product is introduced in this case, and thus it has no incentive to adopt the technology. This result can be attributed to the additional competition brought in by the new product, independent of cannibalization, as we find that this result also arises in the benchmark with additional competition only, but not in the benchmark with cannibalization only (as suggested by Corollary 1). Essentially, the new product equips Firm 2 to profit from the market, but it can also create a burden for Firm 2.

Only if Firm 1 has incentive to share its technology and Firm 2 has incentive to adopt the technology, would the new product be introduced to the market. Interestingly, we find that in some cases Firm 1 has incentive to share but Firm 2 has no incentive to adopt—when the existing product’s valuation is high but the new product’s valuation would be low, Firm 1 is willing to share its technology, but Firm 2 has no incentive to adopt the technology. Combining the two firms’ incentive conditions in Proposition 2 leads to the conditions under which the new product would be introduced in equilibrium, as summarized in Proposition 3.

**Proposition 3.** In equilibrium, the new product would be introduced to the market if and only if \( \bar{q}(\theta) \leq q \leq \hat{q}(\theta) \), where

\[
\hat{q}(\theta) = \begin{cases} 
1 + 11\theta + \sqrt{4\theta + 4\theta^2 + 17\theta^4} t & \text{if } \theta < \frac{231 + 52\sqrt{33}}{1087} \\
\frac{4(37 - 10\theta) - 6\sqrt{6(13 + 12\theta - 8\theta^2)}}{31(1 - \theta)^2} t & \text{if } \frac{231 + 52\sqrt{33}}{1087} < \theta \leq \frac{311}{635} \\
19 + 8\theta - 3\sqrt{9(1 + 2\theta - 2\theta^2)} (1 - \theta)^2 t & \text{if } \frac{311}{635} < \theta 
\end{cases}
\]

Moreover, \( \bar{q}(\theta) \) decreases in \( \theta \), and \( \hat{q}(\theta) \) increases in \( \theta \) when \( \theta \leq \frac{231 + 52\sqrt{33}}{1087} \) and then decreases.

As illustrated in Figure 5, in equilibrium, the new product would be introduced only when the existing product valuation is low but the new product’s valuation is not too low, when the existing

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product valuation is fairly high but the new product’s valuation is not too high, or when the existing product valuation is high but the new product’s valuation is neither too high nor too low. The lower bound in the conditions in Proposition 3 represents Firm 1’s incentive to share its technology, below which Firm 1 has no incentive to share. As indicated by the small circle on \( \hat{q}(\theta) \) in the figure, the upper bound contains a kink. The condition before the kink ensures Firm 2 has incentive to adopt the technology, and the condition after the kink represents Firm 1’s incentive, above which Firm 1 has no incentive to share.

One implication of Proposition 3, as illustrated in Figure 5, is that when the existing product’s valuation is very high—specifically, when \( q > \frac{185-4\sqrt{33}}{62}\) t—the two firms’ incentives cannot be aligned and the new product would never be introduced to the market, regardless of the new product’s valuation or the technology transfer rate. In this case, if the new product’s valuation is high, the entry of the new product can create significant additional competition pressure on Firm 1, seriously affecting the sales of its high-price product. If the new product’s valuation is low, the potential use of the new product can create burden for Firm 2, because Firm 1 anticipates Firm 2 using the new product to serve the market and strategically reacts, but the new product itself does not create much value, hurting Firm 2.

While Firm 2 in general might prefer a high technology transfer rate (or high new-product valuation given the existing products’ valuation), the lower and upper bounds in Firm 1’s incentive
conditions in Figure 5 being downward-sloped suggests that Firm 1 might have different preferences, depending on the existing product’s valuation. Corollary 2 summarizes how Firm 1’s profit changes with the technology transfer rate in equilibrium.

**Corollary 2.** *In the equilibrium where the new product is introduced to market, Firm 1’s profit increases in \( \theta \) when \( q \leq \frac{3t}{1+q} \); otherwise, its profit (weakly) decreases in \( \theta \).*

Conventional wisdom might suggest that a high technology transfer rate can equip a firm’s rival with a competitive new product, intensifying the interfirm competition and negatively affecting the firm’s performance on the market. When the existing product’s valuation is high, our finding, as summarized in Proposition 3 and illustrated in Figure 5, confirms such a concern for Firm 1. In particular, the downward-sloped upper bound of Firm 1’s incentive conditions (as illustrated in Figure 4(a)) suggests that Firm 1 prefers a low technology transfer rate under which the rival’s new product, if introduced, is in a competitively disadvantaged position. However, when the existing product’s valuation is low, our findings suggest the opposite—Firm 1 prefers a high technology transfer rate. In particular, the downward-sloped lower bound of Firm 1’s incentive conditions suggests that, given the existing product’s (low) valuation, Firm 1 prefers not to share its technology when technology transfer rate is low, whereas when technology transfer rate is high, Firm 1 shares. Even beyond this boundary in the incentive condition, Corollary 2 indicates that Firm 1’s profit increases in \( \theta \) to a large extent when the existing product’s valuation is low. The driving force for such a counterintuitive finding is again Firm 2’s cannibalization concern upon the new product introduction and the resulting *price boosting* effect for Firm 1.

### 5.3 Effects of the New Product on Social Welfare and Consumer Surplus

We next examine the effects of technology sharing and adoption on social welfare and consumer surplus. Social welfare is the total value created by product consumption. For example, for a consumer in Submarket \( \bar{i} \) purchasing Product 1, if \( x \) is the distance between the consumer and Product 1, the value created by the consumption is \( q - xt \). When the new product is introduced to the market, social welfare can be formulated as

\[
sw = \frac{1}{3} \left( \int_0^{m_{12}} (q - xt) dx + \int_0^{1} (q - (1 - x)t) dx \right) + \frac{1}{3} \sum_{i=1}^{2} \left( \int_0^{m_{i3}} (q - xt) dx + \int_0^{1} [\theta q - (1 - x)t] dx \right), \tag{15}
\]
where \( m_{12} \) and \( m_{i3} \) are defined in Equations (2) and (8), respectively. In the absence of the new product, social welfare can be similarly formulated. Although product prices do not directly affect social welfare (because payments are an internal transfer from consumers to firms from a social planner’s perspective), product prices affect the locations of the marginal consumers, which in turn affect social welfare. Consumer surplus is the total consumer net utility from purchasing the products and is the social welfare minus the firms’ profits. Proposition 4 summarizes how the introduction of the new product affects social welfare and consumer surplus.

**Proposition 4.** When the new product is introduced, in equilibrium,

(a) social welfare increases if and only if \( q \leq \tilde{q}(\theta) \), where

\[
\tilde{q}(\theta) = \begin{cases} 
\frac{2(58+50\theta+5\sqrt{109+94\theta+169\theta^2})t}{3(31+50\theta-25\theta^2)} & \text{if } \theta \leq \frac{119+30\sqrt{322}}{1183} \\
\frac{(464+55\theta+5\sqrt{381+499\theta+17437\theta^2})t}{347+650\theta-225\theta^2} & \text{if } \frac{119+30\sqrt{322}}{1183} < \theta \leq \frac{1+\sqrt{61}}{18} \\
\frac{(16-22\theta-\sqrt{165-522\theta+421\theta^2})t}{13-26\theta+9\theta^2} & \text{if } \frac{1+\sqrt{61}}{18} < \theta \leq \frac{1413+208\sqrt{6}}{2693} \\
\frac{8(16-3\sqrt{5})t}{101(1-\theta)} & \text{if } \frac{1413+208\sqrt{6}}{2693} < \theta
\end{cases}
\]

(b) consumer surplus increases if and only if \( q < \frac{11+25\theta-15\sqrt{49+54\theta+230\theta^2}t}{-47-50\theta+25\theta^2} \).

We might expect social welfare to increase with the new product entry. As illustrated by Figure 6, Proposition 4 shows that such an expectation is not always met. The new-product entry can decrease social welfare when the existing product’s valuation is high but the new product’s...
valuation is low. The introduction of the new product might affect social welfare in three different ways: (1) When the existing product’s valuation is low, the new product can cover some consumers who are left out of the market in its absence, which can increase social welfare. (2) When the existing product’s valuation is high, the new product might further segment consumers and better serve some of the existing consumers who have low degrees of fit with the existing products, which can increase social welfare. (3) Because of the firms’ strategic pricing when the new product is introduced, its introduction can cause some allocation distortion between the products, which could reduce social welfare. For example, when the new product is introduced, as shown in Proposition 1, Firm 2 increases its existing product’s price. Even although Firm 1 might also raise its price as the reaction to Firm 2’s price change, Firm 1’s increase might be less than Firm 2’s. Consequently, some consumers who have better fit with Product 2 choose to purchase Product 1 in equilibrium, causing product misallocation. Such a misallocation arises with the introduction of the new product, regardless of the existing product’s valuation.

The social benefits from the additional consumer coverage and further consumer segmentation can be salient such that social welfare can be boosted by the introduction of the new product to a large extent. However, as illustrated in Figure 6, when the existing products’ valuation is high but the technology transfer rate is low, surprisingly, the introduction of the new product can reduce social welfare. In this case, the benefit from additional consumer coverage is small (because of the high value associated with the existing products), and the benefit from further consumer segmentation is small (because of the low value associated with the new product). As a result, the efficiency loss caused by misallocation can dominate such that social welfare decreases because of the introduction of the new product.

The increase in consumer surplus can only occur when social welfare increases because social welfare is the sum of the firms’ profits and consumer surplus. In equilibrium, the new product would be introduced only if doing so is profitable to both firms. Therefore, consumers can benefit from the introduction of the new product only if the total pie (social welfare) is enlarged and the firms transfer a proportion of their gains to consumers. Because consumer surplus measures the aggregate net consumer utility, consumer surplus can increase either because of additional market coverage or because of reduced product prices. As illustrated in Figure 6, consumer surplus increases with the introduction of the new product only if the existing products’ valuation is low (below the
dashed line). In this case, considerable consumers are left out of the market in the absence of the new product. Although both firms raise their existing products’ prices when the new product is introduced (as suggested by Proposition 1), the benefit from additional market coverage can be pronounced such that consumer surplus increases.

6 Comparison to Technology Sharing with a Third-Party Firm

In our main model, we consider technology sharing to a rival with the new product owned by the rival (if introduced). In practice, sometimes firms with proprietary technology open their technology to encourage new-firm entry into the market (Conner, 1995; Niculescu et al., 2018), which is essentially technology sharing with third parties. In this section, we consider the case where Product 3, if introduced, is owned by a third-party firm, Firm 3. Everything else stays the same as in the main model. We first examine the condition under which Firm 1 has incentive to share its technology with Firm 3 and then compare the equilibrium outcomes between technology sharing with a rival and with a third-party firm.

In the subgame without Product 3, the price competition equilibrium remains the same as in Lemma 1. When Product 3 is introduced, the demand for each product is the same as in Equation (9), and each firm’s profit is \( \pi_i = p_i d_i, i \in \{1, 2, 3\} \). As in the main model, we can derive the equilibrium for this subgame. Comparing the equilibrium outcomes in the two subgames, we conclude the conditions under which Firm 1 has incentive to share its technology and Product 3 is introduced to the market in equilibrium.

**Proposition 5.** In the case with technology sharing with a third-party firm, in equilibrium, Firm 1 has incentive to share its technology and the new product would be introduced to the market if and only if \( \bar{q}'(\theta) \leq q \leq \hat{q}'(\theta) \), where \( \bar{q}'(\theta) = \frac{18t}{9 + 3\theta} \), and

\[
\hat{q}'(\theta) = \begin{cases} 
\frac{55 + 20\theta - 5\sqrt{3(27 + 56\theta - 86\theta^2)}}{4(1 - \theta)^2} t & \text{if } \theta \leq \frac{1}{3} (14 - 5\sqrt{6}) \\
\frac{4 + 9\sqrt{\pi} - (10 - \sqrt{\pi})\theta}{2(5 + 2\theta - \theta^2)} t & \text{if } \frac{1}{3} (14 - 5\sqrt{6}) < \theta.
\end{cases}
\]

Moreover, both \( \bar{q}'(\theta) \) and \( \hat{q}'(\theta) \) decrease in \( \theta \).

Figure 7 illustrates the conditions in Proposition 5 under which technology sharing with the third-party firm arises in equilibrium. First, different from the main model, the third-party firm
always has incentive to adopt the technology (if shared), because it has nothing to lose from the adoption. Second, only when the existing product valuation is low and the new product’s valuation is not too low or when the existing product valuation is fairly high and the new product’s valuation is not too high, does Firm 1 also have incentive to share its technology and thus the new product would be introduced in equilibrium. In contrast to the main model, this case lacks the benefit of the cannibalization externality. The only driving force for Firm 1’s incentive lies in the additional competition engendered by the new product. As in the benchmark with additional competition only, the entry of the new product could alter the demand elasticity. The condition that the new product’s valuation is not too low when the existing product’s valuation is low ensures that the new product competes with the existing products. The condition that the new product’s valuation is not too high when the existing product valuation is fairly high ensures that the new product decreases the demand elasticity for the existing products. When both conditions are satisfied, the incumbent firms raise their prices (between $q'(\theta)$ and the dashed line in Figure 7). Although the new product takes away some demand from the incumbent firms, the tendency to increase their prices induced by the decreased demand elasticity softens the competition in Submarket 12, benefiting the incumbent firms. When the existing product’s valuation is not too high, the demand loss can be compensated by the benefit of reduced competition in Submarket 12. Consequently, Firm 1 (as well as Firm 2) benefits from the new product entry, and Firm 1 has incentive to share its technology.
Comparing the conditions in Propositions 3 and 5, we can derive the conditions under which technology sharing to a rival and to a third-party firm can both arise as an equilibrium and conditions under which it does not arise.

**Proposition 6.** When \( \bar{q}(\theta) \leq q \leq \underline{q}'(\theta) \), Firm 1 is willing to share its technology in both the case of technology sharing with a rival and the case of technology sharing with a third-party firm. When \( \bar{q}(\theta) < q \leq \bar{q}'(\theta) \) or when \( \underline{q}'(\theta) < q \leq \bar{q}(\theta) \), Firm 1 is willing to share its technology in the case of technology sharing with a rival but not in the case of technology sharing with a third-party firm.

As illustrated by Figure 8, Proposition 6 shows that the new-product introduction is more likely to occur when technology sharing is with a rival than when technology sharing is with a third party. The main driving force for such a result is that the benefit from the cannibalization externality exists only in the former case, not in the latter. For example, when the existing product’s valuation is high (i.e., \( \bar{q}'(\theta) < q \)), an independent new product would create too much competition pressure on the existing firms and hurt them. In contrast, when the new product is owned by Firm 2, the firm has no incentive to use the new product to aggressively compete with Firm 1 because of cannibalization concern, which gives rise to room to benefit Firm 1.

It is worth noting that, although the additional competition brought in by the new product might benefit Firm 1 in both the main model (as suggested by Corollary 1) and the case of technology

![Figure 8: Comparison of Conditions for Technology Sharing with a Rival and with a Third Party](image)
sharing with a third party (as suggested by Proposition 6), the way such a benefit takes place is different. In the main-model benchmark with additional competition only, the additional competition might decrease the demand elasticity in Submarket \( \bar{1}3 \) for Firm 1 and thus it tends to induce Firm 1 to charge a higher price, which in turn can reduce the competition against Firm 2 in Submarket \( 12 \). In contrast, in the case of technology sharing with a third party, the additional competition can decrease the demand elasticities in Submarkets \( 13 \) and \( 23 \) for both firms simultaneously. These reductions reinforce each other and reduce the competition between the two firms in Submarket \( 12 \). Consequently, a new-product introduction is more likely to occur in the case of technology sharing with a third party than in the main-model benchmark with additional competition only. Figure 8 illustrates such a comparison in which technology sharing for the benchmark occurs only in the region between \( q'(\theta) \) and the dashed line.

The difference in the effect of the additional competition caused by the new product in the main model and the case of technology sharing with a third party is also evidenced by the equilibrium demand change for Firm 1. In the case of technology sharing with a third party, Firm 1’s demand is always decreased upon the new-product introduction because the new product takes away some demand. In contrast, in the main model (as well as in the benchmark with additional competition only), we find that Firm 1’s demand can increase upon the new-product introduction. This is because, in our main model, Products 2 and 3 are owned by Firm 2, and their prices are jointly determined by it. As a result, Firm 1 can lose demand to the rival’s new product, but can gain demand from the rival in the existing market. The gain could be greater than the loss because Firm 1, competing in two markets with Firm 2 equipped with two products, might reduce its product price more significantly than Firm 2 reduces Product 2’s price.

Additionally, similar to the main model, we can also derive the social welfare and consumer surplus for the case of technology sharing with a third party. The effects of the new-product entry in equilibrium share similar patterns with the main model—the new-product entry increases social welfare except when the existing product’s valuation is high and the new product’s valuation is low, and the new-product entry enhances consumer surplus only when the existing product’s valuation is low. Further, in the region where the new product would be introduced in both the main model and this case with technology sharing with a third party, we find that social welfare is more likely to increase under the main model. This is because when Products 2 and 3 are owned by the same
firm and their prices are jointly determined by it, compared to being owned by different firms, the misallocation issue can be mitigated.

7 Conclusion

We develop a game-theoretic model to examine a firm’s incentive to share its proprietary technology and help its rival develop a new product, along with the rival’s incentive to adopt the technology (if shared). Although an introduction of the new product by the rival brings in additional competition pressure on the firm, it also makes the rival a multiproduct firm for which cannibalization consideration plays a role in its pricing. We find that the rival’s cannibalization concern might soften the competition in the existing product market, which imposes a positive externality on the focal firm and gives rise to its incentive to share the technology. Surprisingly, we find that the rival does not always adopt the technology because, anticipating the entry of the new product and the additional competition it brings, the focal firm would lower its product price, which can intensify the competition in the existing product market. We derive the conditions which induce the focal firm to share its technology and induce the rival to adopt the technology. The new product would be introduced to the market in equilibrium when the two firms’ incentives are aligned or, more specifically, when the existing product valuation is low but the technology transfer rate is not too low, when the existing product valuation is fairly high but the technology transfer rate is not too high, or when the existing product valuation is high but the technology transfer rate is neither too high nor too low. We show that social welfare increases with the new-product introduction to a large extent except when the existing product’s valuation is high but the technology transfer rate is low. The new-product introduction increases consumer surplus only when the existing product valuation is low. Compared to technology sharing with an independent third party, the focal firm is more likely to share its technology with the rival.

Our study has several managerial implications. First, our study reveals potential benefit of technology sharing from a competition-altering perspective, which calls for firms’ attention in their strategic technology-sharing decisions. It has been long recognized that technology sharing might increase network effects or expand market size, providing firms’ incentive to share. Our study shows that, even without these effects, a firm might want to share its technology and help its rival develop
a new product, because the firm might benefit from the change in the demand elasticity caused by an introduction of a new product and also enjoys the positive externality imposed by the rival’s cannibalization concern in pricing its products. This new perspective, in line with the *frenemies* paradigm, deserves firms’ particular consideration, as conventional business wisdom might suggest that firms should keep and protect their proprietary technologies especially from their competitors.

Second, our results provide guidelines that help firms make strategic decisions of whether a firm should share its technology with a rival to help the rival develop a new product and whether the rival should adopt the technology in a competitive landscape. The firms should of course not blindly share or adopt technology. A firm should share its technology only if the new product alters the competition landscape when the existing product’s valuation is low, or if the new product does not impose high competition pressure against its product when the existing product’s valuation is high. Meanwhile, technology adoption is not always beneficial to the rival: The rival should not adopt the technology when the existing product’s valuation is high and the new product’s valuation is low. Our study underscores the importance of the proper evaluation of the competitive situations with and without the potential entry of a new product for both involved parties. Because the valuation of a new product would be largely determined by the technology transfer rate, our study suggests that both the focal firm and rival may want to establish collaboration to ensure a proper level of technology would be transferred such that a win-win situation would result.

Third, this research also has implications for social planners, such as the Federal Trade Commission. One might expect that increased product variety should increase social welfare, and thus the introduction of a new product resulting from technology sharing with a rival should benefit society. Our finding reveals this is not always the case, which calls for social planners’ attention. In particular, the new-product entry can decrease social welfare when the existing product’s valuation is high and the new product’s valuation is low, which would lead to significant product misallocation from the introduction of the new product. Other than that, social welfare can be increased to a large extent even beyond the cases in which both firms have incentive to introduce the new product. In other words, the lack of either firm’s incentive might cause social welfare loss. In these cases, introducing government subsidy programs can boost the firms’ incentives and restore the welfare benefit.

Our study also highlights that even when firms have incentive to share and adopt technolo-
gies and social welfare is increased by the new-product introduction enabled by the technology, consumers might not benefit—consumer surplus increases only if the existing products’ valuation is low. Therefore, if consumer surplus is a concern, social planners may consider subsidizing consumers through such financial incentives as tax credits.

References


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A Appendix

A.1 Proof of Lemma 1

Proof. We distinguish two cases based on the value of $q$.

(a) $q \leq 2t$: When $q$ is relatively low, such that some consumers in Submarkets 13 and 23 do not purchase but the marginal consumer in Submarket 12 derives positive utility, the profit functions for the two firms are

$$
\begin{align*}
\pi_1 &= p_1 \left( \frac{1}{3} \frac{t-p_1+p_2}{2t} + \frac{1}{3} \frac{q-p_1}{t} \right) \\
\pi_2 &= p_2 \left( \frac{1}{3} \frac{t-p_2+p_1}{2t} + \frac{1}{3} \frac{q-p_2}{t} \right)
\end{align*}
$$

(16)
We drive the first-order conditions as

\[
\begin{cases}
\frac{\partial \pi_1}{\partial p_1} = \frac{1}{3} \frac{t-2p_1+p_2}{2t} + \frac{1}{3} \frac{q-2p_1}{t} = 0 \\
\frac{\partial \pi_2}{\partial p_2} = \frac{1}{3} \frac{t-2p_2+p_1}{2t} + \frac{1}{3} \frac{q-2p_2}{t} = 0
\end{cases}
\]

Solving this system of equations, we can derive \( p_i^* = \frac{1}{5} (2q + t) \) and \( d_i^* = \frac{1}{10t} (2q + t) \), \( i \in \{1, 2\} \).

Substituting \( p_i^* \) into Equation (16), we have the equilibrium profits as specified in the lemma.

To ensure that some consumers in Submarkets \( \overline{13} \) and \( 23 \) do not purchase requires \( q - t - p_i^* \leq 0 \), or, equivalently, \( q \leq 2t \). To ensure that the marginal consumer in Submarket \( \overline{12} \) derives positive utility requires \( q - \frac{1}{2} t - p_i^* \geq 0 \), or, equivalently, \( q \geq \frac{7}{6} t \) (the assumption imposed in the model).

(b) \( q > 2t \): When \( q \) is large, all consumers in Submarkets \( \overline{13} \) and \( 23 \) purchase. The firms have incentive to charge a price high enough such that the consumers with the highest misfit cost in Submarkets \( \overline{13} \) and \( 23 \) derive zero utility; that is, \( q - t - p_i = 0 \), leading to \( p_i^* = q - t \), \( d_i^* = \frac{1}{2} \), and \( \pi_i^* = \frac{1}{2} (q - t) \). We can verify neither firm has incentive to deviate. \( \square \)

A.2 Proof of Lemma 2

Proof. We distinguish four cases based on the value of \( q \).

(a) \( \frac{13t}{3 + 2q} < q \): When \( 0 \leq m_{12}, m_{13}, m_{23} \leq 1 \) and the marginal consumers derive positive utilities, the demand functions in Equation (9) are well behaved. We derive the first-order conditions of Equation (10) as

\[
\begin{cases}
\frac{\partial \pi_1}{\partial p_1} = -\frac{4p_1+p_2+p_3+(1-\theta)q+2t}{6t} = 0 \\
\frac{\partial \pi_2}{\partial p_2} = \frac{p_1-4p_2+2p_3+(1-\theta)q+2t}{6t} = 0 \\
\frac{\partial \pi_3}{\partial p_3} = \frac{p_1+2p_2-4p_3-2(1-\theta)q+2t}{6t} = 0
\end{cases}
\]

Solving this system of equations, we can derive \( p_i^* \) as specified in the lemma. We can verify that \( 0 \leq m_{12}, m_{13}, m_{23} \leq 1 \) within this region, which also ensures that the marginal consumers located at \( m_{12}, m_{13}, \) and \( m_{23} \) derive positive utility. We can verify that neither firm has profitable deviation.

(b) \( \frac{2t}{1+\theta} < q \leq \frac{13t}{3+2q} \): In this case, marginal consumer \( m_{23} \) derives zero utility because \( q \leq \frac{13t}{3+2q} \). When Firm 2 prices the products such that marginal consumer \( m_{23} \) derives zero utility, \( \frac{q-p_2}{t} + \frac{\theta q - p_3}{t} = 1 \), or, equivalently, \( p_2 + p_3 = q + \theta q - t \). Using the Lagrange-multiplier method, we have \( L(p_2, p_3, \lambda) = \pi_1 + \lambda(q + \theta q - t + p_2 - p_3) \). By solving the first-order condition of \( L(p_2, p_3, \lambda) \),

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we can derive \( p_2^* \) and \( p_3^* \) as specified in the lemma, which is independent of \( p_1 \). By the first-order condition of \( \pi_1 \) in Equation (10), we can derive \( p_1^* \) as specified in the lemma. The condition \( \frac{3t}{1+\theta} < q \) ensures that marginal consumer \( m_{13} \) derives positive utility. We can verify that marginal consumer \( m_{12} \) derives positive utility, \( 0 \leq m_{12}, m_{13}, m_{23} \leq 1 \), and neither firm has incentive to deviate from \((p_1^*, p_2^*, p_3^*)\).

(c) \( \frac{11t}{7+5\theta} < q \leq \frac{3t}{1+\theta} \): In this case, marginal consumers \( m_{13} \) and \( m_{23} \) derive zero utility in equilibrium. Because \( \pi_2 \) shares the same optimization problem as in case (b), \( p_2^* \) and \( p_3^* \) stay the same as in case (b). When Firm 1 prices its product such that marginal consumer \( m_{13} \) derives zero utility, \( \frac{2-\theta}{t} + \frac{\theta q-p_3}{t} = 1 \), or, equivalently, \( p_1 + p_3 = q + \theta q - t \). Therefore, \( p_1^* = p_2^* \) in equilibrium. The condition \( \frac{11t}{7+5\theta} < q \) ensures that Firm 1 has no incentive to increase the price of Product 1 because the marginal loss from a price increase is greater than the marginal benefit. Meanwhile, Firm 1 has no incentive to decrease the price of Product 1 because the marginal loss of a price decrease is greater than the marginal benefit under \( q \leq \frac{3t}{1+\theta} \).

(d) \( q \leq \frac{11t}{7+5\theta} \): In this case, the two firms compete in Submarket \( \bar{1}2 \) but not in Submarket \( \bar{1}3 \). Consequently, as in case (a) in the proof of Lemma 1, \( p_1^* = p_2^* = \frac{1}{5}(2q + t) \). Firm 2 either charges \( p_3^* = \frac{1}{5}(-6t + 3q + 5\theta q) \) to cover all of the residual demands in Submarkets \( \bar{1}3 \) and \( 23 \) or charges a monopoly price \( p_3^* = \frac{\theta q}{2} \) for Product 3, whichever is higher. As a result, \( d_1^*, d_2^* \), and \( \pi_1^* \) stay the same as in case (a) of Lemma 1, and \( \pi_2^* \) consists of the revenue from Product 2 (the same as that of Product 1) and the revenue from Product 3. We can verify that neither firm has incentive to deviate.

Substituting \( p_i^* \) into Equations (9) and (10), we can derive the equilibrium demands and profits as specified in the lemma.

\( \square \)

A.3 Proof of Proposition 1

Proof. Based on Lemmas 1 and 2, we compare the equilibrium prices and demands before and after the introduction of Product 3 for different cases.

(a) The case with \( q \leq 2t \):

(a.1) When \( q \leq \bar{q}(\theta) \), as shown in Lemma 2, the introduction of Product 3 affects neither Product 1’s or 2’s price or demand. That is, \( \hat{p}_i^* = p_i^* \), and \( \hat{d}_i^* = d_i^* \), where \( i \in \{1, 2\} \).

(a.2) When \( \bar{q}(\theta) < q \leq \frac{3t}{1+\theta} \), we can verify that \( \hat{p}_i^* > p_i^* \) and \( \hat{d}_i^* < d_i^* \).
(a.3) When $\frac{3t}{1+\theta} < q$, similarly, we can verify that $\hat{p}_i^* > p_i^*$ and $\hat{d}_i^* < d_i^*$ within the region $q \leq 2t$.

(b) The case with $q > 2t$:

(b.1) When $q \leq \frac{3t}{1+\theta}$, we can verify that $\hat{p}_i^* > p_i^*$ and $\hat{d}_i^* < d_i^*$.

(b.2) When $\frac{3t}{1+\theta} < q \leq \frac{19t}{4+2\theta}$, we can verify that $\hat{p}_2^* > p_2^*$ and $\hat{d}_2^* < d_2^*$. We have

$$\hat{p}_1^* - p_1^* = \frac{1}{4}(t + 2q) - (q - t) = \frac{5}{4}t - \frac{q}{2}$$
$$\hat{d}_1^* - d_1^* = \frac{1}{12t}(t + 2q) - \frac{1}{2} = \frac{9}{12}t - \frac{5}{12}$$

Therefore, $\hat{p}_1^* > p_1^*$ if and only if $q \leq \frac{5t}{2}$, and $\hat{d}_1^* < d_1^*$ if and only if $q \leq \frac{5t}{2}$.

(b.3) When $\frac{19t}{4+2\theta} < q$, we can verify that $\hat{p}_2^* > p_2^*$ and $\hat{d}_2^* < d_2^*$. We have

$$\hat{p}_1^* - p_1^* = \frac{1}{6}(8t + q - \theta q) - (q - t) = \frac{1}{6}(14t - 5q - \theta q)$$
$$\hat{d}_1^* - d_1^* = \frac{1}{18t}(8t + q - \theta q) - \frac{1}{2} = \frac{1}{18t}(q - \theta q) - \frac{1}{18}$$

Therefore, $\hat{p}_1^* > p_1^*$ if and only if $q \leq \frac{14t}{5+\theta}$, and $\hat{d}_1^* < d_1^*$ if and only if $q \leq \frac{t}{1+\theta}$.

Altogether, we have the comparison results as in Proposition 1.}

A.4 Proof of Proposition 2

Proof. Based on Lemmas 1 and 2, we compare the firms’ equilibrium profits before and after the introduction of Product 3 for different cases.

(a) The case with $q \leq 2t$:

(a.1) When $q \leq \bar{q}(\theta)$, as shown in Lemma 2, the introduction of Product 3 affects neither Product 1’s nor Product 2’s profit. Therefore, $\hat{\pi}_1^* = \pi_1^*; \hat{\pi}_2^* > \pi_2^*$ because Firm 2 obtains additional revenue from Product 3.

(a.2) When $\bar{q}(\theta) < q \leq \frac{3t}{1+\theta}$, we can verify that $\hat{\pi}_i^* > \pi_i^*$, where $i \in \{1, 2\}$.

(a.3) When $\frac{3t}{1+\theta} < q$, similarly, we can verify that $\hat{\pi}_i^* > \pi_i^*$.

(b) The case with $q > 2t$:

(b.1) When $q \leq \frac{3t}{1+\theta}$, we can verify that $\hat{\pi}_i^* > \pi_i^*$.
(b.2) When \( \frac{3\theta}{1+\theta} < \frac{13\theta}{4+2\theta} \), we can verify that \( \hat{\pi}_1^* \geq \pi_1^* \). We have

\[
\hat{\pi}_2^* - \pi_2^* = \frac{1}{48}\left[-22t^2 + 2(13 + 11\theta)qt - (1 + 10\theta - 3\theta^2)q^2\right] - \frac{1}{2}(q - t)
\]

\[
= \frac{1}{48}(-1 - 10\theta + 3\theta^2)(q - \frac{1 + 11\theta + \sqrt{3 + 42\theta + 115\theta^2}}{1 + 10\theta - 3\theta^2}t)(q - \frac{1 + 11\theta + \sqrt{3 + 42\theta + 115\theta^2}}{1 + 10\theta - 3\theta^2}t)
\]

Because the term in the first bracket is negative and that in the second bracket is positive, \( \hat{\pi}_2^* \geq \pi_2^* \) if and only if \( q \leq \frac{1 + 11\theta + \sqrt{3 + 42\theta + 115\theta^2}}{1 + 10\theta - 3\theta^2}t \).

(b.3) When \( \frac{13\theta}{4+2\theta} < q \), we have

\[
\hat{\pi}_1^* - \pi_1^* = \frac{1}{108}(8t + q - \theta q)^2 - \frac{1}{2}(q - t)
\]

\[
= \frac{1}{108}(1 - \theta)^2(q - \frac{19 + 8\theta + 3\sqrt{3(9 + 20\theta - 2\theta^2)}}{(1 - \theta)^2}t)(q - \frac{19 + 8\theta + 3\sqrt{3(9 + 20\theta - 2\theta^2)}}{(1 - \theta)^2}t)
\]

Because the term in the second bracket is negative, \( \hat{\pi}_1^* \geq \pi_1^* \) if and only if \( q \leq \frac{19 + 8\theta + 3\sqrt{3(9 + 20\theta - 2\theta^2)}}{(1 - \theta)^2}t \).

\[
\hat{\pi}_2^* - \pi_2^* = \frac{1}{432}\left[4(10t - \theta q)^2 + 27(1 - \theta)^2q^2\right] - \frac{1}{2}(q - t)
\]

\[
= \frac{31}{432}(1 - \theta)^2(q - \frac{4(37 - 10\theta) + 6\sqrt{6(13 + 122\theta - 81\theta^2)}}{31(1 - \theta)^2}t)(q - \frac{4(37 - 10\theta) + 6\sqrt{6(13 + 122\theta - 81\theta^2)}}{31(1 - \theta)^2}t)
\]

Because the term in the second bracket is negative, \( \hat{\pi}_2^* \geq \pi_2^* \) if and only if \( q \leq \frac{4(37 - 10\theta) + 6\sqrt{6(13 + 122\theta - 81\theta^2)}}{31(1 - \theta)^2}t \).

Altogether, we have the comparison results as in Proposition 2.

A.5 Proof of Proposition 3

Proof. Combining the conditions in Proposition 2 leads to the result in this proposition. Moreover, by Equation (14), \( \bar{q}(\theta) \) decreases in \( \theta \). We can verify that \( \bar{q}(\theta) \) increases in \( \theta \) if \( \theta \leq \frac{231 + 52\sqrt{33}}{1087} \) but decreases otherwise.

A.6 Proof of Corollary 2

Proof. By Lemma 2, if \( \bar{q}(\theta) < q \leq \frac{3\theta}{1+\theta} \), we can verify that \( \hat{\pi}_1^* \) increases in \( \theta \); If \( \frac{3\theta}{1+\theta} < q \leq \frac{13\theta}{4+2\theta} \), \( \hat{\pi}_1^* \) is independent of \( \theta \); If \( \frac{13\theta}{4+2\theta} < \bar{q}(\theta) \), \( \partial \hat{\pi}_1^*/\partial \theta = -\frac{1}{44t}(8t + q - \theta q) < 0 \). We therefore have the results in Corollary 2.
A.7 Proof of Proposition 4

Proof. In the absence of Product 3, we have

\[
sw = \frac{1}{3} \left( \int_0^{m_{12}} (q - xt) dx + \int_{m_{12}}^1 [q - (1 - x)t] dx \right) + \frac{1}{3} \sum_{i=1}^2 \int_0^{q - x_i} (q - xt) dx, \tag{18}
\]

where \( m_{12} \) is defined in Equation (2). Substituting the equilibrium prices from Equation (5) into Equation (18), we have

\[
sw^* = \begin{cases} 
q - \frac{5}{12} t & \text{if } 2t < q \\
\frac{1}{300} \left[ -29t + 84 \left( q + \frac{q^2}{t} \right) \right] & \text{otherwise}.
\end{cases}
\]

Because \( cs^* = sw^* - \pi_1^* - \pi_2^* \), by substituting in the equilibrium profits from Equation (7), we have

\[
\begin{align*}
cs^* = \begin{cases} 
\frac{7}{12} t & \text{if } 2t < q \\
\frac{1}{300} \left[ -41t + 36 \left( q + \frac{q^2}{t} \right) \right] & \text{otherwise}.
\end{cases}
\end{align*}
\]

Similarly, substituting the equilibrium prices from Equation (11) into Equation (15), we have

\[
\begin{align*}
\hat{sw}^* = \begin{cases} 
-\frac{232t^2 + 101q^2(-1 + \theta)^2 + 32qt(19 + 8\theta)}{864t} & \text{if } \frac{13t}{4 + 2\theta} < q \\
-\frac{33t^2 + 4qt(16 + 11\theta) + q^2(13 + \theta(-26 + 99)\theta))}{96t} & \text{if } \frac{3t}{1 + \theta} < q \leq \frac{13t}{4 + 2\theta} \\
-\frac{6t^2 + 8qt(2 + \theta) + 3q^2(-1 + \theta)^2}{24t} & \text{if } \bar{q}(\theta) < q \leq \frac{3t}{1 + \theta}.
\end{cases}
\end{align*}
\]

By substituting the equilibrium profits from Equation (13) into \( \hat{cs}^* = \hat{sw}^* - \hat{\pi}_1^* - \hat{\pi}_2^* \), we have

\[
\begin{align*}
\hat{cs}^* = \begin{cases} 
-\frac{1544t^2 + 31q^2(-1 + \theta)^2 + 32qt(20 + 7\theta)}{864t} & \text{if } \frac{13t}{4 + 2\theta} < q \\
\frac{4qt + 9t^2 + q^2(7 + 3(-2 + \theta)\theta)}{96t} & \text{if } \frac{3t}{1 + \theta} < q \leq \frac{13t}{4 + 2\theta} \\
\frac{6t^2 - 2qt(-1 + \theta) + q^2(-1 + \theta)^2}{24t} & \text{if } \bar{q}(\theta) < q \leq \frac{3t}{1 + \theta}.
\end{cases}
\end{align*}
\]

We next compare social welfare and consumer surplus within \( \bar{q}(\theta) \leq q \leq \hat{q}(\theta) \).

(a) The case with \( q \leq 2t \):

Electronic copy available at: https://ssrn.com/abstract=4259573
(a.1) When \( \tilde{q}(\theta) < q \leq \frac{3t}{1+\theta}, \)

\[
s\tilde{w}^* - sw^* = \frac{-6t^2 + 8qt(2 + \theta) + q^2(-1 + \theta)^2}{24t} - \frac{1}{300} \left[ -29t + 84(q + \frac{q^2}{7}) \right]
\]

\[
= \left( \frac{(1-\theta)^2}{8t} - \frac{7}{25t} \right) \left( q - \frac{2(58 + 50\theta - 5\sqrt[3]{9} + 49\theta + 169\theta^2)}{3(31 + 50\theta - 25\theta^2)} \right) \left( q - \frac{2(58 + 50\theta + 5\sqrt[3]{9} + 49\theta + 169\theta^2)}{3(31 + 50\theta - 25\theta^2)} \right)
\]

Because the term in the first bracket is negative and the term in second bracket is positive, \( s\tilde{w}^* > sw^* \) if and only if \( q < \frac{2(58 + 50\theta + 5\sqrt[3]{9} + 49\theta + 169\theta^2)}{3(31 + 50\theta - 25\theta^2)} t. \)

\[
\hat{c}s^* - cs^* = \frac{6t^2 - 2q(1 + \theta) + q^2(-1 + \theta)^2}{24t} - \frac{1}{300} \left[ -41t + 36(q + \frac{q^2}{7}) \right]
\]

\[
= \left( \frac{(1-\theta)^2}{24t} - \frac{3}{25t} \right) \left( q - \frac{(11 + 25\theta + 15\sqrt[3]{9} + 54\theta - 23\theta^2)}{47 - 50\theta + 25\theta^2} \right) \left( q - \frac{(11 + 25\theta - 15\sqrt[3]{9} + 54\theta - 23\theta^2)}{47 - 50\theta + 25\theta^2} \right)
\]

Because the term in the first bracket is negative and the term in second bracket is positive, \( \hat{c}s^* > cs^* \) if and only if \( q < \frac{11 + 25\theta - 15\sqrt[3]{9} + 54\theta - 23\theta^2}{47 - 50\theta + 25\theta^2} t. \)

(a.2) When \( \frac{3t}{1+\theta} < q \leq \frac{13t}{4+2\theta}, \) we can verify that \( \hat{c}s^* < cs^* \).

\[
s\tilde{w}^* - sw^* = \frac{-33t^2 + 4qt(16 + 11\theta) + q^2(13 + \theta(-26 + 9\theta))}{96t} - \frac{1}{300} \left[ -29t + 84(q + \frac{q^2}{7}) \right]
\]

\[
= \left( \frac{13 - 26\theta + 9\theta^2}{96t} - \frac{7}{25t} \right) \left( q - \frac{(64 + 55\theta + 5\sqrt[9]{381} + 49\theta + 1743\theta^2)}{347 + 650\theta - 225\theta^2} \right) \left( q - \frac{(64 + 55\theta + 5\sqrt[9]{381} + 49\theta + 1743\theta^2)}{347 + 650\theta - 225\theta^2} \right)
\]

Because the term in the first bracket is negative and the term in second bracket is positive, \( s\tilde{w}^* > sw^* \) if and only if \( q < \frac{64 + 55\theta + 5\sqrt[9]{381} + 49\theta + 1743\theta^2}{347 + 650\theta - 225\theta^2} t. \)

(b) The case with \( q > 2t: \)

(b.1) When \( \tilde{q}(\theta) < q \leq \frac{3t}{1+\theta}, \) we can verify that \( \hat{c}s^* < cs^* \) and \( s\tilde{w}^* < sw^*. \)

(b.2) When \( \frac{3t}{1+\theta} < q \leq \frac{13t}{4+2\theta}, \) we can verify that \( \hat{c}s^* < cs^* \).

\[
s\tilde{w}^* - sw^* = \frac{-33t^2 + 4qt(16 + 11\theta) + q^2(13 + \theta(-26 + 9\theta))}{96t} - \left( q - \frac{5}{12} t \right)
\]

\[
= \left( \frac{13 - 26\theta + 9\theta^2}{96t} \right) \left( q - \frac{16 - 22\theta + \sqrt{165 - 522\theta + 421\theta^2}}{13 - 26\theta + 9\theta^2} \right) \left( q - \frac{16 - 22\theta - \sqrt{165 - 522\theta + 421\theta^2}}{13 - 26\theta + 9\theta^2} \right)
\]

If the term in the first bracket is positive, \( s\tilde{w}^* < sw^*. \) If that term is negative, we can verify that the term in the second bracket is positive, and thus \( s\tilde{w}^* > sw^* \) if and only if \( q < \frac{16 - 22\theta - \sqrt{165 - 522\theta + 421\theta^2}}{13 - 26\theta + 9\theta^2} t. \)

(b.3) When \( \frac{13t}{4+2\theta} < q, \) we can verify that \( \hat{c}s^* < cs^* \).

\[
s\tilde{w}^* - sw^* = \frac{-23t^2 + 101q^2(-1 + \theta)^2 + 32qt(19 + 8\theta)}{864t} - \left( q - \frac{5}{12} t \right)
\]

\[
= \left( \frac{10t(1-\theta)^2}{864t} \right) \left( q - \frac{8(16 + 3\sqrt{3}6)}{101(1-\theta)} \right) \left( q - \frac{8(16 - 3\sqrt{3}6)}{101(1-\theta)} \right)
\]
Because the term in the second bracket is negative, $sw^* > sw^*$ if and only if $q < \frac{8(16-3\sqrt{5})t}{101(1-\theta)^2}$.

Altogether, we have the comparison results as in Proposition 4.
B  Online Appendix A: Proofs of the Results in Section 6

When Firm 1 shares its technology with Firm 3, the equilibrium outcome can be derived as follows.

Lemma 3. If Firm 3 introduces Product 3 into the market, the equilibrium prices of the products are

\[
(p_i^*, p_3^*) = \begin{cases} 
\left( \frac{1}{5}(5t + q - \theta q), \frac{1}{5}(5t - 2q + 2\theta q) \right) & \text{if } \frac{15t}{6+4q} < q \\
\left( \frac{1}{3}(-3t + 3q + \theta q), \frac{2\theta q}{3} \right) & \text{if } \frac{18t}{9+5q} < q \leq \frac{15t}{6+4q} \\
\frac{1}{5}(2q + t), \max \left\{ \frac{1}{5}(-6t + 3q + 5\theta q), \frac{\theta q}{2} \right\} & \text{if } q \geq \frac{18t}{9+5q},
\end{cases}
\]

the equilibrium demands of the products are

\[
(d_i^*, d_3^*) = \begin{cases} 
\left( \frac{1}{15t}(5t + q - \theta q), \frac{1}{15t}(5t - 2q + 2\theta q) \right) & \text{if } \frac{15t}{6+4q} < q \\
\left( \frac{1}{18t}(9t - 2\theta q), \frac{2\theta q}{9t} \right) & \text{if } \frac{18t}{9+5q} < q \leq \frac{15t}{6+4q} \\
\frac{1}{15t}(2q + t), \min \left\{ \frac{1}{5t}(4t - 2q), \frac{\theta q}{3t} \right\} & \text{if } q \geq \frac{18t}{9+5q},
\end{cases}
\]

and the equilibrium profits for the firms are

\[
(\pi_i^*, \pi_3^*) = \begin{cases} 
\left( \frac{1}{5t}(5t + q - \theta q)^2, \frac{1}{5t}(5t - 2q + 2\theta q)^2 \right) & \text{if } \frac{15t}{6+4q} < q \\
\left( \frac{1}{5t}(9t - 2\theta q)(-3t + 3q + \theta q), \frac{4\theta q^2}{27t} \right) & \text{if } \frac{18t}{9+5q} < q \leq \frac{15t}{6+4q} \\
\frac{1}{5t}(2q + t)^2, \max \left\{ \frac{1}{25t}(4t - 2q)(-6t + 3q + 5\theta q), \frac{\theta q}{6t} \right\} & \text{if } q \geq \frac{18t}{9+5q},
\end{cases}
\]

where } i \in \{1, 2\}.

Proof. We distinguish three cases based on the values of q and \( \theta \).

(a) \( \frac{15t}{6+4q} < q \): When } 0 \leq m_{12}, m_{13}, m_{23} \leq 1 \text{ and the marginal consumers all derive positive utilities, the demand functions in Equation (9) are well behaved. We derive the first-order conditions of the firms’ profit functions, } \pi_j = p_j d_j, \text{ where } j \in \{1, 2, 3\}, \text{ as

\[
\begin{align*}
\frac{\partial \pi_1}{\partial p_1} &= \frac{-4p_1 + p_2 + p_3 + (1-\theta)q + 2t}{6t} = 0 \\
\frac{\partial \pi_2}{\partial p_2} &= \frac{p_1 - 4p_2 + p_3 + (1-\theta)q + 2t}{6t} = 0 \\
\frac{\partial \pi_3}{\partial p_3} &= \frac{p_1 + p_2 - 4p_3 - 2(1-\theta)q + 2t}{6t} = 0
\end{align*}
\]

Solving this system of equations, we can derive } p_j^* \text{ as specified in the lemma’s first case. We can verify that } 0 \leq m_{12}, m_{13}, m_{23} \leq 1, \text{ marginal consumers } m_{13} \text{ and } m_{23} \text{ derive positive utilities under
the condition \( \frac{15t}{\theta+5t} < q \), and marginal consumer \( m_{12} \) also derives positive utility.

(b) \( \frac{18t}{\theta+5t} < q \leq \frac{15t}{\theta+4t} \): In this case, marginal consumers \( m_{13} \) and \( m_{23} \) derive zero utility because \( q \leq \frac{15t}{\theta+4t} \). When Firms 1 and 2 price their products such that marginal consumers \( m_{13} \) and \( m_{23} \) derive zero utilities, \( \frac{q-p_1}{t} + \frac{\theta q-p_3}{t} = 1 \) and \( \frac{q-p_2}{t} + \frac{\theta q-p_3}{t} = 1 \), or, equivalently, \( p_1 + p_3 = q + \theta q - t \) and \( p_2 + p_3 = q + \theta q - t \). Using the Lagrange-multiplier method, we have \( L_1(p_1, \lambda) = \pi_1 + \lambda_1(q + \theta q - t - p_1 - p_3) \) and \( L_2(p_2, \lambda) = \pi_2 + \lambda_2(q + \theta q - t - p_2 - p_3) \). By solving the following system of equations,

\[
\begin{align*}
\frac{\partial L_1}{\partial p_1} &= \frac{-4p_1+p_2+p_3+(1-\theta)q+2t}{6t} - \lambda_1 = 0 \\
\frac{\partial L_2}{\partial p_2} &= \frac{p_1-4p_2+p_3+(1-\theta)q+2t}{6t} - \lambda_2 = 0 \\
\frac{\partial L_3}{\partial p_3} &= \frac{p_1+p_2-4p_3-2(1-\theta)q+2t}{6t} = 0 \\
q + \theta q - t - p_1 - p_3 &= 0 \\
q + \theta q - t - p_2 - p_3 &= 0
\end{align*}
\]

we can derive \( (p_1^*, p_2^*, p_3^*) \) as specified in the lemma’s second case. The condition \( \frac{18t}{\theta+5t} < q \) ensures that neither Firm 1 nor 2 has incentive to increase product prices, and condition \( q \leq \frac{15t}{\theta+4t} \) ensures that neither Firm 1 nor 2 has incentive to decrease product prices. We can verify that marginal consumer \( m_{12} \) derives positive utility, \( 0 < m_{12}, m_{13}, m_{23} \leq 1 \), and Firm 3 has no incentive to deviate from \( p_3^* \) under these two conditions.

(c) \( q \leq \frac{18t}{\theta+5t} \): In this case, Firms 1 and 2 compete with each other but they do not compete against Firm 3. Consequently, as in case (a) in the proof of Lemma 1, \( p_1^* = p_2^* = \frac{1}{5}(2q + t) \). Firm 3 either charges \( p_3^* = \frac{1}{4}(-6t + 3q + 5\theta q) \) to cover all of the residual demands in Submarkets 13 and 23 or charges a monopoly price \( p_3^* = \frac{\theta q}{2} \) for Product 3, whichever is higher. We can verify that no firm has incentive to deviate.

Substituting \( p_j^* \) into the demand functions and the firms’ profit functions, we can derive the equilibrium demands and profits as specified in the lemma.

In the following, we use regular notations (e.g., \( p_1^* \)) for the equilibrium outcome in the absence of the new product and the notations with hats and primes (e.g., \( \hat{p}_1^* \)) for the equilibrium outcome in the presence of the new product.
B.1 Proof of Proposition 5

Proof. Based on Lemmas 1 and 3, we compare the equilibrium profit of Firm 1 before and after the introduction of Product 3.

(a) The case with \( q \leq 2t \):

(a.1) When \( q \leq \hat{q}'(\theta) = \frac{18t}{9+3\theta} \), as shown in Lemma 3, the introduction of Product 3 does not affect Product 1’s profit. Therefore, \( \hat{\pi}'_1^* = \pi_1^* \).

(a.2) When \( \hat{q}'(\theta) < q \leq \frac{15t}{6+4\theta} \), we can verify that \( \hat{\pi}'_1^* \geq \pi_1^* \).

(a.3) When \( \frac{15t}{6+4\theta} < q \),

\[
\hat{\pi}'_1^* - \pi_1^* = \frac{1}{75t} (5t + q - \theta q)^2 - \frac{1}{50t} (2q + t)^2 \\
= \left[ \frac{1}{75t} (1 - \theta)^2 - \frac{2}{25t} \right] \left( q - \frac{(4-10\theta+\sqrt{6}(9+\theta))t}{2(5+2\theta-\theta^2)} \right)^2 \\
= \left[ \frac{1}{75t} (1 - \theta)^2 - \frac{2}{25t} \right] \left( q - \frac{(4-10\theta+\sqrt{6}(9+\theta))t}{2(5+2\theta-\theta^2)} \right)
\]

Because the term in the first bracket is negative and we can verify that the term in the second bracket is positive, \( \hat{\pi}'_1^* \geq \pi_1^* \) if and only if \( q \leq \frac{(4-10\theta+\sqrt{6}(9+\theta))t}{2(5+2\theta-\theta^2)} \).

(b) The case with \( q > 2t \):

\[
\hat{\pi}'_1^* - \pi_1^* = \frac{1}{75t} (5t + q - \theta q)^2 - \frac{1}{2}(q - t) \\
= \frac{1}{75t} (1 - \theta)^2 \left( q - \frac{(55+20\theta+5\sqrt{3(27+56\theta-8\theta^2)})t}{4(1-\theta)^2} \right) \left( q - \frac{(55+20\theta+5\sqrt{3(27+56\theta-8\theta^2)})t}{4(1-\theta)^2} \right)
\]

Because the term in the second bracket is negative, \( \hat{\pi}'_1^* \geq \pi_1^* \) if and only if \( q \leq \frac{(55+20\theta+5\sqrt{3(27+56\theta-8\theta^2)})t}{4(1-\theta)^2} \).

Altogether, we have the results as in Proposition 5.

B.2 Proof of Proposition 6

Proof. Combining the conditions in Propositions 3 and 5 leads to the results in this proposition.
C. Online Appendix B: Proof of the Results for Benchmark Cases

We first present the equilibrium results for the two benchmarks and then prove Corollary 1.

C.1 Equilibrium for Benchmark with Cannibalization Only

**Lemma 4.** In the benchmark with cannibalization only, the equilibrium prices of the products \((p_1^*, p_2^*, p_3^*)\) are

\[
\begin{cases}
(q - t, \frac{1}{79} (7q - 4t + 2\theta q), \frac{1}{110} (3q - 6t + 8\theta q)) & \text{if } \frac{66t}{33-29} < q \\
(\frac{1}{35} (26q + 7t + 2\theta q), \frac{1}{79} (38q - 17t + 12\theta q), \frac{1}{59} (21q - 42t + 47\theta q)) & \text{if } \max\{\frac{12t}{6+59}, \frac{49t}{54-149}\} < q \leq \frac{66t}{33-29} \\
(\frac{1}{7}, \frac{1}{7} (10q - 7t), \frac{1}{7} (-3q + 7\theta q)) & \text{if } q \leq \frac{49t}{54-149}
\end{cases}
\]

the equilibrium demands of the products \((d_1^*, d_2^*, d_3^*)\) are

\[
\begin{cases}
\left(\frac{1}{60t} (-3q + 36t + 2\theta q), \frac{1}{20t} (3q + 4t - 2\theta q), \frac{1}{10t} (-3q + 6t + 2\theta q)\right) & \text{if } \frac{66t}{33-29} < q \\
\left(\frac{1}{118t} (26q + 7t + 2\theta q), \frac{1}{354t} (30q + 117t - 34\theta q), \frac{1}{59t} (-7q + 14t + 4\theta q)\right) & \text{if } \max\{\frac{12t}{6+59}, \frac{49t}{54-149}\} < q \leq \frac{66t}{33-29} \\
\left(\frac{2q}{77}, \frac{2}{3}, -\frac{2q}{77}, \frac{q}{77}\right) & \text{if } q \leq \frac{49t}{54-149}
\end{cases}
\]

and the equilibrium profits for the two firms \((\pi_1^*, \pi_2^*)\) are

\[
\begin{cases}
\left(\frac{q-t}{60t} (-3q+36t+2\theta q), -\frac{24t^2+q^2(3-2\theta)^2+24q(1+\theta)}{120t}\right) & \text{if } \frac{66t}{33-29} < q \\
\left(\frac{26q+7t+2\theta q}{6962t}, -\frac{5517t^2+2q^2(-1-9\theta)(-129+40\theta)+2q(3732+2461\theta)}{20886t}\right) & \text{if } \max\{\frac{12t}{6+59}, \frac{49t}{54-149}\} < q \leq \frac{66t}{33-29} \\
\left(\frac{8q^2}{39t}, \frac{1274q^2-686q^2(23+7\theta)}{1029t}\right) & \text{if } q \leq \frac{49t}{54-149}
\end{cases}
\]

**Proof.** When Product 3 serves only Submarket \(23\) (but not Submarket \(13\)), because no competition exists between Products 1 and 3, Firm 2 has incentive to charge high prices for Products 2 and 3 such that either the marginal consumer \(m_{23}\) derives zero utility or Submarket \(23\) is not fully covered. We distinguish four cases for the interfirm competition.

(a) \(\frac{66t}{33-29} < q\): When \(q\) is large, all consumers in Submarket \(13\) purchase. Firm 1 has incentive to charge a price high enough such that the consumer with the highest misfit cost in Submarket \(13\) derive zero utility (i.e., \(p_1^* = q - t\)), while Firm 2 chooses the optimal prices for Products 2 and 3 to compete against Product 1 and to just fully serve Submarket \(23\). When \(0 \leq m_{12}, m_{23} \leq 1\) and
marginal consumer $m_{12}$ derives positive utility, the demand functions for the products are

$$
\begin{align*}
  d_1 &= \frac{1}{3} m_{12} + \frac{1}{3} \\
  d_2 &= \frac{1}{3} (1 - m_{12}) + \frac{1}{3} m_{23} \\
  d_3 &= \frac{1}{3} (1 - m_{23})
\end{align*}
$$

Because marginal consumer $m_{23}$ derives zero utility, $\frac{2-p_3}{t} + \frac{\theta q - p_2}{t} = 1$, or, equivalently, $p_2 + p_3 = q + \theta q - t$. Using the Lagrange-multiplier method, we have $L_2(p_2, p_3, \lambda) = \pi_2 + \lambda(q + \theta q - t - p_2 - p_3)$—in which $\pi_2 = p_2 d_2 + p_3 d_3$—and $\pi_1 = p_1^* d_1$. By the first-order conditions for $L_2(p_2, p_3, \lambda)$, we have

$$
\begin{align*}
  \frac{\partial L_2}{\partial p_2} &= \frac{-4p_2 + p_1^* + 2p_3 + (1-\theta)q + 2t}{6t} - \lambda = 0 \\
  \frac{\partial L_2}{\partial p_3} &= \frac{2p_2 - 2p_3 - (1-\theta)q + t}{6t} - \lambda = 0 \\
  q + \theta q - t - p_2 - p_3 &= 0
\end{align*}
$$

Solving the above system of equations, we have $p_2^* = \frac{1}{10} (7q - 4t + 2q)$ and $p_3^* = \frac{1}{10} (3q - 6t + 8\theta q)$. We can verify that $0 \leq m_{12}, m_{23} \leq 1$ within this region, which also ensures that the marginal consumer located at $m_{12}$ derives positive utility. We can verify that neither firm has profitable deviation.

(b) $\max\{\frac{12t + 5q}{54 + 14q}, \frac{49t}{33 - 2q}\} < q \leq \frac{66t}{33 - 2q}$: In this case, because the value of $q$ is not large enough (i.e., $q \leq \frac{66t}{33 - 2q}$), Firm 1 has incentive to charge a price higher than $q - t$ to only partially serve Submarket 13. Therefore, the demand functions for the three products are

$$
\begin{align*}
  d_1 &= \frac{1}{3} m_{12} + \frac{1}{3} \frac{q-p_1}{t} \\
  d_2 &= \frac{1}{3} (1 - m_{12}) + \frac{1}{3} m_{23} \\
  d_3 &= \frac{1}{3} (1 - m_{23})
\end{align*}
$$

(19)

By the first-order conditions for $\pi_1 = p_1 d_1$ and $L_2(p_2, p_3, \lambda)$, we have

$$
\begin{align*}
  \frac{\partial \pi_1}{\partial p_1} &= \frac{t - 6p_1 + p_2 + 2q}{6t} = 0 \\
  \frac{\partial L_2}{\partial p_2} &= \frac{-4p_2 + p_1 + 2p_3 + (1-\theta)q + 2t}{6t} - \lambda = 0 \\
  \frac{\partial L_2}{\partial p_3} &= \frac{2p_2 - 2p_3 - (1-\theta)q + t}{6t} - \lambda = 0 \\
  q + \theta q - t - p_2 - p_3 &= 0
\end{align*}
$$
Solving this system of equations, we can derive \( p_1^* = \frac{1}{39} (26q + 7t + 2\theta q) \), \( p_2^* = \frac{1}{39} (38q - 17t + 12\theta q) \), and \( p_3^* = \frac{1}{39} (21q - 42t + 47\theta q) \). We can verify that \( 0 \leq m_{12}, m_{23} \leq 1 \) and \( 0 \leq \frac{q - p^*_1}{t} \leq 1 \) within this region. The condition \( \frac{49t}{54 + 2q} < q \) ensures that the marginal consumer located at \( m_{12} \) derives positive utility. We can verify that the condition \( \frac{12q}{6 + 5\theta} < q \) ensures that Firm 2 has no incentive to increase the price of Product 2 or Product 3 to have Submarket 23 not fully covered.

(c) \( q \leq \frac{49t}{54 + 2q} \): Because \( q \leq \frac{49t}{54 + 2q} \), marginal consumer \( m_{12} \) derives zero utility, \( \frac{q - p_1}{t} + \frac{q - p_2}{t} = 1 \), or, equivalently, \( p_1 + p_2 = 2q - t \). Using the Lagrange-multiplier method, we have \( L_2(p_2, p_3, \lambda_1, \lambda_2) = \pi_2 + \pi_1(2q - t - p_1 - p_2) + \lambda_2(q + \theta q - t - p_2 - p_3) \) in which \( \pi_2 = p_2d_2 + p_3d_3 \), and \( \pi_1 = p_1d_1 \), with \( d_1, d_2, \) and \( d_3 \) specified as in Equation (19). By solving the following first-order conditions for \( \pi_1 \) and \( L_2(p_2, p_3, \lambda_1, \lambda_2) \),

\[
\begin{align*}
\frac{\partial \pi_1}{\partial p_1} &= \frac{t - 6p_1 + p_2 + 2q}{6t} = 0 \\
\frac{\partial L_2}{\partial p_2} &= -4p_2 + p_1 + 2p_3 + (1 - \theta)q + 2t - \lambda_1 - \lambda_2 = 0 \\
\frac{\partial L_2}{\partial p_3} &= 2p_2 - 2p_3 - (1 - \theta)q - t - \lambda_2 = 0 \\
q + \theta q - t - p_2 - p_3 &= 0 \\
2q - t - p_1 - p_2 &= 0
\end{align*}
\]

we can derive \( p_1^* = \frac{4}{7} q \), \( p_2^* = \frac{1}{7} (10q - 7t) \), and \( p_3^* = \frac{1}{7} (-3q + 7\theta q) \). We can verify that \( 0 \leq m_{12}, m_{23} \leq 1 \) and \( 0 \leq \frac{q - p^*_1}{t} \leq 1 \) within this region, and neither firm has profitable deviation.

(d) \( q \leq \frac{12q}{6 + 5\theta} \): In this case, Firm 2 charge prices for Product 2 and Product 3 such that Submarket 23 is not fully covered, and thus Submarket 23 is no longer competitive.

Substituting the equilibrium prices into the corresponding demand and profit functions, we can derive the equilibrium demands and profits as in the lemma. \( \square \)

### C.2 Equilibrium for Benchmark with Additional Competition Only

**Lemma 5.** In the benchmark with additional competition only, the equilibrium prices of the products \( (p_1^*, p_2^*, p_3^*) \) are

\[
\begin{align*}
&\begin{cases}
(\frac{1}{7} (3q + 3t - \theta q), q - t, \frac{1}{7} (-2q + 5t + 3\theta q)) & \text{if } \frac{52t}{25 + \theta} < q \\
(\frac{1}{20} (5q + 16t - 3\theta q), \frac{1}{10} (15q + 12t - \theta q), \frac{1}{40} (-15q + 36t + 17\theta q)) & \text{if } \frac{108t}{35 + 29\theta} < q \leq \frac{52t}{25 + \theta} \\
(\frac{1}{3} (3q - 3t + \theta q), \frac{1}{18} (9q + 9\theta q), \frac{2}{3} \theta q) & \text{if } \frac{18t}{9 + 5\theta} < q \leq \frac{108t}{35 + 29\theta}
\end{cases}
\end{align*}
\]
the equilibrium demands of the products \((d_1^*, d_2^*, d_3^*)\) are

\[
\begin{align*}
&\begin{cases}
\frac{1}{2t} (3q + 3t - \theta q), \frac{1}{2t} (31t - 4q - \theta q), \frac{1}{2t} (-2q + 5t + 3\theta q) & \text{if } \frac{52t}{25+\theta} < q \\
\frac{1}{60t} (5q + 16t - 3\theta q), \frac{1}{80t} (15q + 12t - \theta q), \frac{1}{20t} (-15q + 36t + 17\theta q) & \text{if } \frac{108t}{45+29\theta} < q \leq \frac{52t}{25+\theta} \\
\frac{1}{108t} (-9q + 72t - 17\theta q), \frac{1}{96t} (9q + \theta q), \frac{1}{18t} (\theta q) & \text{if } \frac{18t}{9+5\theta} < q \leq \frac{108t}{45+29\theta}
\end{cases}
\end{align*}
\]

and the equilibrium profits for the two firms \((\pi_1^*, \pi_2^*)\) are

\[
\begin{align*}
&\begin{cases}
\frac{1}{117t} (3q + 3t - \theta q)^2, -192t^2 + q^2(-3+\theta)(8+9\theta)+qt(225+37\theta) & \text{if } \frac{52t}{25+\theta} < q \\
\frac{1}{1200t} (5q + 16t - 3\theta q)^2, \frac{432t^2+q^2(225+\theta(-150+73\theta))+288q\theta}{2400t} & \text{if } \frac{108t}{45+29\theta} < q \leq \frac{52t}{25+\theta} \\
\frac{1}{324t} (-3q+3t-\theta q)(-72t+9q+17\theta q), q^2(81+18\theta+49\theta^2) & \text{if } \frac{18t}{9+5\theta} < q \leq \frac{108t}{45+29\theta}
\end{cases}
\end{align*}
\]

Proof. We distinguish four cases for the firms’ competition.

(a) \(\frac{52t}{25+\theta} < q\): When \(q\) is large, all consumers in Submarket 23 purchase. Firm 2 has incentive to charge a price high enough for Product 2 such that the consumer with the highest misfit cost in Submarket 23 derives zero utility; that is, \(p_2^* = q - t\). When \(0 \leq m_{12}, m_{13} \leq 1\) and the marginal consumers \(m_{12}\) and \(m_{13}\) derive positive utilities, the demand functions for the three products are

\[
\begin{align*}
&d_1 = \frac{1}{3} m_{12} + \frac{1}{3} m_{13} \\
&d_2 = \frac{1}{3} (1 - m_{12}) + \frac{1}{3} \\
&d_3 = \frac{1}{3} (1 - m_{13})
\end{align*}
\]

By the first-order conditions for the firms’ profit functions (i.e., \(\pi_1 = p_1d_1\) and \(\pi_2 = p_2d_2 + p_3d_3\), we get

\[
\begin{align*}
&\frac{\partial \pi_1}{\partial p_1} = \frac{-4p_1 + p_2^* + p_3 + (1-\theta)q + 2t}{6t} = 0 \\
&\frac{\partial \pi_2}{\partial p_2} = \frac{p_2 - 2p_3 - (1-\theta)q + t}{6t} = 0
\end{align*}
\]

Solving this system of equations, we can derive \(p_1^* = \frac{1}{t}(3q + 3t - \theta q)\) and \(p_3^* = \frac{1}{t}(-2q + 5t + 3\theta q)\).

We can verify that \(0 \leq m_{12}, m_{13} \leq 1\) within this region, which also ensures that the marginal consumers located at \(m_{12}\) and \(m_{13}\) both derive positive utility. We can verify that neither firm has profitable deviation.

(b) \(\frac{108t}{45+29\theta} < q \leq \frac{52t}{25+\theta}\): In this case, the value of \(q\) is not large enough, and Firm 2 has incentive to charge a price higher than \(q - t\) for Product 2 to only partially serve Submarket 23. Therefore,
By the first-order conditions for the firms’ profit functions, we have

\[
\begin{align*}
\frac{\partial \pi_1}{\partial p_1} &= -\frac{4p_1 + p_2 + p_3 + (1-\theta)q + 2t}{6t} = 0 \\
\frac{\partial \pi_2}{\partial p_2} &= \frac{t - 6p_2 + p_1 + 2q}{6t} = 0 \\
\frac{\partial \pi_2}{\partial p_3} &= \frac{p_3 - 2p_3 - (1-\theta)q + t}{6t} = 0
\end{align*}
\]

Solving this system of equations, we can derive \( p_1^* = \frac{1}{20} (5q + 16t - 3\theta q) \), \( p_2^* = \frac{1}{40} (15q + 12t - \theta q) \), and \( p_3^* = \frac{1}{40} (-15q + 36t + 17\theta q) \). We can verify that \( 0 \leq m_{12}, m_{13} \leq 1 \) and \( 0 \leq \frac{q-p_3^*}{t} \leq 1 \) within this region, which also ensures that the marginal consumers located at \( m_{12} \) and \( m_{13} \) both derive positive utility. We can verify that neither firm has profitable deviation.

(c) \( \frac{18t}{9+5\theta} < q \leq \frac{108t}{45+29\theta} \). Because \( q \leq \frac{108t}{45+29\theta} \), marginal consumer \( m_{13} \) derives zero utility, \( \frac{q-p_1}{t} + \frac{\theta q - p_1}{t} = 1 \), or, equivalently, \( p_1 + p_3 = q + \theta q - t \). Using the Lagrange-multiplier method, we have \( L_1(p_1, \lambda) = \pi_1 + \lambda(q + \theta q - t - p_1 - p_3) \) in which \( \pi_1 = p_1d_1 \), and \( \pi_2 = p_2d_2 + p_3d_3 \), with \( d_1, d_2, \) and \( d_3 \) specified as in Equation (20). By solving the the first-order conditions for \( L_1(p_1, \lambda) \) and \( \pi_2 \),

\[
\begin{align*}
\frac{\partial L_1}{\partial p_1} &= -\frac{4p_1 + p_2 + p_3 + (1-\theta)q + 2t}{6t} - \lambda = 0 \\
\frac{\partial L_1}{\partial p_2} &= \frac{t - 6p_2 + p_1 + 2q}{6t} = 0 \\
\frac{\partial L_1}{\partial p_3} &= \frac{p_3 - 2p_3 - (1-\theta)q + t}{6t} = 0 \\
q + \theta q - t - p_1 - p_3 &= 0
\end{align*}
\]

we can derive \( p_1^* = \frac{1}{3} (3q - 3t + \theta q) \), \( p_2^* = \frac{1}{18} (9q + \theta q) \), and \( p_3^* = \frac{2}{3} \theta q \). We can verify that \( 0 \leq m_{12}, m_{13} \leq 1 \) and \( 0 \leq \frac{q-p_3^*}{t} \leq 1 \) within this region, which also ensures that the marginal consumer located at \( m_{12} \) derives positive utility. We can verify that neither firm has profitable deviation.

(d) \( q \leq \frac{18t}{9+5\theta} \): In this case, the two firms compete in Submarket \( \bar{I}_2 \), but Submarket \( \bar{I}_3 \) is no longer competitive.

Substituting the equilibrium prices into the corresponding demand and profit functions, we can
derive the equilibrium demands and profits as specified in the lemma.

C.3 Proof of Corollary 1

Proof. (a) Based on Lemmas 1 and 4, we can verify that both Product 2’s price and Firm 2’s profit become higher after the introduction of Product 3.

(b) Based on Lemmas 1 and 5, when \( q \leq 2t \), after the introduction of Product 3, if \( \frac{108t}{45+29\theta} < q \leq \frac{52t}{25+\theta} \), we can verify that Product 1’s price increases but Firm 1’s profit decreases. If \( \frac{18t}{9+5\theta} < q \leq \frac{108t}{45+29\theta} \), we can verify that Product 1’s price increases. The difference between Firm 1’s profits with and without Product 3’s entry is

\[
\frac{(-3q+3t-\theta q)(-72t+9q+176\theta q)}{324t} - \frac{1}{50t} (2q + t)^2 = \left[ \frac{(-3-\theta)(9+17\theta)}{324t} - \frac{4}{50t} \right] \left( q - \frac{18t}{9+5\theta} \right) \left( q - \frac{309t}{147+85\theta} \right)
\]

Because the term in the first bracket is negative and the term in the second bracket is positive, Firm 1’s profit increases with the entry of Product 3 if and only if \( q \leq \frac{309t}{147+85\theta} \).

When \( q > 2t \), if \( \frac{108t}{45+29\theta} < q \leq \frac{52t}{25+\theta} \), we can verify that the price of Product 1 increases but Firm 1’s profit decreases. If \( q > \frac{52t}{25+\theta} \), the difference between the prices of Product 1 with and without Product 3 is

\[
\frac{1}{7} (3q + 3t - \theta q) - (q - t) = \frac{1}{7} (-4q + 10t - \theta q).
\]

Therefore, the price of Product 1 increases if and only if \( q \leq \frac{10t}{4+\theta} \). Moreover, we can verify that Firm 1’s profit decreases.

Altogether, we have the comparison results as in Corollary 1. \( \square \)