This paper studies surprise clearance as an innovative business model to increase store profit and reduce food waste. A store holding surprise clearance sells “surprise bags” composed of surplus food that would otherwise go to waste. At the time of ordering, consumers are uncertain about the quantity of food items included in a surprise bag. We model surprise clearance and compare it with both no clearance and transparent clearance, which sets a transparent unit clearance price based on the amount of surplus food after regular sales.

We find that surprise clearance achieves the highest store profit and induces the most store production among the three selling schemes. While both clearance schemes have the ability to eliminate store waste, the store effectively does so under surprise clearance but deliberately does not under transparent clearance. In fact, transparent clearance may generate even more store waste than no clearance. Further, both clearance schemes exacerbate the problem of consumer waste compared to no clearance, and despite zero store waste, surprise clearance generates the most consumer waste among the three schemes. If clearance sales target a consumer segment with a sufficiently low valuation of consumption, both clearance schemes reduce total food waste and surprise clearance represents a win-win solution that maximizes store profit and minimizes total waste among the three schemes. However, there are circumstances under which both clearance schemes fail to reduce total waste, and surprise clearance may end up with the most total waste among the three schemes.

Key words: food waste, inventory, clearance sales, surprise bag, quantity uncertainty
grocers, restaurants, and bakeries) is almost 40% (Gunders and Bloom 2017). To tackle food waste at the end stages, innovative, technology-based, and market-driven solutions have been proposed and implemented in recent years. Our paper centers around one such innovation: Too Good to Go.

Too Good to Go (TGTG) is a mobile-application-based service that aims to reduce food waste by connecting consumers to stores and restaurants with surplus food that would otherwise be sent to landfills. Launched in Denmark in 2015, TGTG has since expanded to more than 17 countries, including the UK, France, Germany, and the US, amassing more than 62 million users worldwide (Washington Post 2022). A key ingenuity of TGTG is that it allows stores to sell “surprise bags” composed of leftover food. Consumers pre-order surprise bags in advance and pick up their orders at a specified time window (New York Times 2022), often at the end of the business day. Importantly, at the time of ordering, consumers do not know exactly “the amount of food inside each bag” (McLean 2021) and may be “surprised” by the bag content (e.g., Figure 1 shows that one app user gets as many as 36 bagels in a surprise bag). The quantity of food items a surprise bag contains may vary depending on how much surplus food is available on a given day at the store. Since TGTG enables clearance sales with a surprise element, we refer to it as surprise clearance.

![An Example of the Content of a Surprise Bag](https://t.ly/LBiUW)

*Note.* A TGTG user posted on Reddit that a surprise bag from a bagel shop in Philadelphia contained 36 bagels. Source: [https://t.ly/LBiUW](https://t.ly/LBiUW)

Other food waste apps, such as Flashfood (in North America) and Karma (in Europe), also enable stores to sell surplus food at a discounted price, but unlike TGTG, they take a more classical approach to clearance and put emphasis on “transparency” (Lamb 2018). Stores use these apps to set a discounted price for each unsold food item, possibly based on the amount of leftover inventory.
Users browsing the app observe the unit price for each product and decide for themselves how much to purchase. We refer to such a scheme as transparent clearance.

Advocates of these (innovative) clearance schemes argue that they reduce food waste, boost store profit, feed the food-insecure, and fill a unique niche by selling small volumes of perishable food typically not suitable for donations to food banks (New York Times 2022, Washington Post 2022). However, skeptics raise the question of whether food sold through clearance sales is truly “a leftover or effectively a planned production” (Yu et al. 2022), especially in the case of surprise clearance, where consumers reserve surprise bags in advance. Further, skeptics doubt whether these clearance schemes can address food waste from consumers who face uncertain consumption needs and cannot do perfect meal planning (Washington Post 2022). These competing views beg our research questions: Will these clearance schemes be effective in reducing food waste (including both store waste and consumer waste) when stores strategically determine production quantities? Which clearance scheme is better at food waste reduction (if at all) and which one is more profitable?

To address these questions, we develop an analytical modeling framework of three selling schemes: no clearance, transparent clearance, and surprise clearance. In our model, a profit-maximizing store selling perishable food products determines in advance how much to produce when facing uncertainty about the market size. The market comprises two types of utility-maximizing consumers with uncertain consumption needs. High-end consumers decide how much to purchase in the regular phase, whereas low-end consumers cannot afford to purchase in the regular phase and may only order if there is a discount in the clearance phase. Under transparent clearance, the store sets the (discounted) unit clearance price based on the amount of leftover inventory after regular sales, and low-end consumers determine how much to purchase. Under surprise clearance, the store sets the price for the surprise bag in advance, and low-end consumers determine whether to pre-order a surprise bag without knowing the exact amount of food it may contain. After regular sales, if there is no inventory left, all surprise-bag orders are canceled and refunded. Otherwise, the store evenly distributes the leftover inventory among the surprise bags ordered before consumers pick them up. Due to the perishable nature of the food product, any inventory the store produces but does not sell (after clearance) represents store waste, and any units consumers purchase but do not consume represent consumer waste. Total food waste is the sum of store waste and consumer waste.

Our three-way comparison shows that both clearance schemes boost store profit (relative to no clearance) and surprise clearance further outperforms the transparent clearance in profit even though it lacks the pricing flexibility of transparent clearance. While the surprise-bag price is fixed in advance, surprise clearance implements contingent pricing “in disguise.” If there is much (little) leftover inventory after regular sales, the store allocates a large (small) amount to each surprise bag, which implies that each product unit in the surprise bag is effectively sold at a low (high)
price. As a result, surprise clearance mimics transparent clearance in that the unit price in surprise clearance is essentially also contingent on the amount of leftover inventory after regular sales. Besides the hidden benefit of contingent pricing, surprise clearance also has an extra advantage of implementing *quantity discounts* “in disguise.” In transparent clearance, the store charges a constant unit clearance price regardless of a consumer’s purchase quantity. However, it fails to account for consumers’ diminishing marginal utility (i.e., the utility of consuming an extra unit declines as the total consumption amount increases), thereby limiting the store’s ability to extract consumer surplus. By contrast, in surprise clearance, the price of a surprise bag can be calibrated to consumers’ expected valuation of the bag, allowing the store to extract more consumer surplus. Hence, surprise clearance mimics an outcome that would be achieved by quantity discounts, where the unit clearance price decreases in the quantity purchased in a way that perfectly matches consumers’ diminishing marginal utility. We find that because surprise clearance is able to extract more value from leftover inventory, it also spawns more store production than transparent clearance, which itself prompts the store to produce more than no clearance.

In addition to the profit advantage, surprise clearance also completely eliminates store waste by packing all leftover inventory into surprise bags, which maximizes the sale value of a surprise bag. By contrast, transparent clearance may fail to achieve zero store waste even though it has the ability to do so, because clearing the leftover inventory when there is plenty would require a clearance price that is too low to be optimal for the store. In fact, when both the production cost and low-end consumers’ consumption valuation are high, transparent clearance generates even more store waste than no clearance. In this case, transparent clearance induces the store to produce more but reluctant to charge a low enough clearance price to get rid of the increased leftover inventory, causing store waste to climb. While the two clearance schemes may have varying success in reducing store waste, both increase consumer waste. Clearance sales transfer more inventory from the store to consumers, which ultimately leads to more consumer waste. In fact, surprise clearance generates the most consumer waste (although it boasts the least store waste) among the three schemes because it engenders the most inventory and transfers all of it to consumers.

The possibility that store waste and consumer waste move in opposite directions complicates the comparison of total waste. Nevertheless, we show that relative to no clearance, both clearance schemes reduce total waste when low-end consumers have low valuation of consumption. In such a case, the revenue potential of clearance sales is limited by low-end consumers’ weak purchasing power and thus the production quantity is not much different across the three schemes. Further, an unsold unit is guaranteed to be wasted but a purchased unit is not. Hence, the more inventory a scheme is able to transfer from the store to consumers, the less total waste it generates. It follows that in this case, both clearance schemes generate less total waste than no clearance, and surprise
clearance is the one with the least total waste. Thus, surprise clearance can be a “win-win” solution that simultaneously maximizes store profit and minimizes total waste among the three schemes.

However, both clearance schemes can backfire and generate more total waste than no clearance, and surprise clearance can even generate the most total waste among the three schemes. This happens when the production cost is high, in which case, the production quantity is limited across the schemes and most inventory is thus transferred to high-end consumers during regular sales. Hence, total waste is predominated by high-end consumers’ food waste. Since introducing clearance sales spurs production, more high-end demand is fulfilled, which eventually translates into more food waste from high-end consumers. As a consequence, surprise clearance, by inducing the most store production, also leads to the highest total waste in such a case.

Our paper presents the first analysis of surprise clearance’s profit and food-waste performance relative to transparent clearance and no clearance. We show that while this novel scheme can be a promising “win-win” solution, it may also fail (miserably) in the mission to reduce food waste, despite its deceiving success in eliminating store waste.

2. Literature

Our paper contributes to the literature on food waste, perishable inventory, and opaque selling.

**Food Waste.** There is a growing literature on food waste in operations management (OM) and related fields. Akka¸s and Gaur (2022) present an OM research agenda on this topic. Lee and Tongarläk (2017) propose a by-product strategy to mitigate food waste in a retail grocer setting. Akka¸s and Honhon (2022) and Akka¸s and Honhon (2023) study shipment policies for shelf-life and food-waste management. They demonstrate that optimizing shipping age thresholds can drastically reduce food waste. Crama et al. (2023), Kazaz et al. (2023), and Han et al. (2023) study retail ugly produce and its impact on food waste. Keskin et al. (2023) show that blockchain-enabled freshness transparency can increase a fresh produce retailer’s profit and reduce food waste. Lu (2023) shows that the adoption of sensor technologies to monitor spoilage conditions may unintentionally increase food waste by inducing a larger purchase quantity. Sanders (2023) empirically demonstrates the value of dynamic pricing in reducing grocery-store waste.

To the best of our knowledge, we are the first to formally study surprise clearance as a novel scheme to reduce food waste. Similar to Sanders (2023), our research shows that clearance sales (either transparent clearance or surprise clearance) can reduce store waste, but different from Sanders (2023), we also consider consumer waste and show that clearance sales increase consumer waste and may ultimately increase total food waste despite a potential reduction in store waste.

While the above papers study food waste at various stages of the food supply chain, they do not account for food waste generated by end consumers. However, it is estimated that consumers (i.e.,...
households) generate the largest share (43%) of food waste (Gunders and Bloom 2017). Like ours, several papers explicitly capture consumer food waste. Wu and Honhon (2023) study the impact of the Buy-one-get-one-free (BOGOF) promotions on retail profit and food waste. They find that the increase in consumer waste is often offset by the decrease in store waste. As a result, BOGOF promotions often constitute “win-win propositions” by simultaneously increasing the store profit and decreasing total waste. Similar to Wu and Honhon (2023), the two clearance schemes studied in our paper can also be win-win propositions, but we also identify conditions under which they increase total food waste despite their claimed mission to reduce food waste.

Belavina et al. (2017) compare the per-order model and subscription model for an online grocery retailer. They find that the subscription model reduces food waste but increases delivery emissions. Belavina (2021) studies the impact of grocery-store density on the food waste generated at stores and by households. The author shows that an increase in store density may increase store waste but reduces consumer waste. Similar to Belavina (2021), our paper also highlights the importance of accounting for consumer waste and cautions against a narrow focus on store waste alone.

On a modeling note, Wu and Honhon (2023) model consumer waste as an exogenous proportion of total demand. By contrast, Belavina et al. (2017) and Belavina (2021) explicitly model consumers’ ordering decisions and thus provide a micro-foundation for consumer waste. In their models, consumer waste arises due to consumers’ uncertain consumption needs at the time of ordering. We follow a similar modeling approach by explicitly capturing both the store’s production decision and individual consumers’ (basket) ordering decisions.

**Perishable Inventory Management.** Our paper also builds on the literature of pricing and inventory management of perishable goods. Dana and Petruzzi (2001) capture consumers’ binary ordering decisions in the regular phase of a newsvendor model, whereas Cachon and Kök (2007) endogenize the clearance price in the salvage phase of a newsvendor model. Chen et al. (2014) develop an effective heuristic policy for joint pricing and inventory control of a perishable product. Li et al. (2016) study joint inventory replenishment and clearance sales of perishable goods under a last-in-first-out issuing rule. They find that in the optimal policy, clearance sales may occur either when the inventory level is high or low. Su and Zhang (2008) study a newsvendor that faces consumers who have an incentive wait for the clearance sales with an exogenous clearance price. Yin et al. (2009) examine the impact of inventory display formats on markdown pricing, whereas Chen et al. (2021) study inventory disclosure policies for online sales of limited inventory. Li and Zhang (2013) study whether a seller should accept preorders in selling a perishable product to facilitate inventory planning and price discrimination. Hu et al. (2016) investigate joint inventory and markdown management for perishable products under intertemporal demand substitution. They show that it is optimal to markdown when the amount of leftover inventory is higher than
a certain threshold, and dispose all otherwise. Unlike our work, the papers above do not consider the food-waste angle, do not model consumers’ basket ordering decisions, and most importantly, do not study the novel surprise-clearance scheme in which the store sells surprise bags that contain an uncertain quantity of product units.

**Opaque Selling.** The surprise-clearance scheme studied in our paper bears a resemblance to opaque selling (e.g., Fay and Xie 2008, 2010). In a canonical setting of opaque selling, a buyer does not know the exact identity of the product prior to purchase and may get any one of a set of multiple distinct items (e.g., a T-shirt whose color is uncertain or a hotel room whose location is unknown). A number of papers study opaque selling as a way to dispose of inventory/capacity with the assumption of a fixed, exogenous amount of inventory/capacity. Jerath et al. (2010) compare opaque selling with last-minute selling and show that opaque selling becomes more preferred over last-minute selling as the probability of having high demand increases. Li et al. (2020) show that revealing inventory information in opaque selling results in higher customer surplus, but lower revenue for the firm. A few papers endogenize the firm’s inventory decisions. Fay and Xie (2015) show that the firm should order less inventory (relative to the case without opaque selling) if costs are very low but more inventory otherwise. Ren and Huang (2022) examine opaque selling under vertical differentiation and find that with endogenous inventory, opaque selling prompts the firm to procure fewer high-quality products than under last-minute selling.

The opaque-selling literature invariably focuses on unit-demand consumers, and the uncertainty lies in the type of the product but not the quantity of products. By contrast, consumers in our model have multi-unit demand and face quantity uncertainty (we additionally consider product-type uncertainty alongside quantity uncertainty in an extension in §5.2). Thus, our work contributes to this literature by identifying the value of quantity uncertainty in surprise clearance.

### 3. Model

A store (e.g., a bagel shop) sells a perishable food product to a population of consumers over a (short) planning horizon (e.g., a day) in two phases: a regular phase (e.g., before 5pm), followed by a clearance phase intended to salvage surplus food (e.g., after 5pm). The total number of potential consumers, referred to as the market size and denoted by $\mathcal{M}$, is a random variable over support $[0, \bar{M}]$ with a cumulative distribution function $F(\cdot)$, a continuous probability density function $f(\cdot)$ and a finite mean $\mu = \mathbb{E}[\mathcal{M}]$. Let $\bar{F}(x) = 1 - F(x)$ and $F^{-1}(\cdot)$ be the inverse function of $F(x)$ for $x \in [0, \bar{M}]$. The store is unable to produce products on the fly as consumers come, but rather must only produce at the beginning of the horizon before $\mathcal{M}$ is realized. The store’s unit production cost is $c$ and the regular price for each unit is $r$ with $r > c$ (we will endogenize $r$ in §5.1). Due to the perishable nature of the product, any unsold inventory at the end of the horizon (after clearance) constitutes store waste.
Consumers are infinitesimal and heterogeneous. An $\alpha$ proportion of consumers are high-end consumers (type $h$), and the rest of consumers are low-end consumers (type $l$) that may correspond to food-insecure, under-served or lower-income households (Washington Post 2022). A high-end consumer’s valuation of consuming each product unit sold in the regular phase is $v_h$ with $v_h > r$. High-end consumers are not interested in products sold in the clearance phase possibly because they are highly sensitive to food freshness of which clearance products fall short or because they are averse to the additional hassle cost of buying clearance products (e.g., a restrictive buying or pickup window). (We later relax this assumption in §5.3.) A low-end consumer’s valuation of consuming each product unit is $v_l$ with $v_l < c$ to capture these consumers’ limited willingness to pay (as they may not have the best means) and also to preclude the pathological case of the store purposefully stocking for a clearance sale rather than using it to salvage surplus food. Each consumer has uncertain consumption needs due to potential consumption shocks, e.g., changes in dining plans, an unexpectedly good or bad appetite, unforeseen meal arrangements (WRAP 2007, Glanz 2008). Each consumer’s consumption amount $D$ is independently drawn from a uniform distribution over $[0, \bar{D}]$. Any unconsumed units in excess of a consumer’s demand constitute consumer waste.

Next, we introduce three different (clearance) selling schemes: (1) no clearance in §3.1; (2) transparent clearance in §3.2; (3) surprise clearance in §3.3.

### 3.1. Benchmark: No Clearance (NC)

In the case of no clearance, the store only sells in the regular phase and discards any leftover inventory after regular sales. A consumer of type $i$ determines the purchase quantity $q_i$ in the regular phase to maximize her expected utility (conditional on her purchase order being fulfilled), which equals the expected value she derives from consumption less the purchase cost:

$$\max_{q_i \geq 0} \frac{v_i E_D[\min\{D, q_i\}]}{\min\{D, q_i\}} - r q_i. \tag{1}$$

Thus, the high-end consumer’s optimal purchase quantity, denoted by $q_R$, is

$$q_R \equiv q^*_h = \frac{v_h r}{v_h} \bar{D}, \tag{2}$$

whereas the low-end consumer’s optimal purchase quantity is zero, i.e., $q^*_l = 0$. In other words, low-end consumers cannot afford to purchase in the regular phase and may purchase only when the product is on sale in the clearance phase (as we will see in §3.2 and §3.3).

On another note, in the expected utility function (1), define

$$U(q) \triangleq E_D[\min\{D, q\}].$$
That is, \( U(q) \) is the expected number of units consumed if a consumer gets \( q \) units. It is not difficult to verify that \( U(q) \) is weakly increasing and concave in \( q \), suggesting that (1) a consumer is expected to consume more if she gets more units; (2) as a consumer gets more units, each additional unit is less likely to be consumed, and hence less valued by the consumer. Thus, the expected utility function of consumers demonstrates diminishing marginal utility.

Next, we turn to the store’s problem. At the beginning of the horizon, the store chooses the production quantity \( Q \) to maximize its expected profit, which equals the expected revenue from regular sales less the production cost:

\[
\pi^{NC} = \max_Q r \mathbb{E}_\mathcal{M} \left[ \min \{ q_R \alpha, Q \} \right] - cQ.
\]

Let the store’s optimal production quantity be \( Q^{NC} \). For notational convenience, denote \( k_1 \equiv q_R \alpha \).

### 3.1.1. Food Waste

The total food waste consists of store waste and consumer waste. Store waste stems from leftover inventory unsold in regular sales. Thus, the expected store waste \( SW^{NC} \) is:

\[
SW^{NC} = \mathbb{E}_\mathcal{M} \left( (Q^{NC} - k_1) + \right),
\]

where \( x^+ \equiv \max \{ 0, x \} \).

We next turn to consumer waste. Low-end consumers do not purchase and thus do not generate waste. A high-end consumer’s waste consists of the units she purchases in excess of her realized demand. For each high-end consumer, her expected individual waste, \( IW(q_R) \), as a function of the quantity she acquires \( q_R \), is

\[
IW(q_R) = \mathbb{E}_D \left[ (q_R - D^+) \right].
\]

Hence, the expected total consumer waste \( CW^{NC} \) is equal to the expected value of the individual waste per consumer times the number of consumers served:

\[
CW^{NC} = \mathbb{E}_\mathcal{M} \left[ IW(q_R) \cdot \frac{\min \{ k_1, Q^{NC} \}}{q_R} \right].
\]

The expected total food waste \( TW^{NC} \) is

\[
TW^{NC} = SW^{NC} + CW^{NC}. \tag{3}
\]

### 3.2. Transparent Clearance (TC)

In the case of transparent clearance, the store sells in both the regular phase and the clearance phase. The sequence of events is illustrated by Figure 2.

The store first produces \( Q \) units and then high-end consumers purchase in the regular phase. After regular sales, the amount of unsold leftover inventory is \( I = [Q - k_1] + \). If \( I > 0 \), then the
remaining inventory is put up for clearance sales (e.g., after 5pm). The store sets the (transparent) unit clearance price according to the amount of leftover inventory \( I = Q - k_1 \mathcal{M} \), or equivalently according to the realized market size \( \mathcal{M} = \frac{Q - I}{k_1} \). Adapting the price to the market condition is consistent with the practice that food waste mobile applications such as Flashfood are increasingly applying dynamic pricing algorithms (NRC 2022). Denote the clearance price by \( p(\mathcal{M}) \). Thus, for a given \( p(\mathcal{M}) \), a low-end consumer chooses the purchase quantity to maximize her expected utility, similar to what a high-end consumer does in the regular phase. Thus, parallel to Equation (1), a low-end consumer’s utility-maximizing purchase quantity in the clearance phase \( q_C(p(\mathcal{M})) \) satisfies \( q_C(p(\mathcal{M})) \in \arg \max_{q \geq 0} \left[ v_l U(q) - p(\mathcal{M})q \right] \). Hence, parallel to Equation (2), we have

\[
q_C(p(\mathcal{M})) = \frac{(v_l - p(\mathcal{M}))^+}{v_l} \bar{D}.
\]

Clearly, a higher clearance price leads to a lower purchase quantity. Moreover, as long as \( v_l > p(\mathcal{M}) \), a low-end consumer will purchase, and her expected utility is positive.

When there is leftover inventory after regular sales (i.e., \( Q > k_1 \mathcal{M} \)), the store in the clearance phase sets the clearance price \( p(\mathcal{M}) \) to maximize its revenue:

\[
\pi_C(\mathcal{M}|Q) = \max_{p(\mathcal{M})} \quad p(\mathcal{M}) \min \left\{ (1 - \alpha) \mathcal{M} \cdot q_C(p(\mathcal{M})), [Q - k_1 \mathcal{M}]^+ \right\}.
\]

In the revenue function (4), the total sales volume is the minimum of \((1 - \alpha) \mathcal{M} \cdot q_C(p(\mathcal{M}))\), the total demand induced by price \( p(\mathcal{M}) \), and the amount of inventory available after regular sales, \([Q - k_1 \mathcal{M}]^+\). Let \( S_C(\mathcal{M}|Q) = \min \left\{ (1 - \alpha) \mathcal{M} \cdot q_C(p^*(\mathcal{M})), [Q - k_1 \mathcal{M}]^+ \right\} \) be the the total sales volume in the clearance phase under the optimal price \( p^*(\mathcal{M}) \).

At the beginning of the horizon, the store chooses production quantity \( Q \) to maximize its expected total profit over the entire horizon, which equals the expected revenue from regular sales less the production cost plus the expected revenue from transparent clearance:

\[
\pi^{TC} = \max_Q \quad r \mathbb{E}_\mathcal{M} \left[ \min \{ k_1 \mathcal{M}, Q \} \right] - cQ + \mathbb{E}_\mathcal{M} \left[ \pi_C(\mathcal{M}|Q) \cdot 1_{\{Q > k_1 \mathcal{M} \}} \right].
\]

Denote the store’s optimal production quantity under transparent clearance by \( Q^{TC} \). 

\[ \text{Figure 2} \quad \text{Sequence of Events in Transparent Clearance} \]
### 3.2.1. Food Waste

The total food waste consists of store waste and consumer waste. Store waste stems from leftover inventory unsold after clearance sales. Thus, the expected store waste $SW^{TC}$ is:

$$SW^{TC} = \mathbb{E}_M \left[ ((Q^{TC} - k_1M)^+ - S_C(M|Q^{TC})) \cdot 1_{\{Q^{TC} > k_1M\}} \right].$$

Consumer waste consists of high-end consumers’ waste and low-end consumers’ waste. The expected high-end consumers’ waste is similar to that defined in (3). Low-end consumers generate waste only if $Q^{TC} > k_1M$, in which case, conditioned on $M$, the expected low-end consumers’ waste is equal to the expected individual waste per low-end consumer, $I_W(q_C(p^*(M)))$, times the number of low-end consumers served, $\frac{S_C(M|Q^{TC})}{q_C(p^*(M))}$; taking the expectation with respect to $M$ gives the unconditional expected low-end consumers’ waste. Hence, the expected total consumer waste, $CW^{TC}$, is

$$CW^{TC} = \mathbb{E}_M \left[ IW(q_R) \cdot \min \left\{ \frac{k_1M, Q^{TC}}{q_R} \right\} \right] + \mathbb{E}_M \left[ IW(q_C(p^*(M))) \cdot \frac{S_C(M|Q^{TC})}{q_C(p^*(M))} \cdot 1_{\{Q^{TC} > k_1M\}} \right].$$

The expected total food waste $TW^{TC}$ is

$$TW^{TC} = SW^{TC} + CW^{TC}.$$

### 3.3. Surprise Clearance (SC)

In the case of surprise clearance, besides regular sales, the store sells “surprise bags” in the clearance phase. Surprise bags are composed of unsold units after regular sales and the “surprise” comes from the fact that the number of units a surprise bag contains (i.e., the surprise-bag size) is ex-ante uncertain due to uncertainty in the market size (and thus uncertainty in the amount of leftover inventory after regular sales). The sequence of events is illustrated by Figure 3.

At the beginning of the horizon, the store produces $Q$ units and sets the price of the surprise bag $s$. Then, high-end consumers decide the quantity to purchase at the regular price $r$, whereas
low-end consumers decide whether to pre-order surprise bags that they can later pick up during a specified time window at the end of the business day (e.g., 5pm-5:30pm). It is worth noting that consistent with practice, if a consumer decides to order a surprise bag, she must do so by a certain cutoff time (e.g., 5pm) to give the store adequate time to prepare the order.

After regular sales, if no inventory is left, then all orders of surprise bags will be canceled and consumers who pre-order will receive a full refund. Otherwise, the store evenly distributes the leftover inventory among the surprise bags ordered. For example, if 60 units are left and the store receives 15 surprise-bag orders, then the store puts 60/15 = 4 units in each surprise bag. Then, consumers come to the store to pick up their surprise bags during the pickup window.

From a consumer’s perspective, the market size \( \hat{M} \) follows a size-biased distribution of \( M \) and thus has a probability density function \( \hat{f}(x) = xf(x)/\mu \) (Deneckere and Peck 1995, Dana and Petruzzi 2001, Cachon and Feldman 2015). The intuition for the size bias is that a consumer is more likely to come from a large market than a small market and thus the likelihood that a consumer sees a particular market size should be proportional to the size of the market. Given \( (Q, s) \), a low-end consumer pre-orders a surprise bag if and only if the expected utility of doing so is non-negative:

\[
\mathbb{E}_{\hat{M}} \left[ \left( v_l U \left( \frac{Q - k_1 \hat{M}}{\xi(\hat{M})} \right) - s \right) \cdot 1_{\{Q > k_1 \hat{M} \}} \right] \geq 0. \tag{6}
\]

We explain the utility function in Condition (6) as follows. A consumer will get a surprise bag and be charged for it if and only if there is leftover inventory after regular sales, i.e., \( Q > k_1 \hat{M} \). In such a case, an ordering consumer pays \( s \), and the amount included in a surprise bag is the total amount of inventory left, \( Q - k_1 \hat{M} \), divided by the number of bags ordered, denoted by \( \xi(\hat{M}) \), which is a function of market size \( \hat{M} \) and each consumer’s ordering decision. In equilibrium, the profit-maximizing store will sell surprise bags and thus Condition (6) holds. Thus, low-end consumers all purchase surprise bags, which implies \( \xi(\hat{M}) = (1 - \alpha) \hat{M} \).

At the beginning of the horizon, the store chooses production quantity \( Q \) and surprise-bag price \( s \) to maximize its expected total profit over the entire horizon, which equals the expected revenue from regular sales less the production cost plus the expected revenue from selling surprise bags in the clearance phase, subject to low-end consumers’ participation constraint (6):

\[
\pi^{SC} = \max_{Q, s} r \mathbb{E}_M \left[ \min\{k_1 M, Q\} \right] - cQ + s \mathbb{E}_M \left[ (1 - \alpha) M \cdot 1_{\{Q > k_1 M\}} \right], \quad \tag{7a}
\]

s.t. \( \mathbb{E}_{\hat{M}} \left[ \left( v_l U \left( \frac{Q - k_1 \hat{M}}{(1 - \alpha) \hat{M}} \right) - s \right) \cdot 1_{\{Q > k_1 \hat{M} \}} \right] \geq 0. \tag{7b} \)

Denote the store’s optimal production quantity under surprise clearance by \( Q^{SC} \).

In practice, stores typically sell surprise bags on a daily basis. Thus, repeat consumers can use past experience (Dana and Petruzzi 2001) to estimate the expected size of the surprise bag, whereas new consumers can learn it from social media sharing (New York Times 2022).

Electronic copy available at: https://ssrn.com/abstract=4573386
3.3.1. **Food Waste** The total food waste consists of store waste and consumer waste. Since any leftover inventory after clearance sales will be put into surprise bags, store waste is zero, i.e.,

\[ SW^{SC} = 0. \]

Consumer waste consists of high-end consumers’ waste and low-end consumers’ waste. The expected high-end consumers’ waste is similar to that defined in (3). Low-end consumers generate waste only if \( Q^{SC} > k_1 M \), in which case, conditioned on \( M \), the expected low-end consumers’ waste is equal to the expected individual waste per low-end consumer, \( IW \left( \frac{Q^{SC} - k_1 M}{(1-\alpha)M} \right) \), times the number of low-end consumers who get surprise bags, \((1-\alpha)M\); taking the expectation with respect to \( M \) gives the unconditional expected low-end consumers’ waste. Hence, the expected total consumer waste, \( CW^{SC} \), is

\[
CW^{SC} = E_M \left[ IW(q_R) \cdot \min \{ k_1 M, Q^{SC} \} / q_R \right] + E_M \left[ IW \left( \frac{Q^{SC} - k_1 M}{(1-\alpha)M} \right) \cdot (1-\alpha)M \cdot 1_{\{Q^{SC} > k_1 M\}} \right].
\]

The expected total food waste \( TW^{SC} \) is

\[
TW^{SC} = SW^{SC} + CW^{SC} = CW^{SC}.
\]

4. **Results**

In this section, we first characterize the store’s optimal quantity and pricing decisions in the three selling schemes introduced in §3. Then, in §4.1, we compare the store’s optimal profits and production quantities in the three schemes; in §4.2, we conduct a comparison of food waste.

Lemma 1 characterizes the store’s clearance pricing decisions in the two clearance schemes, respectively, for a given production quantity.

**Lemma 1** *(Clearance Pricing).* Given production quantity \( Q > 0 \):

(i) under transparent clearance, the store’s optimal clearance price \( p^*(M|Q) \) for realized market size \( M \) is

\[
p(M|Q) = \begin{cases} 
  v_l / 2, & 0 < M \leq \frac{Q}{k_1 + D(1-\alpha)/2}; \\
  v_l \left[ 1 - \frac{Q - k_1 M}{D(1-\alpha)M} \right], & \frac{Q}{k_1 + D(1-\alpha)/2} < M < Q/k_1;
\end{cases}
\]

(ii) under surprise clearance, the store’s optimal price of the surprise bag, \( s(Q) \), is

\[
s(Q) = \frac{v_l \int_0^{Q/k_1} \bar{U} \left( \frac{Q - x}{D(1-\alpha)x} \right) x f(x) dx}{\int_0^{Q/k_1} x f(x) dx}.
\]
Lemma 1-(i) solves the store’s clearance-phase problem in (4) and shows how the store should set the clearance price in response to the realized market size under transparent clearance. When the realized market size is below a threshold, the store should charge a constant clearance price independent of the realized market size. In this case, high-end consumers’ demand is weak in the regular phase, creating plenty of leftover inventory after regular sales. The store could set a clearance price low enough to induce sufficient demand from low-end consumers that clears all inventory, but doing so would require a price that is too low to be optimal. Instead, the store charges a higher clearance price that induces a smaller purchase quantity. This deliberate choice of the store results in unsold inventory even after clearance, which becomes the store’s food waste.

However, when the realized market size is above a threshold (but not high enough to deplete all inventory in the regular phase), the store should charge a clearance price that is increasing in the realized market size. In this case, demand is strong in the regular phase, leading to little leftover inventory after regular sales. Thus, it is optimal for the store to set an inventory-clearing price. Since all units are sold, the store does not generate any food waste in this case.

Putting the two cases together suggests that the clearance price is non-decreasing in the realized market size. Since a larger realized market size translates into less leftover inventory after regular sales, it follows that the store should set a (weakly) lower clearance price with more leftover inventory after the regular sales. This structural property is consistent with the commonly observed practice of a deep (shallow) discount being taken if there is excessive (limited) remaining inventory.

Lemma 1-(ii) specifies the store’s optimal price of the surprise bag, which, unlike the transparent-clearance price, is set prior to market-size realization. The optimal surprise-bag price is such that every low-end consumer buys a surprise bag and expects zero utility. A lower price would not induce more purchases and thus hurt revenue, whereas a higher price would cause low-end consumers to not purchase at all. Hence, the store sets the surprise-bag price to fully extract surplus of low-end consumers while making sure they still purchase. Unlike transparent clearance, surprise clearance prompts the store to sell all inventory and thus eliminates store waste. The store has no incentive to withhold inventory from consumers in this case because doing so would only reduce the surprise-bag size, which would compel a lower surprise-bag price and ultimately hurt the store’s profit.

Building on Lemma 1, Proposition 1 characterizes the store’s optimal production quantity in each of the three selling schemes.

**Proposition 1 (Production Quantities).**

(i) In no clearance, the optimal production quantity is $Q^{NC} = k_1 F^{-1} \left( \frac{r-c}{v_l} \right)$.

(ii) In transparent clearance, the optimal production quantity $Q^{TC} \in (0, k_1 \bar{M})$ uniquely solves:

\[
0 = r - c + (v_l - r) F\left( \frac{Q^{TC}}{k_1} \right) - \frac{2v_l Q^{TC}}{D(1-\alpha)} \int_{Q^{TC}/k_1}^{\bar{M}} \frac{F(x)}{x^2} dx.
\]
(iii) In surprise clearance, the optimal production quantity $Q^{SC} \in (0, k_1 \bar{M})$ uniquely solves:

$$r - c + (v_l - r)F\left(\frac{Q^{SC}}{k_1}\right) - \frac{v_l Q^{SC}}{D(1 - \alpha)} \int_{k_1 + \frac{Q^{SC}}{k_1(1 - \alpha)D}}^{Q^{SC}/k_1} \frac{F(x)}{x^2} dx = 0.$$  

Proposition 1-(i) shows that in no clearance, the store’s optimal production quantity essentially follows the newsvendor formula. However, parts (ii) and (iii) of Proposition 1 show that in the two clearance schemes, solving for the optimal production quantities is more complicated and closed-form expressions do not exist in general.

Note that if the unit salvage value were $v_l$, then the optimal production quantity would be $\bar{Q} = k_1 F^{-1}\left(\frac{r-c}{r-v_l}\right)$ according to the newsvendor formula. Clearly, the optimal production quantity in the no-clearance scheme (where the salvage value is zero) is less than this hypothetical quantity (i.e., $Q^{NC} < \bar{Q}$). It is not difficult to verify that the optimal production quantities in both clearance schemes are also less than this hypothetical quantity (i.e., $Q^{TC} < \bar{Q}$ and $Q^{SC} < \bar{Q}$). Although low-end consumers value each consumed unit at $v_l$, they may not always consume all units bought due to uncertain food needs. Anticipating this, low-end consumers expect diminishing marginal utility, just like their high-end counterparts (as noted in §3.1). Therefore, the effective unit salvage value cannot reach $v_l$ in either clearance scheme, causing the store to be more conservative in production.

### 4.1. Comparison of Profits and Production Quantities

Next, we compare in Proposition 2 the maximum profits of the three selling schemes under their respective optimal production quantities.

**PROPOSITION 2 (Profit Comparison).** Surprise clearance yields the highest profit, followed by transparent clearance, and no clearance yields the lowest profit, i.e., $\pi^{NC} < \pi^{TC} < \pi^{SC}$.

Proposition 2 presents a clear profit ordering of the three selling schemes. It is intuitive that both clearance schemes outperform the no-clearance scheme as the clearance phase creates a second selling opportunity by targeting a market segment (i.e., low-end consumers) that would otherwise not afford to buy. Nevertheless, the profit comparison of the two clearance schemes is less straightforward. Proposition 2 indicates that surprise clearance always outperforms transparent clearance. In fact, this is true not only under the optimal production quantity chosen for each respective scheme, but also true under any given positive production quantity. The profit dominance of surprise clearance may be surprising because it lacks the pricing flexibility of transparent clearance. The price of a surprise bag is set prior to the realization of the market size, whereas the unit clearance price is contingent on the remaining inventory after the regular sales in transparent clearance.

However, surprise clearance implements *contingent pricing “in disguise.”* While the surprise-bag price is determined before market-size realization, the number of product units to be allocated
to each bag is determined after market-size realization. If the market size turns out to be small (large), then there will be plenty of (little) leftover inventory to clear, and the store will allocate many (few) product units to a surprise bag, which implies that each product unit in the surprise bag is effectively sold at a low (high) price since the price of the whole surprise bag is fixed in advance. As a result, surprise clearance mimics transparent clearance in that the unit price in surprise clearance is essentially also contingent on the amount of leftover inventory after regular sales, despite a fixed upfront price for the surprise bag.

Besides the hidden benefit of contingent pricing, surprise clearance also has an extra advantage of implementing *quantity discounts* “in disguise.” In transparent clearance, the store charges a constant unit clearance price regardless of a consumer’s purchase quantity. While it is easy to implement and thus commonly practiced, it fails to account for consumers’ diminishing marginal utility (i.e., consumer valuation of each additional product unit is not constant in the purchase quantity). Therefore, a constant unit clearance price—even if it is adjusted to the amount of leftover inventory (and the realized market size)—is still not tailored to consumers’ actual willingness to pay for each product unit, thereby limiting the store’s ability to extract consumer surplus. This is reflected in our model by the fact that low-end consumers always have a positive expected net utility from purchase at the time of ordering. By contrast, surprise clearance allows the store to fully extract consumer surplus in the sense that a low-end consumer’s expected net utility from purchasing a surprise bag is zero (as pointed out in the explanation of Lemma 1-(ii)). This implies that the price of a surprise bag is calibrated precisely to the expected valuation of the bag. Hence, surprise clearance mimics an outcome that would be achieved by quantity discounts, where the unit clearance price decreases in the quantity purchased in a way that perfectly matches the diminishing marginal utility of low-end consumers.

From a practical standpoint, surprise clearance reaps the benefits of both contingent pricing and quantity discounts without the implementation hassle of either. That is, the store does not need to adjust clearance prices every day to the amount of leftover inventory (as one would in contingent pricing) and the store does not need to charge a different unit clearance price for each additional unit sold (as one would in quantity discounts). Rather, surprise clearance has the convenience of charging a single, fixed price for the surprise bag.

Proposition 3 compares the optimal production quantities of the three selling schemes.

**Proposition 3 (Production-Quantity Comparison).** *Surprise clearance has the highest optimal production quantity, followed by transparent clearance, and no clearance has the lowest optimal production quantity, i.e.,* $Q^{NC} < Q^{TC} < Q^{SC}$.
By Proposition 3, the optimal production quantities for the three selling schemes are also nicely ordered. It is intuitive that the store would like to produce more in a clearance scheme than it would with no clearance: the clearance phase enables the store to salvage at least part of its unsold inventory, which reduces the cost of excess and stimulates production. As for which clearance scheme induces more production, note from the explanation after Proposition 2 that surprise clearance is more profitable than transparent clearance because it is able to extract more value from unsold inventory after regular sales. This further allays the concern of overproduction, encouraging the store to produce even more aggressively in surprise clearance than in transparent clearance.

4.2. Food Waste Comparison

This subsection conducts food waste comparison. Proposition 4 compares both the store waste and consumer waste of the three selling schemes.

**Proposition 4 (Comparison of Store and Consumer Waste).**

(i) **Store waste (SW):** Surprise clearance has zero and thus the least store waste, i.e., \( \min\{SW^{NC}, SW^{TC}\} > SW^{SC}(= 0) \). Transparent clearance generates less store waste than no clearance \( (SW^{TC} < SW^{NC}) \) if \( v_l \) is low enough. When the distribution of the market size is uniform over \([0, \bar{M}]\), there exists a threshold \( \bar{c} \in (0, r) \) such that

(a) if \( c < \bar{c} \), transparent clearance generates less store waste than no clearance \( (SW^{TC} < SW^{NC}) \);

(b) if \( c \in (\bar{c}, r) \), there exists a threshold \( \bar{v}_l \in (0, c) \) such that transparent clearance generates less store waste than no clearance \( (SW^{TC} < SW^{NC}) \) if \( v_l < \bar{v}_l \) and transparent clearance generates more store waste than no clearance \( (SW^{TC} > SW^{NC}) \) if \( v_l \in (\bar{v}_l, c) \).

(ii) **Consumer waste (CW):** Surprise clearance generates the most consumer waste, followed by transparent clearance, and no clearance generates the least consumer waste, i.e., \( CW^{NC} < CW^{TC} < CW^{SC} \).

In minimizing store waste, surprise clearance is a clear winner because any unsold inventory after the regular sales is packed into surprise bags. However, the comparison between no clearance and transparent clearance is more involved. If the store’s production quantity did not change with the selling scheme, then transparent clearance would clearly generate lower store waste than no clearance because the former could transform at least part of the leftover inventory (which would be store waste in the no-clearance scheme) into clearance sales. However, the profit-oriented store produces more in transparent clearance (according to Proposition 3)—which increases the expected amount of leftover inventory after regular sales—but is not always willing to set a low-enough price to sell all leftover inventory during clearance (according to Lemma 1).

Proposition 4-(i) shows that if low-end consumers’ valuation of consumption \( v_l \) is low, then transparent clearance indeed reduces store waste relative to no clearance. In this case, the revenue..
potential from clearance sales is limited since low-end consumers’ willingness to pay for clearance products is low; therefore, the store is not keen on vastly ramping up production as a result of the clearance opportunity, making the upward pressure of more production on store waste a secondary concern. By contrast, if \( v_l \) is high (which implies production cost \( c \) is high since \( v_l < c \)), then knowing clearance will be more lucrative, the store will be more aggressive with its production, making the upward pressure of more production a dominant force. In this case, transparent clearance ends up generating more store waste than no clearance, despite its claimed intention to reduce food waste. All told, the store-waste comparison points to surprise clearance as a more robust approach to eliminate food waste at the store than transparent clearance, an approach less influenced by the store’s incentive to adapt its production decision to clearance sales.

The comparison of consumer waste paints a different picture. Proposition 4-(ii) presents a clear ordering of consumer waste in the three selling schemes. No clearance generates the least consumer waste among the three, because, relative to no clearance, clearance sales increase the amount of food wasted by both low-end and high-end consumers: on the one hand, without clearance, low-end consumers do not afford to buy and generate zero waste. As clearance sales make the product affordable, they choose to purchase and start consumption, ultimately generating their own food waste; on the other hand, clearance sales increase store production, so more high-end demand is fulfilled, resulting in more food waste from high-end consumers.

As for the consumer-waste comparison between transparent clearance and surprise clearance, note that in surprise clearance, all the leftover inventory after regular sales is put into surprise bags for low-end consumers, whereas in transparent clearance, the store may deliberately choose to sell only a portion of the leftover inventory to low-end consumers by setting a high clearance price (see Lemma 1). This implies that even for a fixed production quantity, a low-end consumer gets more product units on average in surprise clearance, which translates into more waste. The fact that surprise clearance spawns more production than transparent clearance only strengthens this effect. As a result, surprise clearance engenders the most consumer waste among the three selling schemes, even though it boasts the least store waste (in fact, zero store waste).

Proposition 4 indicates a tension between store waste and consumer waste: while the two clearance schemes may reduce food waste for the store (relative to no clearance), they pass food waste on to consumers. This makes it unclear how the total food wastes (i.e., the sum of store waste and consumer waste) compare across the three selling schemes.

Lemma 2 compares the total waste of the three selling schemes for a given production quantity.

**Lemma 2 (Total Waste for a Given Production Quantity).** For a given production quantity \( Q > 0 \), surprise clearance generates the least total waste, followed by transparent clearance, and no clearance generates the most total waste, i.e., \( TW^{SC}(Q) < TW^{TC}(Q) < TW^{NC}(Q) \).
Lemma 2 suggests that if the store does not strategically adjust its production quantity to the selling scheme, then both clearance schemes are effective in reducing the total waste (despite an increase in consumer waste), with surprise clearance being the most effective with the least total waste. If the production quantity is fixed, high-end consumers’ food waste is the same across the three schemes. Hence, the difference in total food waste is driven by the difference in food waste generated by the leftover inventory after regular sales. In evaluating contributors to food waste at this stage, the guiding principle is that a unit left unsold at the store is “worse” than a unit in the hands of a consumer. This is because the former will be food waste for sure whereas the latter may be consumed by the consumer and thus is not guaranteed to turn into food waste. Therefore, for the same amount of leftover inventory after regular sales, whichever scheme transfers more units from the store to consumers will lead to less total waste. By this logic, surprise clearance is clearly the best as it transfers all the leftover inventory after regular sales to consumers; no clearance is clearly the worst as no leftover inventory is transferred; transparent clearance falls in between as it transfers some, but not necessarily all the leftover inventory. This result shows that while surprise clearance brings about more food waste at the consumer level, it can be offset by less food waste at the store level, and therefore can still be a promising alternative to transparent clearance, which itself is better than no clearance at food waste reduction.

Nevertheless, the caveats for the above argument are that (1) high-end consumers generate the same amount of food waste and that (2) the amount of leftover inventory after regular sales is held unchanged, both of which hinge on the production quantity being held constant across the three selling schemes. Yet, the production quantity is endogenously chosen by the store and may differ across the clearance schemes, making it unclear as to how the total food wastes compare once the store’s production decision is taken into account.

Proposition 5 compares the total food waste across the three selling schemes under the optimal production quantities.

**Proposition 5 (Comparison of Total Waste).**

(i) If \( v_i \) is low enough, surprise clearance generates the least total waste, followed by transparent clearance, and no clearance generates the most total waste, i.e., \( T W^{SC} < T W^{TC} < T W^{NC} \).

(ii) If \( c \) is high enough, surprise clearance generates the most total waste, followed by transparent clearance, and no clearance generates the least total waste; \( T W^{SC} > T W^{TC} > T W^{NC} \).

Two countervailing forces drive the comparison of total food wastes among the three selling schemes. On the one hand, as explained after Lemma 2, a product unit is more likely to be transferred from the store to consumers under surprise clearance than under the other two schemes. This “inventory transfer” effect favors surprise clearance in the race to minimize total waste. On the
other hand, as shown in Proposition 3, more product units are produced under surprise clearance than under the other two schemes. This “strategic production” effect disfavors surprise clearance as far as minimizing total waste is concerned.

Proposition 5-(i) shows that the ordering of total food waste in Lemma 2 is preserved under optimal production quantities when low-end consumers’ valuation of consumption $v_l$ is low. In this case, low-end consumers’ lukewarm interest implies clearance sales have limited revenue potential; therefore, the store will not drastically increase production just because clearance is available. Thus, the production quantity is roughly the same across the three schemes. Hence, the strategic production effect gives way to the inventory transfer effect. Surprise clearance, which leads in inventory transfer, becomes the scheme that minimizes the total food waste among the three, followed by transparent clearance, whereas no clearance, which does not transfer any leftover inventory, ends up with the most total food waste.

However, clearance sales may increase total food waste. Proposition 5-(ii) shows that the ordering of total food waste in Lemma 2 is reversed when the production cost $c$ is high. In this case, the profit margin of regular sales is thin and the store chooses a moderate production quantity in all three selling schemes. This implies that most inventory is sold through regular sales to high-end consumers and there is not much left for low-end consumers. Hence, total food waste is predominated by high-end consumers’ food waste. The more the store produces, the more likely high-end consumers’ orders are fulfilled, and the more food goes unconsumed. Thus, total waste is dictated by the strategic production effect. Since surprise clearance induces the most production, it generates the most total food waste, followed by transparent clearance, whereas no clearance, with the lowest production quantity, will generate the least total waste.

Putting together Propositions 2 and 5 yields Corollary 1.

**Corollary 1 (Win-Win).** If $v_l$ is low enough, surprise clearance results in higher profit and less total food waste than both no clearance and transparent clearance.

Corollary 1 shows that as a novel selling scheme, surprise clearance can be a win-win proposition both for maximizing store profit and for minimizing food waste. A sufficient condition for this win-win outcome to arise is that the consumer segment targeted by clearance sales cannot be too keen on the store’s products. However, as argued earlier, surprise clearance is not always the “go-to” scheme for food waste reduction. While it eliminates store waste, it transfers food waste from the store to consumers and causes the store to produce more, potentially generating more food waste in total than both transparent clearance and no clearance.
4.3. Numerical Observations

We supplement our analytical results with a numerical investigation in Figure 4, which presents a comparison of total food waste in the \((c,v_l)\) space satisfying \(v_l < c < r < v_h\). Region I (low \(v_l\), consistent with the condition in Proposition 5-(i)) represents the market condition under which surprise clearance is a “win-win” proposition with the highest store profit and the least total food waste at the same time among the three selling schemes. In Region II, surprise clearance concedes its title as the least-waste scheme to transparent clearance, but is still able to reduce food waste relative to no clearance. In Region III, surprise clearance falls short of the goal to reduce food waste and becomes the scheme with the most food waste; yet, at least transparent clearance is still effective in food waste reduction relative to no clearance. Region IV (high \(c\), consistent with the condition in Proposition 5-(ii)) sees a reversal of what occurs in Region I; neither clearance scheme is able to reduce food waste, and surprise clearance winds up with the most food waste.

In sum, we observe that each of the three selling schemes can be the winner in food waste reduction. In Region I (the light-gray region), surprise clearance generates the least waste; in Regions II and III (the white region), transparent clearance generates the least waste; in Region IV (the dark-gray region), no clearance generates the least waste. Another observation is that surprise clearance generates the most waste in Regions III and IV, whereas no clearance generates the most waste in Regions I and II; transparent clearance seems a bit more robust for controlling food waste in that it does not appear to generate the most waste in any instances. The general trend we observe is that clearance schemes tend to perform well in reducing food waste when \(v_l\) and \(c\) are
low, but tend to backfire when \(v_l\) and \(c\) are high. Surprise clearance seem to be more susceptible to the increase of \(v_l\) and \(c\) than transparent clearance, possibly because surprise clearance gives rise to the highest production quantity and is subject to a stronger strategic production effect.

5. Extensions

This section studies three extensions: we study an endogenous regular price in §5.1, multiple product types in §5.2, and the threat of cannibalization in §5.3. Each extension relaxes one assumption of the base model in §3 at a time while keeping all other aspects of the model unchanged.

5.1. Endogenous Regular Price

This extension endogenizes the regular price \(r\), i.e., the store optimally chooses \(r\) at the beginning of the horizon in conjunction with production quantity \(Q\) to maximize its expected profit. Proposition 6 shows that the profit ordering established in Proposition 2 continues to hold.

**Proposition 6 (Profit Comparison).** When regular price \(r\) is endogenized, surprise clearance yields the highest profit, followed by transparent clearance, and no clearance yields the lowest profit, i.e., \(\pi^{NC} < \pi^{TC} < \pi^{SC}\).

As explained earlier, surprise clearance beats transparent clearance in store profit for any given \((r,Q)\). Thus, if the store in surprise clearance chooses the \((r,Q)\) pair that is optimal for transparent clearance, it already achieves a higher profit than that of transparent clearance, and optimizing over \((r,Q)\) only increases profit even further. Likewise, we can show that transparent clearance continues to outperform no clearance in profit when the store chooses the optimal regular price.

We next turn to the comparison of total food waste. We analytically show in Proposition 7 that the food-waste ordering in the first part of Proposition 5 remains valid.

**Proposition 7 (Comparison of Total Food Waste).** When regular price \(r\) is endogenized, if \(v_l\) is low enough, surprise clearance generates the least total waste, followed by transparent clearance, and no clearance generates the most total waste, i.e., \(TW^{SC} < TW^{TC} < TW^{NC}\).

If \(v_l\) is low, the limited revenue potential from clearance sales implies that the store is not interested in making any major adjustment (to the selling scheme) of either its production decision or pricing decision during the regular phase. Hence, any strategic effect on either the production or pricing front is secondary to the inventory transfer effect, which favors clearance sales in general, and surprise clearance in particular. As a result, consistent with Proposition 5, surprise clearance achieves the least total food waste among the three selling schemes in this case.

It follows from Propositions 6 and 7 that Corollary 1—which suggests that surprise clearance can be a win-win proposition—also extends to the case of an endogenous regular price. Yet, our
numerical study shows that as in the base model, both surprise clearance and transparent clearance can backfire in food waste reduction under an endogenous regular price, and in particular, surprise clearance can be the scheme that generates the most food waste. See Appendix B for more details.

5.2. Multiple Product Types

This extension considers a generalization to multiple types of noninterchangeable products. Note that if products are interchangeable in the eyes of consumers, then it is essentially a setting of a single product type in which consumers only care about how much they consume, not what they consume (e.g., consuming any 5 units generates the same value, whether they be 4 bagels plus 1 doughnuts or 2 bagels plus 3 doughnuts). Thus, our base model captures both the case of a single product type or that of multiple types of interchangeable products (i.e., pooling across types).

Now, consider a store selling \( n \) noninterchangeable product types. Each consumer’s demand for product type \( i \) is a random variable \( D_i \) independently drawn from a uniform distribution over \([0, \bar{D}_i]\). A consumer of type \( \theta \in \{h, l\} \) derives value \( v_{\theta,i,t} \) from consuming a unit of product type \( i \) in phase \( t \in \{R, C\} \), where \( R \) is the regular phase and \( C \), the clearance phase. For a consumer of type \( \theta \in \{h, l\} \), her expected valuation of a basket of \( q_i \) units of product type \( i \) for \( i = 1, \ldots, n \) in period \( t \) is \( \sum_{i=1}^{n} v_{\theta,i,R} E_{D_i}[\min\{D_i,q_i\}] \triangleq \sum_{i=1}^{n} v_{\theta,i,R} U_i(q_i) \). Note that this (intuitive) functional form of consumer valuation implies non-interchangeability of product types because consumer valuation of a basket depends on the quantity of each product type in the basket, not just the total quantity in the basket, i.e., a consumer cannot use a product of type 1 to fulfill demand of product type 2. For notational convenience, let \( v_{h,i} = v_{h,i,R} \) and \( v_{l,i} = v_{l,i,C} \) for \( i = 1, \ldots, n \). As in the base model, we focus on the setting \( v_{h,i} > r_i > c_i > v_{l,i} \) for \( i = 1, \ldots, n \); high-end consumers are not interested in clearance products, whereas low-end consumers cannot afford regular products.

For surprise clearance, the store sells a mixed surprise bag that may contain leftover inventory of any product types after regular sales. Given production quantity \( Q_i \) for product type \( i \) and a realized market size \( x \), the amount of product type \( i \) included in a surprise bag (when low-end consumers buy surprise bags) is \( \left[ Q_i - k_{1,i} x \right]^+ \) divided by \( (1 - \alpha) x \), where \( k_{1,i} = (\alpha v_{h,i} - r_i) \bar{D}_i \). Thus, surprise bags will be successfully sold if at least one product type has leftover inventory after regular sales, i.e., \( Q_i > k_{1,i} x \) for at least one \( i \) (otherwise, consumers who pre-order surprise bags will be issued a refund). Depending on the realized market size, the number of product types a surprise bag contains may vary. A smaller market size implies more leftover inventory and thus potentially more product types in a surprise bag. Therefore, a consumer who pre-orders a surprise bag faces not only quantity uncertainty but also product-type uncertainty about the surprise bag’s content.
Let the unit production cost and regular price for product type $i$ be $c_i$ and $r_i$, respectively. At the beginning of the horizon, the store chooses production quantities $Q_1, \ldots, Q_n$ and surprise-bag price $s$ to maximize its expected total profit:

$$\pi^{SC} = \max_{Q_1, \ldots, Q_n, s} \left( \sum_{i=1}^{n} (r_i \mathbb{E}_{M}[\min\{k_{1,i}M, Q_i\}] - c_i Q_i) \right) + s \mathbb{E}_{M} \left[ (1 - \alpha)M \cdot 1_{\cup_{i=1}^{n}(Q_i > k_{1,i}M)} \right], \quad (10a)$$

subject to

$$\mathbb{E}_{\mathcal{M}} \left( \sum_{i=1}^{n} v_i, U_i \left( \frac{[Q_i - k_{1,i}M]^+}{(1 - \alpha)M} \right) - s \right) \cdot 1_{\cup_{i=1}^{n}(Q_i > k_{1,i}M)} \geq 0. \quad (10b)$$

Let the store’s optimal production quantity for product $i$ be $Q_i^{SC}$ and the optimal surprise-bag price be $s_i^{SC}$ in this mixed surprise-clearance scheme. With slight abuse of notation, let $\pi^{SC}$ denote the store’s profit under the optimal decisions.

An alternative surprise-clearance scheme would sell a separate surprise bag for each product type. In this case, the firm solves a version of Problem (7a)-(7b) for each product type. Let the optimal production quantity and the optimal surprise-bag price for product type $i$ be $Q_i^*$ and $s_i^*$, respectively. Under these optimal decisions, let the profit from product type $i$ be $\pi_i^*$. Thus, the total store profit is $\sum_{i=1}^{n} \pi_i^*$. Proposition 8 compares this separate surprise-clearance scheme with the mixed surprise-clearance scheme introduced earlier.

**Proposition 8.** The mixed surprise-clearance scheme and the separate surprise-clearance scheme are equivalent in the sense that $Q_i^{SC} = Q_i^*$ for $i = 1, \ldots, n$ and $\pi^{SC} = \sum_{i=1}^{n} \pi_i^*$. Further, $s^{SC} = \frac{\sum_{i=1}^{n} s_i^* \mathbb{E}_{\mathcal{M}}[M \cdot 1_{\cup_{i=1}^{n}(Q_i > k_{1,i}M)}]}{\mathbb{E}_{\mathcal{M}}[M \cdot 1_{\cup_{i=1}^{n}(Q_i > k_{1,i}M)}]} \leq \sum_{i=1}^{n} s_i^*$.

Proposition 8 establishes the profit equivalence between that the mixed surprise-clearance scheme and the separate surprise-clearance scheme. It is not difficult to see the parallel in the process. In the mixed case, a surprise bag contains units of a particular product type if that product type has leftover inventory after regular sales. Likewise, in the separate case, a surprise-bag order of a particular product type will be honored (i.e., not canceled) exactly when that product type is still in stock. This equivalence implies the store’s problem is effectively decoupled across product types. Hence, the store makes the same production decisions in both surprise-clearance schemes, which further implies that the total food waste is also the same between the two schemes.

Interestingly, while the store profit in the mixed scheme is the sum of each product type’s profit in the separate scheme ($\pi^{SC} = \sum_{i=1}^{n} \pi_i^*$), the surprise-bag price in the mixed case is weakly less than the sum of each product type’s surprise-bag price in the separate case ($s^{SC} \leq \sum_{i=1}^{n} s_i^*$). To see why, note that in the separate case, consumers are refunded the surprise-bag price of a sold-out product type, whereas in the mixed case, consumers are refunded the surprise-bag price only if all product types are out of stock. Hence, whenever some product types are sold out and some are not, consumers still pay the surprise-bag price $s^{SC}$ in the mixed case, but do not need to pay the
full price $\sum_{i=1}^{n} s_i^*$ in the separate case. This implies that in order for consumers to find the two surprise-clearance schemes equivalent in expectation, the store must set $s^{SC} \leq \sum_{i=1}^{n} s_i^*$.

Notably, $s^{SC}$ is the price a low-end consumer pays up front for ordering a mixed surprise bag in the mixed case and $\sum_{i=1}^{n} s_i^*$ is the total price a low-end consumer pays up front for ordering all the surprise bags (one for each product type) in the separate case. While the expected net payment for surprise bags is the same, the result $s^{SC} \leq \sum_{i=1}^{n} s_i^*$ implies that the mixed case involves a lower initial payment and a lower likelihood of cancellation than if surprise bags are sold separately for each product type. Fewer cancellations and refunds are a clear practical advantage in favor of selling mixed surprise bags. Besides, in practice, selling a single mixed surprise bag that may contain products of different types is much more convenient than selling a separate surprise bag for each product type. All of these may explain the popularity of the mixing approach in practice.

For no clearance and traditional clearance, it is straightforward that the store’s problem is also decoupled across product types. Since the problem is decoupled across product types in all three selling schemes, all our results for the base model with a single product type generalize to the case of multiple product types.

5.3. Threat of Cannibalization

This extension studies a case in which clearance sales may threaten to cannibalize regular sales. We assume high-end consumers’ valuation of consuming a clearance product is $\delta v_h$, $\delta \in [0,1]$. Note that our base model corresponds to a case of $\delta = 0$. If $\delta > 0$, then high-end consumers may have an incentive to opt for clearance products instead of buying at the regular price in the two clearance schemes, which may hurt the store’s profitability. To guard against this cannibalization threat, the store needs to ensure that high-end consumers will not switch to clearance by imposing an additional incentive constraint on its profit maximization problem for each clearance scheme (following the modeling approach of Su and Zhang 2008). Such an incentive constraint governs the store’s production quantity $Q$ and is of the form: $u_R(Q) \geq u^X_C(Q)$, where $u_R(Q)$ is a high-end consumer’s expected utility of purchasing in the regular phase, and $u^X_C(Q)$ is a high-end consumer’s expected utility if she were to purchase clearance products in clearance scheme $X \in \{TC, SC\}$. The incentive constraint requires that purchasing in the regular phase yields a weakly higher expected utility than switching to the clearance channel for high-end consumers. To be specific, a high-end consumer’s expected utility of purchasing in the regular phase is

$$u_R(Q) = \mathbb{E}_M \left[ \frac{\min \{k_1\hat{M}, Q\}}{k_1\hat{M}} \right] (v_h U(q_R) - rq_R),$$

where $q_R$ is a high-end consumer’s optimal order quantity in the regular phase as specified in (2) and $\mathbb{E}_M \left[ \frac{\min \{k_1\hat{M}, Q\}}{k_1\hat{M}} \right]$ is the order fill rate, i.e., the probability that a high-end consumer will see her order fulfilled (Dana and Petruzzi 2001).
For transparent clearance, if a high-end consumer were to switch to the clearance channel, then her expected utility would be

$$u_{TC}^{C}(Q) = \mathbb{E}_{\tilde{M}} \left[ \max_q \left( \delta v_h U(q) - p(\tilde{M}|Q)q \right) \cdot 1_{\{Q > k_1 \tilde{M}\}} \right],$$

where $p(\tilde{M}|Q)$ is given by Equation (8) in Lemma 1.

For surprise clearance, if a high-end consumer were to pre-order a surprise bag, then her expected utility would be

$$u_{SC}^{C}(Q) = \mathbb{E}_{\tilde{M}} \left[ \left( \delta v_h U \left( \frac{Q - k_1 \tilde{M}}{1 - \alpha} \tilde{M} \right) - s(Q) \right) \cdot 1_{\{Q > k_1 \tilde{M}\}} \right],$$

where $s(Q)$ is given by Equation (9) in Lemma 1.

Thus, the store’s problem in transparent clearance is adapted from (5) to

$$\pi_{TC} = \max_{Q} r\mathbb{E}_M[\min\{k_1 M, Q\}] - cQ + \mathbb{E}_M \left[ \pi_C(M|Q) \cdot 1_{\{Q > k_1 M\}} \right],$$

subject to $u_R(Q) \geq u_{TC}^C(Q)$.

The store’s problem in surprise clearance is adapted from (7a)-(7b) to

$$\pi_{SC} = \max_{Q, s, y \in \{0, 1\}} r\mathbb{E}_M[\min\{k_1 M, Q\}] - cQ + y \cdot s\mathbb{E}_M \left[ (1 - \alpha) M \cdot 1_{\{Q > k_1 M\}} \right],$$

subject to $u_R(Q) \geq u_{SC}^C(Q) \geq 0$,

where $y$ is a binary decision variable indicating whether the store decides to sell surprise bags at the beginning of the horizon.

Proposition 9 conducts a profit comparison of the three selling schemes.

**Proposition 9 (Profit Comparison).** When $\delta > 0$, surprise clearance yields the highest profit among the three selling schemes, i.e., $\pi_{SC} \geq \max\{\pi_{NC}, \pi_{TC}\}$.

In transparent clearance, the store sets the clearance price after the regular sales, at which point, engaging in clearance sales is an optimal strategy for the store (i.e., subgame perfect). Since the store cannot commit in advance to not selling in the clearance phase yet high-end consumers may have an incentive to wait for clearance, there is no guarantee that the store’s total profit will be at least as high as that of no clearance. In other words, the store may have to compete with its future self, who cannot help but launch transparent clearance whenever there is leftover inventory.

By contrast, in surprise clearance, the store can always choose not to (pre)sell surprise bags at all. As a result, the store’s profit can never be lower than that of no clearance. Additionally, Proposition 9 shows that surprise clearance always profit-dominates transparent clearance and thus continues to be the most profitable scheme out of the three even after the threat of cannibalization is accounted for.
for. Recall that surprise clearance better exploits consumers’ diminishing marginal utility and thus extracts more consumer surplus than transparent clearance. Hence, high-end consumers would gain less from pre-ordering surprise bags than from waiting for transparent clearance. Therefore, surprise clearance faces a lesser threat of cannibalization and is thus more profitable.

We next turn to the comparison of total food waste. We analytically show in Proposition 10 that the food-waste ordering in the second part of Proposition 5 is preserved.

**Proposition 10 (Comparison of Total Waste).** When \( \delta > 0 \), if \( c \) is high enough, surprise clearance generates the most total waste, followed by transparent clearance, and no clearance generates the least total waste; \( TW^{SC} > TW^{TC} > TW^{NC} \).

A high production cost induces a low production quantity. This implies that all the inventory is likely to be sold during regular sales, and there will not be much left afterwards. Hence, consumers who switch to clearance will most likely experience a stockout, which, in turn, reduces consumers’ incentive to switch. Thus, cannibalization is not a cause for concern, and the total waste comparison from Proposition 5 carries over. This implies that even after the threat of cannibalization is accounted for, both clearance schemes can still fail to reduce food waste, and surprise clearance can still be the worst scheme in food waste reduction despite its profit superiority.

We supplement our analytical results with a numerical investigation, summarized in Figures 5 and 6. In addition to confirming that surprise clearance continues to be the most profitable scheme (as analytically shown by Proposition 9), Figure 5 further shows that transparent clearance can fall short of no clearance in profit, especially when the production cost is low. In this case, the store has an incentive to produce a large quantity thanks to the low production cost. This nevertheless gives high-end consumers a strong incentive to wait for clearance because the store most likely will have inventory left for clearance sales. Thus, to stave off such a threat of cannibalization, the store must substantially reduce its production quantity (which reduces the likelihood that clearance sales occur, deterring the waiting behavior). Doing so may eventually undermine the store’s profit when it adopts transparent clearance. While surprise clearance is less affected by the threat of cannibalization (as argued earlier), we observe that when the threat is severe (i.e., \( \delta = 0.9 \)), the store may choose to turn off surprise clearance if the production cost is sufficiently low, in which case surprise clearance degenerates into no clearance.

Figure 6 presents the total food waste comparison. It first confirms the analytical result of Proposition 10 that if the production cost is sufficiently high, the store does not need to worry about cannibalization in either clearance scheme and the result from the base model is preserved, i.e., both clearance schemes fall short of no clearance in food waste reduction, with surprise clearance...
generating the most waste among the three. This can be seen by noticing that Region IV in Figure 6 is hardly changed from its non-cannibalization counterpart in Figure 4.

We also observe that as compared to the case without the threat of cannibalization (illustrated by Figure 4), Region II—in which transparent clearance generates the least waste—encroaches on Region I, in which surprise clearance generates the least waste. Moreover, the encroachment expands as the threat of cannibalization intensifies (i.e., as $\delta$ increases from 0.3 to 0.9). As explained earlier, transparent clearance is more susceptible to the threat of cannibalization than surprise clearance and the store running transparent clearance must reduce its production quantity more...
dramatically to deter consumers from waiting for clearance, a byproduct of which is a more substantial food waste reduction. However, the lower food waste of transparent clearance does come at the expense of a lower store profit (possibly even lower than that of no clearance). All told, the threat of cannibalization causes the “win-win” region (Region I) for surprise clearance to shrink, i.e., the market condition under which surprise clearance is the winner in both maximizing store profit and minimizing food waste among the three selling schemes becomes more stringent as the threat of cannibalization intensifies.

6. Concluding Remarks

This paper studies surprise clearance as an innovative business model to increase store profit and reduce food waste. Surprise clearance sells surprise bags composed of leftover inventory after regular sales. A key feature of a surprise bag is that its content, and in particular, the quantity of food items it includes is uncertain to consumers at the time of ordering. We compare surprise clearance with no clearance and transparent clearance, which sets a transparent unit clearance price contingent on the amount of leftover inventory.

We show that surprise clearance is more effective than transparent clearance in increasing store profit; it also triggers the most store production among the three selling schemes. Moreover, despite its highest production quantity, surprise clearance achieves zero store waste. Transparent clearance also has the potential to achieve zero store waste, but may end up generating more store waste than no clearance under the store’s optimal pricing. Further, both clearance schemes increase consumer waste. In particular, surprise clearance generates the least store waste but the most consumer waste among the three selling schemes, making the comparison of total food waste unclear. We find that both clearance schemes successfully reduce total waste relative to no clearance when clearance sales targets a consumer segment with low valuation of consumption. In such a case, surprise clearance can be a “win-win” solution that achieves both the highest store profit and the least total waste. Nevertheless, both clearance schemes can backfire and generate more total waste than no clearance, and surprise clearance can even generate the most total waste among the three schemes. We also find that surprise clearance is not as susceptible to the threat of cannibalization as transparent clearance, which strengthens its profit advantage over transparent clearance, but weakens its relative effectiveness in reducing food waste.

Overall, our work suggests that the novel surprise-clearance scheme can be a promising alternative to boost store profit and reduce food waste; we also tell a cautionary tale that surprise clearance may not always fulfill its claimed goal to combat food waste despite its ability to attain zero store waste. To that end, our paper highlights the importance of taking a more holistic approach in evaluating food waste, and in particular, the importance of accounting for food waste at various stages of food supply chains, especially that generated by end consumers.
Future research can study more sophisticated inventory allocation strategies in surprise clearance. For example, in our current model, the store honors all surprise-bag orders as long as there is leftover inventory after regular sales. Alternatively, the store can set a lower limit on the surprise bag size, which implies that if there is not too much leftover inventory, then some surprise-bag orders will get canceled to ensure that any surprise bags sold contain at least as many items as specified by the lower limit. Doing so protects consumers against an unpleasant surprise of a bag being too small and may enable the store to charge a higher price. The obvious downside is the loss of refunded orders. This further presents a commitment issue for implementing such a lower limit. When the clearance phase arrives, it is in the store’s best interest to not cancel any orders, and therefore the store will not be able to commit to the lower limit. In other words, the strategy of setting a lower limit cannot survive subgame perfection.

Instead of setting a lower limit, the store could also set an upper limit on the surprise-bag size. This could be helpful in counteracting the threat of cannibalization (complementing the strategy of limiting the initial production quantity considered in §5.3) because limiting the number of items a surprise bag may contain can detract from the appeal of buying surprise bags. Note that this strategy does not suffer from the aforementioned commitment issue because it does not involve cancellation (and surprise bags are pre-ordered). Imposing an (optimal) size limit in surprise clearance will not affect its profit-dominant status as it is already the most profitable scheme among the three. Nevertheless, doing so means possibly fewer items in each bag and more items left unsold. This implies that zero store waste is no longer guaranteed and surprise clearance’s prospect of being the most effective scheme in food waste reduction may further dwindle.

References


