This paper proposes a new form of farm debt that is designed to facilitate profitable changes in land use by highly indebted farms. Growing environmental concerns mean that many farmers will need to change the way they use their land if they are to remain financially viable. However, the debt overhang problem can create situations when profitable land-use changes are not actually in farmers' own best interests. The proposed form of debt features an early repayment provision that alleviates the debt overhang problem by allowing farmers to retain a greater share of the benefits of land-use changes.

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1 Introduction

Growing environmental concerns mean that many farmers will need to change the way they use their land if they are to remain financially viable. Unfortunately, the debt overhang problem can give highly indebted farmers a disincentive to undertake profitable changes in land use. The problem arises because farmers incur all the cost of changing farming practices, but share the benefits with their banks. It is not enough for the benefit of a change in land use to exceed the cost—it must exceed the cost by a sufficiently large margin that the farmer’s share of the benefit exceeds the total cost. Therefore, many value-enhancing changes in land use will not occur. This paper examines this mechanism and proposes an alternative form of debt that reduces the severity of the debt overhang problem, potentially benefitting farmers, banks, and the environment.

A farm is a portfolio of two different assets. The first asset is the “business as usual” (BAU) version of the farm, which is identical to the full farming operation in all respects except that no future change in land use is possible. The second asset is the real option to convert the farming operation to use the land in a different way.1 Traditional debt has the whole portfolio as collateral, so if the farmer defaults on the loan then the bank takes ownership of the BAU farm and the conversion option. The benefit-sharing causing the debt overhang problem occurs because profitable land-use change increases the value of the conversion option. The alternative form of debt proposed here weakens the bank’s claim on the farmer’s conversion option, while retaining its claim on the BAU farming operation. As a result, profitable land-use change has a much smaller effect on the bank’s collateral, so that the bank receives a smaller share of the benefits. This leaves a greater share of the benefits for the farmer, which gives the farmer a stronger incentive to undertake the farm conversion. The bank needs to be compensated for the weakening of its claim on the farm’s conversion option. However, enough value is potentially created by the new lending arrangement to allow an increase in interest rate that is simultaneously large enough to make the bank whole and small enough to enable the farmer to benefit from the change.

Under (idealised) traditional debt arrangements, the farmer continues paying an agreed coupon until the debt is repaid. If the farmer stops paying this coupon, the bank takes ownership of the collateral, in this case the farm. The only change to this arrangement proposed here is the inclusion of an innovative form of early-repayment option. This option allows the farmer to repay the loan early, any time after the change in land use has occurred, by paying the bank an amount equal to the estimated value of the hypothetical BAU farm. The presence of this repayment option means that the original loan’s present value to the bank is primarily determined by the present value of the hypothetical BAU farm. Changes in the value of the conversion option have a negligible effect on the loan’s present value, so the bank receives little benefit from profitable land-use changes. Instead, the farmer keeps most of the benefits and the severity of the debt overhang problem is greatly reduced.

From a bank’s perspective, a farmer exercising the repayment option has much in common with a BAU-farmer defaulting on a traditional loan. In the latter case, the bank receives a farm that it subsequently sells. In the former case, the bank does not take ownership of the farm, but it does end up with cash equal to the farm’s estimated value. Therefore, it is natural to interpret exercising the repayment option as triggering a form of “synthetic” default. However, this synthetic default is a default by a hypothetical BAU farm, because the bank does not receive the conversion option, or even cash equal to the conversion option’s value. It only has a claim on the BAU component of the farming operation.2 A key difference between synthetic and actual

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1This interpretation is analogous to decomposing generic firms into their assets-in-place and growth options, which is common in corporate finance modelling.

2This only applies to the “synthetic” default described here. The farmer still has the option to trigger an actual default, in which case the bank takes ownership of the farming operation as a whole, comprising the BAU farm and the conversion option.
defaults is that the bank does not incur any bankruptcy costs in the case of the former, which contributes to the economic benefits generated by switching to the alternative form of debt. The reduced expected bankruptcy costs will be shared by the bank and the farmer when the new interest rate is set.

If farmers are to receive the benefits of this form of borrowing, then they will need to refinance their existing (traditional) debt with the new variety before they change land use. Farmers and their banks will need to agree on the interest rate applied to the new loan and the mechanism for calculating the level of any future repayment. Provided the new interest rate is not too high, the farmer still benefits from refinancing because the new debt incentivises the farmer to adopt a conversion policy that is closer to the value-maximising one. The new interest rate will reflect the cost to the bank of embedding the repayment option in the new loan. Calculating this rate should not be an insurmountable problem for banks given their expertise in valuing options routinely embedded in other loan products. The other main implementation challenge will be setting the level of the repayment. This will depend on market conditions at the time of repayment, so it cannot be specified as part of the loan contract, but valuation formulas and methods for calculating inputs to these formulas could be. Alternatively, setting the repayment level—that is, estimating the market value of the hypothetical BAU farming operation—could be left to professional valuers. The absence of any conversion options in the hypothetical BAU farm might make the valuation problem simpler, although it will force valuers to use discounted cash flow methods rather than comparable sales approaches.

There is now an extensive literature using real options analysis to investigate agricultural land-use change. The simplest studies focus on a farmer’s exit decision, without detailed modelling of what the post-exit farm will look like (Pieralli et al., 2017). However, most authors focus on irreversible changes to land use that involve specific alternative crops or agricultural activities. In some studies, the decision-maker has non-financial motives (Strange et al., 2019), but the decision-maker is motivated solely by commercial considerations. This is the case for real-options studies of the decisions to convert to organic farming (Kuminoff and Wossink, 2010), to change from traditional agriculture to the cultivation of bio-energy feedstock (Song et al., 2011; Di Corato et al., 2013; Wolbert-Haverkamp and Musshoff, 2014; Regan et al., 2015, 2017; Dumortier et al., 2017), to switch from passive to active subsidised farming (Di Corato and Brady, 2019), and to engage in agroforestry (Reeson et al., 2015; Abdul-Salam et al., 2022). Other researchers have examined the option to switch from agricultural to non-agricultural activities. For example, Gazheli and Di Corato (2013) and Kim et al. (2020) consider the option to use farmland to host solar power generation equipment. Nishihara (2012) analyses the option to convert farmland for generic non-agricultural uses. However, despite the importance of farm debt in farmers’ decision-making, none of these papers considers debt and the incentives it creates. In contrast, examining the role of debt in land-use decision-making is at the heart of the current paper.

Another substantial literature uses real options analysis to investigate the effect of debt overhang on investment timing. Mauer and Ott (2000) initiated this literature with a simple model featuring a firm with the option to expand the scale of its operations. Their model has been extended in many directions, allowing for alternative stochastic processes (Chen and Manso, 2017), flexibility involving various aspects of the investment decision (Nishihara et al., 2019), and alternatives to the standard default arrangements (Pawlina, 2010). This literature has shown that debt overhang problems can be reduced if firms use less debt to finance current investment (Sundareshan et al., 2015), issue convertible debt (Lyandres and Zhdanov, 2014), or performance-

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3 An important second difference is that the “synthetic” default does not result in the farmer losing their home and feeling the stigma associated with financial failure. Thus, farmers are more likely to exercise the repayment option than they would be to default on the loan if they owned just the hypothetical BAU farm.

4 This can even be the case for conservation-based changes in land use. For example, Isik and Yang (2004) analyse a financial-value-maximising farmer who is considering participating in the Conservation Reserve Program in order to receive rental payments for establishing conservation practices.
sensitive debt (Sarkar and Zhang, 2015; Bensoussan et al., 2021), or use a mixture of bank debt and market debt (Gan et al., 2022). Unfortunately, these responses are of limited use in an agricultural setting, where farms tend to be relatively small businesses, often family owned and without access to debt markets. Recently, Guthrie (2024) has shown how sustainability-linked loans can reduce the severity of the debt overhang problem that incentivises highly indebted farms to over-exploit their natural capital. However, as with the earlier papers, Guthrie (2024) focusses on investments that enhance the borrower’s current operations. In contrast, the current paper studies the debt overhang problem in the context of conversion options, which requires a quite different underlying model of the firm and its investment opportunities.

Section 2 describes the theoretical framework that I use to study the new form of debt. Section 3 presents the baseline case corresponding to traditional farm debt and demonstrates the role of debt overhang in delaying value-enhancing changes in land use. Section 4 introduces the proposed new form of debt and shows how it mitigates the debt overhang problem. Section 5 switches the focus from farmer behaviour to the value of the farm. It examines the new debt’s effect on how a farm’s value is allocated between the farmer and the bank. I discuss practical implementation issues in Section 6 and offer some concluding remarks in Section 7.

2 Theoretical framework

Time is continuous and the risk-free interest rate, \( r \), is constant. A farm generates a continuous net cash flow of \( x_t \), which evolves according to the geometric Brownian motion

\[
dx_t = \mu x_t dt + \sigma x_t d\xi_t,
\]

where \( \mu_x \) and \( \sigma_x > 0 \) are constants and \( \xi_t \) is a Wiener process. If the farmer incurs lump-sum expenditure of \( k > 0 \), then the farm replaces the original net cash flow with \( y_t \), which evolves according to the geometric Brownian motion

\[
dy_t = \mu y_t dt + \sigma y_t d\zeta_t,
\]

where \( \mu_y \) and \( \sigma_y > 0 \) are constants and \( \zeta_t \) is a Wiener process. The two shocks are potentially correlated, so that \( (d\xi_t)(d\zeta_t) = \rho dt \) for some constant \( \rho \) satisfying \(-1 < \rho < 1\). Changing land use is irreversible.

The prices of \( x \)- and \( y \)-risk equal the constants \( \lambda_x \) and \( \lambda_y \). That is, the market value of any cash flow stream can be calculated by discounting the expected cash flow using the risk-free interest rate, provided the expected value is calculated using the risk-neutral process

\[
dx_t = (\mu_x - \lambda_x)x_t dt + \sigma x_t d\xi_t,
\]

\[
dy_t = (\mu_y - \lambda_y)y_t dt + \sigma y_t d\zeta_t.
\]

I assume that \( \mu_x < r + \lambda_x \) and \( \mu_y < r + \lambda_y \), so that all present values are defined. In order to focus on the key issues, I ignore taxes.

This is a very stylised model, but it captures the most important features of land-use changes. Many changes require upfront expenditure that is difficult to recover if the change is reversed. For example, planting new forests is expensive, as are building and installing the infrastructure needed to launch agritourism ventures or to generate solar power. Likewise, the net cash flow stream generated by the transformed farm can be quite different from pre-conversion net cash flows. In most cases, this will reflect a change in the farm’s outputs, so that profitability is determined by prices in different commodity markets. The correlation of pre- and post-conversion

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5 Most of the debt-overhang literature focusses on corporate investment. However, the under-investment problem it causes arises in many settings, including investment in home-improvements by mortgaged homeowners (Melzer, 2017), labour-market participation (Donaldson et al., 2019), and resource extraction policies (Wittry, 2021).
cash flows will depend on how radical the transformation is. For example, the correlation will probably be lower for a sheep farm converting to solar generation than a crop farm converting to organic farming practices. In the latter case, organic products might attract a price premium, but the demand and supply shocks that drive price volatility will be similar. The correlation will also depend on the scale of the farm transformation, with the correlation being relatively high for partial land conversions. For example, suppose a dairy farmer is proposing to reduce the herd size and introduce a new cash flow stream by building chalets for an agritourism operation. In this case, \( x_t \) is the cash flow from operating the dairy farm, whereas \( y_t \) is the cash flow from the diversified farming operation. The latter cash flow still has significant exposure to the profitability of dairy farming, so the correlation between \( x_t \) and \( y_t \) will be positive.

Some of the modelling assumptions are restrictive. For example, it would be more realistic to assume that reversing land-use change is merely costly rather than irreversible. A farmer can always clear land of newly planted in trees, remove accommodation constructed for an abandoned agritourism operation, or rebuild dairy infrastructure that was removed during a previous land-use change. The model assumes such reversals are too expensive to be viable future options. Another potentially restrictive assumption is that the cash flows generated by the two potential land uses evolve according to geometric Brownian motion. Alternate stochastic processes are possible, notably those featuring mean reversion. The choice of stochastic process will be consequential for quantitative applications (for example, calculating the increase in interest rate needed to compensate the bank for introducing the repayment option), but I believe the paper’s qualitative insights will apply for a wide variety of stochastic processes.

3 Traditional debt and the debt overhang problem

This section examines the effects of traditional debt on the farmer’s land-conversion decision. The farmer must pay a perpetual, continuous, constant coupon of \( c \) per unit of time in order to service the debt. She can default on the loan at any time, which ends the coupon payments and immediately transfers ownership of the farm to the bank.

3.1 Theory

Until the farmer defaults, she receives a continuous net cash flow that equals \( x_t - c \) before the change in land use and \( y_t - c \) after the change in land use. All cash flows to the farmer terminate when the farmer defaults. The farmer chooses default and land-conversion policies in order to maximise the present value of her ownership stake in the farm.

I solve the farmer’s problem by first supposing the farmer has converted the farm to the alternate land use and has not defaulted. It will be optimal to continue to service the debt until the (new) net cash flow falls below some constant level. The next lemma gives the optimal default threshold and the present value of the farmer’s ownership stake if she adopts this default policy.\(^6\)

\(*\) Lemma 1 (Optimal post-conversion policies) Suppose the farmer has already converted the farm to the alternate land use and that the loan coupon equals \( c \). If she plans to default the first time that \( y \leq \hat{y}_d \), for some constant \( \hat{y}_d \), then her ownership stake is currently worth

\[
\begin{align*}
\text{If } y &\leq \hat{y}_d, \\
\text{then } f_{\text{post}}(y; c) = & \left\{ \begin{array}{ll}
0 & \\
\frac{y}{r + \lambda_y - \mu_y} - \frac{\varepsilon}{r} + \left( \frac{\varepsilon}{r} - \frac{\hat{y}_d}{r + \lambda_y - \mu_y} \right) \left( -\frac{y}{\hat{y}_d} \right) & \text{if } y > \hat{y}_d
\end{array} \right.
\end{align*}
\]

\(^6\)This is equivalent to one of the first models used to determine shareholders’ optimal default policies (Leland, 1994).
where
\[
\gamma = \frac{1}{2} - \frac{\mu_y - \lambda_y}{\sigma_y^2} - \sqrt{\frac{2\rho}{\sigma_x^2} + \left(\frac{1}{2} - \frac{\mu_y - \lambda_y}{\sigma_y^2}\right)^2} < 0.
\]

The optimal post-conversion default threshold is
\[
\hat{y}_d = \frac{\gamma}{\gamma - 1} \cdot \frac{(r + \lambda_y - \mu_y)c}{r}.
\]

Now I step backwards to the before-conversion state. It is optimal to continue to service the debt until either the net cash flows from both land uses are so low that the farmer should default or the alternative land use becomes so profitable that the farmer should convert the farming operation. The next lemma gives the conditions that an optimal default–conversion policy must satisfy.

Lemma 2 (Optimal pre-conversion policies) Let \( f_{\text{pre}}(x, y; c) \) denote the present value of the farmer’s ownership stake, assuming the farmer has neither defaulted on the loan nor changed land use and that the loan coupon equals \( c \). If the farmer adopts an optimal default–conversion policy, then \( f_{\text{pre}} \) satisfies the system of variational inequalities
\[
\begin{align*}
\frac{\partial^2}{\partial x^2} f_{\text{pre}}(x, y; c) + \frac{\partial^2}{\partial y^2} f_{\text{pre}}(x, y; c) &\geq 0, \quad (2) \\
\frac{\partial^2}{\partial x^2} f_{\text{pre}}(x, y; c) + \frac{\partial^2}{\partial y^2} f_{\text{pre}}(x, y; c) &\geq f_{\text{post}}(y; c) - k, \quad (3) \\
0 &\geq x - c + D f_{\text{pre}}(x, y; c) - rf_{\text{pre}}(x, y; c), \quad (4)
\end{align*}
\]

where
\[
D = \frac{1}{2} \sigma_x^2 x^2 \frac{\partial^2}{\partial x^2} + \rho \sigma_x \sigma_y x y \frac{\partial^2}{\partial x \partial y} + \frac{1}{2} \sigma_y^2 y^2 \frac{\partial^2}{\partial y^2} + (\mu_x - \lambda_x) x \frac{\partial}{\partial x} + (\mu_y - \lambda_y) y \frac{\partial}{\partial y}.
\]

All three of these inequalities hold. At each point \((x, y)\), one of them will hold with equality. If (2) holds with equality, then the farmer’s best strategy is to default immediately, if (3) holds with equality, then the farmer’s best strategy is to change to the new land use immediately, and if (4) holds with equality, then she should continue to service the debt and wait.

Lemma 2 divides \((x, y)\)-space into three regions, each one corresponding to a different action being optimal. The point \((x_t, y_t)\), corresponding to the date-\(t\) values of the two measures of land-use profitability, fluctuates randomly in \((x, y)\)-space. As long as this point stays in the waiting region, the farmer should continue to service the debt. As soon as it moves into one of the other regions, the farmer should take the corresponding action immediately. These regions can be found by solving the system of variational inequalities in Lemma 2.

3.2 Example

I solve the variational inequalities (2)–(4) using the projected successive over-relaxation (SOR) method outlined in Appendix B. As numerical methods are essential for solving the farmer’s optimal conversion and default policies, it is necessary to specify values for the model’s parameters. For the example in this section, I choose
\[
\mu_x = \mu_y = 0, \quad \sigma_x = \sigma_y = 0.1, \quad \rho = 0, \quad \lambda_x = \lambda_y = 0, \quad r = 0.04, \quad k = 10.
\]

Note that the two land uses have identical growth rates, risks, and risk premia, so the only difference at any point in time is their current profitability (and therefore the distributions of their future profitability). I consider various values of the loan coupon \( c \).

Figure 1 illustrates the farmer’s optimal conversion–default policy. Each graph corresponds to a different level of debt, with the farm shown in the top-left graph being debt-free. If the
Figure 1: Optimal conversion policy for different levels of traditional debt

Notes. Each graph shows the farmer's optimal policy when the farm is in its original state. If the combination of \((x_t, y_t)\) lies in the dark-grey region, then the farmer should default on the loan immediately. If it lies in the light-grey region, then the farmer should convert to the new land use immediately. Otherwise, the farmer should wait until \((x_t, y_t)\) moves into one of the shaded regions and then act accordingly.
combination of \((x_t, y_t)\) lies in the dark-grey region, then the farmer should default on the loan immediately. If it lies in the light-grey region, then the farmer should convert to the new land use immediately. Otherwise, the farmer should wait until \((x_t, y_t)\) moves into one of the shaded regions and then act accordingly. In terms of Lemma 2, (2) holds with equality in the dark-grey region, (3) holds with equality in the light-grey region, and (4) holds with equality in the unshaded region.

The top left-hand graph in Figure 1 shows the optimal policy for the owner of a debt-free farm. In this case, the farmer only has to decide when to exercise the conversion option. If conversion were a now-or-never decision, then the farmer would convert to the new land use if and only if

\[
\frac{y}{r + \lambda_y - \mu_y} - k > \frac{x}{r + \lambda_x - \mu_x}.
\]

Solving this inequality for \(y\) shows that the “breakeven” investment threshold is

\[
y = (r + \lambda_y - \mu_y) \left( k + \frac{x}{r + \lambda_x - \mu_x} \right).
\]

This is shown by the dotted line in the graph. The boundary of the shaded region lies above the breakeven threshold. Thus, it is optimal for the farmer to wait until \(y\) is significantly greater than the breakeven level before investing. The premium reflects the value of the option to delay conversion and potentially avoid making a costly mistake. In particular, in order for immediate conversion to be optimal, the value of the converted farm, \(y/(r + \lambda_y - \mu_y)\), must exceed the opportunity cost, which is the sum of: the investment expenditure, \(k\); the value of the farming operation assuming its land use never changes, \(x/(r + \lambda_x - \mu_x)\); and the value of the option to delay conversion and reconsider it in the future.

The remaining graphs in Figure 1 show the optimal policies of farmers with different amounts of debt. The dashed curve in each graph shows the optimal conversion threshold when the farm has no debt. As we move down the left-hand column, and then down the right-hand column, the level of debt increases and the (dark grey) default region grows. Consider, first, what happens along the horizontal axis of each graph, where \(y = 0\) and the conversion option is worthless. If the farmer had to make a now-or-never default decision, then it would be optimal to default if and only if \(x/(r + \lambda_x - \mu_x) < c/r\). For the parameter values used in Figure 1, this condition reduces to \(x < c\). The graphs show that when the farmer has the option to delay default by injecting additional cash into the business, it is optimal to keep injecting cash unless \(x\) is significantly below \(c\). In other words, the option to delay default is valuable because it allows the farmer to potentially avoid a costly mistake—profitability may bounce back, leaving a defaulting farmer wishing they had waited a little longer and been able to stay on the farm.

As \(y\) increases, the conversion option becomes more valuable, so that defaulting on the farm’s loan becomes even more costly. Accordingly, the graphs in Figure 1 show the default region becoming slightly narrower as \(y\) increases. However, the farmer’s choice is effectively the same: to default or to continue to operate the farm in its current land use while servicing the debt. This changes once \(y\) is sufficiently large that conversion becomes a viable policy. For moderate levels of \(y\), when defaulting and changing land use have similar payoffs, the farmer waits until the picture becomes clearer. This behaviour corresponds to the regions in the graphs where the unshaded region extends towards the vertical axis. Waiting helps the farmer avoid making a costly mistake, such as defaulting and then observing \(y\) increase, or investing in a farm conversion, only to watch \(y\) fall and regret not defaulting before sinking capital into the conversion. For higher levels of \(y\), the farmer’s choice is effectively between changing land use and continuing to operate the farm in its current land use while servicing the debt.

Comparing the light-grey region in the top left-hand graph with the corresponding regions in the other graphs reveals an under-investment problem. That is, a high debt level creates a disincentive for the farmer to make a profitable change in land use. If \(y\) is sufficiently high, then
there is no noticeable difference between the light-grey regions. That is, if conversion is extremely profitable, then the timing of the farm conversion will be independent of the amount of debt. However, for low levels of \( y \), the farmer will default rather than invest in a profitable land-use change. For moderate levels of \( y \), the farmer will—at best—significantly delay conversion compared to the debt-free case. The economic intuition explaining these results can be found in the debt overhang problem, which can prevent an indebted firm from investing even if the combined increase in the value of a firm’s debt and equity is significantly greater than the cost of undertaking the investment (Myers, 1977). In the situation described in this section, value-enhancing investment (that is, the change in land use) benefits the bank by making default less likely and by increasing the farm’s liquidation value in the event of default, but the bank does not contribute to the cost of the investment. The farmer, who ultimately makes the investment decision, incurs the entire cost of the investment, but has to share the benefits with the bank. If the bank captures a sufficiently large share of the benefits, it is not in the farmer’s best interest to invest.

For high debt levels and high values of \( y \), there is also a weak over-investment problem. This can be seen in the graphs corresponding to coupons of \( c = 4 \) and \( c = 5 \), where the light-grey region extends below the dashed curve in places. In the situations where this occurs, a debt-free farmer would delay the farm conversion in order to reduce the likelihood of changing land use and then seeing the original land use become more profitable than the new one. However, when a farm has high debt levels, the farmer has to consider the risk that default might occur while conversion is delayed, in which case the farmer will lose access to the conversion option. This default risk lowers the value of the real option to delay land conversion below the debt-free case, which leads the indebted farmer to change land-use earlier than an otherwise identical debt-free farmer.\(^7\)

4 A new form of farm debt

Any farm can be interpreted as a portfolio made up of two assets. The first asset is a hypothetical version of the farm that remains in its current land use forever (the “business as usual” or BAU version of the farm) and the second is the real option to change the land use at a time chosen by the farm’s owner. Traditional debt of the type studied in Section 3 is a claim on this portfolio. That is, if the farmer defaults, then the bank takes ownership of the portfolio. In contrast, the new form of debt proposed in this section is (almost) a claim on just the first part of the portfolio. It contains an early-repayment feature that is designed to separate out the two components of the farm portfolio, so that the conversion option is to some extent not part of the loan’s effective collateral. The feature gives the farmer the right to retire the loan completely by paying the bank a lump sum equal to the present value of the hypothetical BAU farm. The farmer can exercise this repayment option any time after changing land use. The bank only has a claim on the converted farm if the farmer cannot pay the value of the BAU farm.

In terms of the model described in Section 2, the farmer must pay a perpetual, continuous, constant coupon of \( c \) per unit of time in order to service the new form of debt. As for traditional debt, the farmer can default on the loan at any time. This terminates the coupon payments, but ownership of the farm (including the conversion option) is immediately transferred to the bank. However, unlike traditional debt, the farmer also has the option to retire the debt at any time after changing land use. If she exercises this option at the arbitrary date \( t \), then she must pay the bank a lump sum of \( x_t/(r + \lambda x - \mu x) \), which equals the value of the hypothetical BAU farm. The farmer retains ownership of the (converted) farm in this case. This structure ensures that

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\(^7\)Boyle and Guthrie (2003) identified a similar mechanism that motivates corporations to invest earlier than standard real-option theory predicts if they face significant risk of liquidity problems that would prevent them from financing delayed investment. That is, corporations may invest early if they have funding lined up rather than wait for a better time to invest and run the risk of not being able to raise the required funds.
the loan is not a claim on the conversion option, except in the event of an actual default.

4.1 Theory

As in the case of a traditional debt contract, I analyse the new contract by considering the situations the farmer faces before and after she exercises the land-conversion option. The solution process starts by solving the farmer’s post-conversion problem.

Suppose the farmer has exercised the conversion option, so that she is receiving a net cash flow of \( y_t - c \) from the farm. She has three options: to default; to repay the loan using the repayment mechanism; and to continue servicing the loan during the next increment of time and then reevaluate the situation. The presence of the repayment option means it is not possible to derive an exact solution as in Section 3.1. Instead, the farmer’s value function and her optimal default–repayment policy must be calculated by solving a system of variational inequalities. The next lemma gives the details.

**Lemma 3 (Optimal post-conversion policies)** Let \( f_{\text{post}}(x, y; c) \) denote the present value of the farmer’s ownership stake, assuming the farmer has already changed land use but has not yet defaulted and that the loan coupon equals \( c \). If the farmer adopts an optimal default–repayment policy, then \( f_{\text{post}} \) satisfies the system of variational inequalities

\[
\begin{align*}
    f_{\text{post}}(x, y; c) &\geq 0, \quad (6) \\
    f_{\text{post}}(x, y; c) &\geq \frac{y}{r + \lambda_y - \mu_y} - \frac{x}{r + \lambda_x - \mu_x}, \quad (7) \\
    0 &\geq y - c + Df_{\text{post}}(x, y; c) - rf_{\text{post}}(x, y; c), \quad (8)
\end{align*}
\]

where \( D \) is defined in equation (5). All three of these inequalities hold. At each point \((x, y)\), one of them will hold with equality. If (6) holds with equality, then the farmer’s best strategy is to default immediately, if (7) holds with equality, then the farmer’s best strategy is to repay the loan immediately, and if (8) holds with equality, then she should continue to service the debt and wait.

Lemma 3 divides \((x, y)\)-space into three regions, each one corresponding to an optimal action for the post-conversion farmer. As long as the point \((x_t, y_t)\) stays in the waiting region, the farmer should continue to service the debt. As soon as it moves into one of the other regions, she should take the corresponding action immediately, either defaulting or exercising the repayment option. These regions can be found by solving the system of variational inequalities in Lemma 3.

As in Section 3.1, the next step in solving the model is backwards to the before-conversion state, when the farmer is receiving a net cash flow of \( x_t - c \) from the farm. As the farmer chooses from the same possible actions as in Section 3.1—default immediately, change land use immediately, and continue to service the debt during the next increment of time—the system of variational inequalities is almost unchanged. The only change is that the farmer’s payoff if she exercises her conversion option is a function of \( x \) and \( y \), rather than just \( y \), consistent with the new post-conversion value function in Lemma 3. The solution to the farmer’s pre-conversion problem is described in the following lemma.

**Lemma 4 (Optimal pre-conversion policies)** Let \( f_{\text{pre}}(x, y) \) denote the present value of the farmer’s ownership stake, assuming the farmer has neither defaulted on the loan nor changed land use and that the loan coupon equals \( c \). If the farmer adopts an optimal default–conversion policy, then \( f_{\text{pre}} \) satisfies the system of variational inequalities

\[
\begin{align*}
    f_{\text{pre}}(x, y; c) &\geq 0, \quad (9) \\
    f_{\text{pre}}(x, y; c) &\geq f_{\text{post}}(x, y; c) - k, \quad (10) \\
    0 &\geq x - c + Df_{\text{pre}}(x, y; c) - rf_{\text{pre}}(x, y; c), \quad (11)
\end{align*}
\]
where $D$ is defined in equation (5). All three of these inequalities hold. At each point $(x, y)$, one of them will hold with equality. If (9) holds with equality, then the farmer’s best strategy is to default immediately, if (10) holds with equality, then the farmer’s best strategy is to change to the new land use immediately, and if (11) holds with equality, then she should continue to service the debt and wait.

Lemma 4 divides $(x, y)$-space into conversion, default, and waiting regions. Calculating an optimal policy requires two rounds of numerical calculations. The first round solves the variational inequalities in Lemma 3 for the post-conversion farm and the second round solves the variational inequalities in Lemma 4 for the pre-conversion farm.

### 4.2 Example

Figure 2 illustrates the farmer’s optimal conversion–default policy when the alternative form of debt is in place. It uses the same parameter values as in Section 3.2, so the only difference from Figure 1 is the inclusion of the repayment option. The format is the same as in Figure 1. That is, each graph shows the farmer’s optimal policy when the farm is in its original state, with the dark-grey region showing the combinations of $(x_t, y_t)$ for which immediate default is best for the farmer and the light-grey region showing the combinations where the farmer should convert to the new land use immediately. The farmer should delay taking either action as long as $(x_t, y_t)$ is in the unshaded region.

The most obvious change is that the default region is smaller than for a traditional loan with the same coupon. The approximately square default region is replaced with one that is approximately triangular. Previously, when $x$ and $y$ were both low, it made sense for the farmer to default rather than convert because a large proportion of the conversion benefits accrued (without compensation) to the bank. Now, thanks to the repayment option, the farmer can change land-use and immediately repay the loan without having to share the benefits of the conversion option. Compared to Figure 1, the light-grey conversion region moves down to fill in the “vacated” part of the default region. Figure 1 shows that $y$ has to be relatively large before conversion is optimal when the farm has traditional debt outstanding. Now, however, with the alternative form of debt in place, immediate conversion can be optimal even if $y$ is relatively small.

The dashed curve in each graph in Figure 2 shows the optimal conversion threshold when the farm has no debt. That is, the value of a debt-free farm is maximised if the farmer initiates the land-use change the first time that $(x_t, y_t)$ lies above this curve. With traditional debt, there was under-investment for low values of $y$ and (for relatively high debt levels) slight over-investment for high values of $y$. Figure 2 shows that using the new form of debt eliminates the under-investment problem, but it does so at the cost of intensifying the over-investment problem. In particular, for high debt levels, the optimal conversion threshold is noticeably lower than the optimal debt-free conversion threshold. The farmer is motivated to change land-use early in order to be able to repay the loan on favourable terms.

---

8 The farmer’s optimal post-conversion default–repayment policies are reported in Appendix C, using graphs with a similar format to Figures 1 and 2.

9 Disallowing pre-conversion loan repayment is largely a modelling choice. Deriving the farmer’s optimal conversion policy if pre-conversion loan repayments were allowed would require calculating the farmer’s repayment payoff for each point $(x, y)$ where the farmer might repay the loan. This would be straightforward if the repayment were funded by an equity injection, but farmers are much more likely to fund repayment by taking out new loans. That is, they would borrow the money needed to repay the loan and then continue to pay the new loan coupon until they either repay the new loan, change land use, or default. Unfortunately, in order to do this, we would need to calculate the new loan coupon, which would involve solving the farmer’s problem for multiple possible new coupons in order to find a coupon that would induce a new lender to participate. To make matters worse, we would need to allow for the new loan to be repaid in the future. The computational complexity of this exercise would be substantial and would take us far beyond the scope of the current paper.
Notes. Each graph shows the farmer’s optimal policy when the farm is in its original state. If the combination of \((x_t, y_t)\) lies in the dark-grey region, then the farmer should default on the loan immediately. If it lies in the light-grey region, then the farmer should convert to the new land use immediately. Otherwise, the farmer should wait until \((x_t, y_t)\) moves into one of the shaded regions and then act accordingly.

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5 Implications for the present value of farm debt and equity

The focus of the paper so far has been on how debt affects when farmers exercise their conversion, default, and (if they exist) repayment options. That is, the focus has been on farmer behaviour. This section considers the effect of debt on farm value. How a farmer is affected by switching to the alternative form of debt depends on the size of any changes to the loan’s interest rate. Such a change is likely because the bank will require compensation for granting the repayment option. We therefore need to look at the situation from the bank’s perspective. In practice, the bank will have to benefit from the change (or at least not be made worse off) in order to agree.

The key consideration for a bank will be the present value of the payments that the farmer will make to the bank in accordance with the loan agreement, less any costs the bank incurs as part of any bankruptcy process. These include the costs of the lawyers, accountants, real estate agents, and others involved in the bankruptcy process. They will also include the effect on the farm’s value of the loss of the farmer’s local knowledge after bankruptcy, as well as the possibility that the farm was poorly managed during the period leading up to default. I model these costs by assuming that immediately after the farmer defaults, the value of the farm to the bank is the present value of an otherwise identical debt-free farm with cash flow parameters $(1 - \theta)x$ and $(1 - \theta)y$, for some constant $\theta$ satisfying $0 \leq \theta < 1$.\(^{10}\) For example, if default occurs after the change in land use, bankruptcy costs the bank $\theta y / (r + \lambda y - \mu y)$. In the numerical example below, I assume that $\theta = 0.25$.\(^{11}\)

Consider the situation after the conversion option has been exercised. If the farmer defaults, then the bank takes ownership of a farm that it can sell for $(1 - \theta)y / (r + \lambda y - \mu y)$. On the other hand, if the farmer repays the loan, the bank receives a lump sum of $x / (r + \lambda x - \mu x)$. These results lead to the following lemma, which specifies the equations that determine the present value of the loan, $\ell_{\text{post}}(x, y; c)$.

**Lemma 5 (Post-conversion value of loan)** Let $\ell_{\text{post}}(x, y; c)$ denote the present value of the loan, assuming the farmer has already changed land use but has not yet defaulted and that the loan has a coupon of $c$. The equation that $\ell_{\text{post}}(x, y; c)$ must satisfy depends on the action the farmer will take at $(x, y)$:

\[
\begin{align*}
\text{Default:} & \quad \ell_{\text{post}}(x, y; c) = \frac{(1 - \theta)y}{r + \lambda y - \mu y}, \\
\text{Repay:} & \quad \ell_{\text{post}}(x, y; c) = \frac{\theta y}{r + \lambda y - \mu y}, \\
\text{Service debt:} & \quad 0 = c + D\ell_{\text{post}}(x, y; c) - r\ell_{\text{post}}(x, y; c),
\end{align*}
\]

where $D$ is defined in equation (5). The second row will not occur for traditional debt.

Now consider the situation before the conversion option has been exercised. If the farmer defaults, then the bank takes ownership of the farm, which it can then sell. The farm will be most valuable to a bidder with no debt, so a sensible predicted sale price is the value of a debt-free farm, adjusted for bankruptcy costs: $f_{\text{pre}}(1 - \theta)x, (1 - \theta)y; 0)$. On the other hand, if the farmer changes land use then the loan will be worth $\ell_{\text{post}}(x, y; c)$ to the bank. These results lead to the following lemma, which specifies the equations that determine the present value of the loan, $\ell_{\text{pre}}(x, y; c)$.

\(^{10}\)As $x_t$ and $y_t$ evolve according to geometric Brown motion, this means that all future values of these variables will be reduced by the same proportion.

\(^{11}\)I am not aware of any estimates of bankruptcy costs for farms, and published estimates of bankruptcy costs for firms in general vary widely. For example, Hennessy and Whited (2007) estimate that bankruptcy costs equal 8% of a firm’s capital for large firms and 15% for small firms. In contrast, Glover (2016) obtains estimates with a mean of 45% of an otherwise equivalent all-equity firm’s value. There is evidence that firms with a lower cost of default choose a higher level of leverage, and so are more likely to default (Glover, 2016). Average observed bankruptcy costs (which correspond to only those firms that default) will therefore be lower than the “true” average bankruptcy cost.
Figure 3: How banks and farmers are affected by the introduction of the repayment option

(a) Present values with traditional and alternative debt

(b) Change in present values

Notes. The top three graphs plot the present values of the farmer’s ownership stake (blue) and the loan (yellow) as functions of $x$, with each graph corresponding to a different level of $y$. The solid curves plot the present values for a traditional loan with a coupon of $c = 5$ and the dashed curves plot the present values for the alternative loan structure with the same coupon. The bottom three graphs plot the difference in the present values for traditional and alternative debt for the same parameter combinations and the same three levels of $y$ as in the top row of graphs.

Lemma 6 (Pre-conversion value of loan) Let $\ell_{\text{pre}}(x, y; c)$ denote the present value of the loan, assuming the farmer has neither defaulted on the loan nor changed land use and that the loan has a coupon of $c$. The equation that $\ell_{\text{pre}}(x, y; c)$ must satisfy depends on the action the farmer will take at $(x, y)$:

- Default: $\ell_{\text{pre}}(x, y; c) = f_{\text{pre}}((1 - \theta)x, (1 - \theta)y; 0)$,
- Convert: $\ell_{\text{pre}}(x, y; c) = \ell_{\text{post}}(x, y; c)$,
- Service debt: $0 = c + D\ell_{\text{pre}}(x, y; c) - r\ell_{\text{pre}}(x, y; c)$,

where $D$ is defined in equation (5).

I illustrate how farmers can potentially gain from refinancing traditional debt with the proposed alternative form by continuing the numerical examples in Sections 3.2 and 4.2. The top row of graphs in Figure 3 plot the present values of the farmer’s ownership stake (blue) and the loan (yellow) as functions of $x$, with each graph corresponding to a different level of $y$. The solid curves plot the present values for a traditional loan with a coupon of $c = 5$ and the dashed curves plot the present values for the alternative loan structure with the same coupon. Consider the case of traditional debt first. In the left-hand graph, if $x$ is sufficiently low, the farmer defaults, leaving the farmer with nothing and the bank with a farm with conversion option intact. In contrast, in the other two graphs the farmer exercises the conversion option if $x$ is sufficiently low. The value of the farmer’s equity is increasing in $y$ and insensitive to $x$.

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in this region, reflecting the fact the farmer’s future cash flows depend only on $y$. The loan’s present value to the bank is very weakly increasing in $y$ and insensitive to $x$, reflecting the fact that default risk is low and determined solely by future values of $y$. Now consider the case of the alternative loan structure. For these values of $y$, there is no possibility of default. Instead, if $x$ is sufficiently low, the farmer exercises the repayment option, which increases the present value of the farmer’s equity by reducing the future debt burden. This adversely affects the loan’s present value to the bank, which is now strongly increasing in $x$ and insensitive to $y$.

Comparing the dashed and solid curves shows that switching to the new form of debt makes the farmer better off and the bank worse off. This is unsurprising: the assumption that the coupon does not change means that the farmer is getting something (the repayment option) for nothing, so of course the farmer is better off. However, the gain is smaller when $x$ is larger (and exercising the repayment option is less likely). The bottom row of graphs in Figure 3 shows the difference in the present values for traditional and alternative debt for the same parameter combinations and the same three levels of $y$ as in the top row of graphs. That is, the blue curve shows how much the farmer gains from introducing the repayment option and the yellow curve shows how much the bank loses in present-value terms. The green curve show the sum of the gains to the farmer and the bank. This curve is positive-valued in all cases, indicating that introducing an (uncompensated) repayment option increases the value of the farmer’s ownership stake by more than it decreases the present value of the loan. Therefore, there is potential for the two sides to agree on terms of the new loan (that is, a higher interest rate) that allow both parties to benefit.

The farmer and the bank must both agree to the change in order for the original debt to be replaced with the alternative loan arrangement. In all the cases illustrated above, introducing the repayment option without adjusting the loan coupon benefits the farmer at least as much as it costs the bank. It is therefore possible to increase the loan coupon sufficiently to compensate the bank and still allow the farmer to be better off than if the original loan arrangement remained in place. The left-hand graph in Figure 4 plots the new level of the loan coupon as a function of $x$ for four different levels of $y$, where $x$ and $y$ are measured at the time of refinancing. In all cases, the original loan has a coupon of $c = 5$ and all parameters take their baseline values. The adjustment that must be made to the loan coupon in order to compensate the bank for introducing the repayment option is larger when $x$ is smaller and $y$ is larger. Both results have simple explanations.

- When $x$ is smaller, the farmer is increasingly likely to exercise the repayment option soon, so granting the repayment option is more costly to the bank. It requires greater up-front compensation in the form of a higher loan coupon. Thus, the adjusted coupon increases when $x$ falls: each curve is downward sloping.

- When $y$ is larger, the conversion option is more valuable. When the bank agrees to the repayment option, it loses most of its claim to the conversion option as collateral. That is, although the bank still receives the conversion option if the farmer defaults, it does not receive it if the farmer undertakes a “synthetic” default by exercising the repayment option. 

\[12\] In some rare situations, not shown in the graphs, the bank gains even though the loan coupon is unchanged. This can occur when the farmer would default with traditional debt in place, but exercises the repayment option when the alternative form of debt is in place. In the latter case, the bank does not receive the conversion option but avoids having to incur any bankruptcy costs. In some cases, the avoided bankruptcy costs exceed the value of the conversion option.

\[13\] For a selection of coupons $c'$, I solve the model to obtain the present values of debt and equity for the alternative form of debt with a coupon of $c'$. For each of the values of $y$ used in Figure 4, I calculate the level of $x$ such that the present value of the farm’s debt equals the present value of the original loan. This gives a combination $(x, y, c')$ that makes the bank whole. These are the points that are plotted in the left-hand graph. The right-hand graph plots the percentage increase in present value of the farm’s equity at each of these points. That is, for each combination of $(x, y)$, I calculate the present value of farm equity at the level of $c'$ that makes the bank whole and compare it with the value of equity with the original debt.

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Notes. The left-hand graph plots the new level of the loan coupon as a function of $x$ for four different levels of $y$, where $x$ and $y$ are measured at the time of refinancing. The right-hand graph plots the percentage increase in the value of the farmer’s ownership stake as a function of $x$ for the same levels of $y$. The calculations assume that the coupon is increased by just enough that the loan’s value to the bank is unaffected by the change in loan structure. In all cases, the original loan has a coupon of $c = 5$ and all parameters take their baseline values.

The right-hand graph in Figure 4 plots the percentage increase in the value of the farmer’s ownership stake as a function of $x$ for four different levels of $y$, where $x$ and $y$ are measured at the time of refinancing. The calculations assume that the loan’s coupon is increased by just enough that the value to the bank of this loan is unaffected by the change in loan structure. That is, the bank is indifferent to the (compensated) introduction of the repayment option. The farmer’s gain from refinancing the loan is sensitive to the level of the two cash flows at the time the refinancing occurs. The relationship between the size of the farmer’s gain and the values of these two cash flows is complex, but the farmer will generally benefit more if the cash flow from the current land use is relatively low.

### 6. Implementation issues

The analysis so far has used a theoretical model of a farm with the option to change land use. This section switches focus to how the new loan arrangements might be implemented in practice.

#### 6.1 What is in it for banks?

Inserting the proposed form of repayment option into traditional farm loan agreements can potentially unlock the value of farms’ conversion options and reduce the probability of default. The more profitable use of existing farmland and the lower expected bankruptcy costs that result create an economic surplus that can be shared by farmers and banks, making both better off. The size of any adjustments to the interest rates paid by farmers determines how this economic surplus is shared. The possibility of future early repayment of loans on terms favourable
to farmers can be factored into interest-rate adjustments, so banks need not lose ex ante by introducing the proposed repayment options.

Banks will benefit from the lower likelihood of farmers defaulting on their loans. When a default occurs, the bank takes ownership of the farm and subsequently puts it up for sale. As the amount of the outstanding loan will typically exceed the sale price, the bank keeps the sale proceeds and incurs the costs of the lawyers, accountants, real estate agents, and others involved in the bankruptcy process. Compounding the bank’s problems, the sale price will be relatively low if the farm was poorly managed during the period leading up to default. This will often be the case as the farmer has weaker incentives (and perhaps less ability) to manage the farm well when bankruptcy is imminent. The bank’s situation is even worse if several farms in a location are struggling financially due to the prospects of a “fire sale” lowering the sale price below the farm’s fundamental value. In sum, bankruptcy costs will be substantial. The alternative form of debt proposed in this paper reduces the probability of default, which reduces the expected value of the bankruptcy costs incurred by banks.

When a farmer with the new form of loan exercises their repayment option, the bank receives an amount of cash equal to the present value of the farm if it continued with the initial land use indefinitely. Therefore, the bank’s payoff is similar to what it would have received if the farmer had defaulted on a traditional loan (and the farm had no conversion option). Moreover, this is a “synthetic” default, which has three important benefits. First, there is no need for the bank to manage the process of moving the farmer off the farm. Second, the valuation underpinning the repayment can be based on a “benchmark” farm, so the bank need not incur any land-mismanagement costs if the farming operation was neglected leading up to the repayment. Third, there is no fire-sale discount embedded in the valuation underpinning the repayment. The bank still incurs some costs—lawyers, valuers, and potentially arbitrators will be involved in the repayment process—but these costs will almost certainly be lower than the costs the bank would incur after a full default.

The bank also benefits from the effects the new form of debt have on the farmer’s conversion policy. It benefits directly due to the farm’s higher liquidation value in the event of post-conversion default and indirectly via the economic surplus created and shared by the farmer and the bank when the interest rate is reset.

6.2 How would the repayment level be set?

Following an actual loan default, ownership of the farm transfers to the lender, who then sells the farm. The transaction price is observable. However, the loan repayment envisaged here only reflects a “synthetic” default. There is no sale, and therefore no transaction price that can be used to value the hypothetical BAU farm. Instead, the two parties must use an estimate of the farm’s value. Successful implementation of the proposed new form of debt hinges on farmers and banks accepting the value of the hypothetical BAU farm that determines the repayment level. This cannot be set in advance, as the value of the hypothetical farm will depend on market conditions on the repayment date. Instead, this value will have to be estimated by professional valuers. In practice, borrower and lender could each select a valuer from a previously agreed list and use their estimates to set the repayment level, with a formal arbitration process available if the parties cannot agree.

The approaches used to value farms fall into two broad categories. Discounted cash flow (DCF) methods estimate farms’ expected future net cash flows and discount these estimates using a risk-adjusted discount rate. Comparable sales methods use transaction prices from recent sales of similar farms to infer valuation multiples (such as dollars per hectare) that can be used to estimate the farm’s likely sale price.

14 Indeed, benchmarking will be necessary if some time has elapsed between the change in land use and the farmer repaying the loan.
• The expected net cash flows estimated as part of DCF valuation typically reflect the farm’s current land use. This is problematic when estimating the market value of a farm because it ignores any real options to change land use. However, this is actually an advantage for the purposes of valuing a hypothetical BAU farm.

• One of the main features of comparable sales approaches is that transaction prices include the value of any conversion options in the farm “portfolio” being acquired by the purchaser. However, while this is an advantage when estimating the market value of an actual farm, it is a distinct disadvantage when valuing a hypothetical BAU farm.

Valuers will therefore need to rely on DCF approaches to value the hypothetical BAU farm. If they use a comparable sales approach, then they will need to either restrict their sample of recent transactions to those farms with minimal conversion options or somehow adjust observed prices to remove the effect of any conversion options. Given that experienced valuers will have worked during periods with low transaction volumes, the increased emphasis on DCF-based farm values needed to value hypothetical BAU farms should not pose serious problems. Excluding conversion options from the farm being valued might actually make the hypothetical farm easier to value.

6.3 How would the interest rate be set?

The repayment option is valuable to the farmer and costly to the bank. The interest rate applied to the new form of debt will therefore be higher than the rate applied to otherwise identical traditional debt. The question that has to be resolved is: how much higher?

Two different situations might arise. In the first, a bank is issuing a new loan to a farmer. Competition between banks will mean the interest rate is set so that the loan’s present value to the bank equals the amount being loaned to the farmer. That is, the interest rate will be market-determined, reflecting the risks of default and early repayment. In the second case, a bank is replacing existing traditional debt with the new form of debt, so the interest rate needs to be set such that the bank is no worse off compared to the case if the existing loan continues.

In both cases, the bank’s key task is calculating the level of the interest rate that compensates it for granting the farmer an early repayment option. This is a fairly standard problem for banks, as they already offer loan products with repayment and other embedded options as part of their wider operations. Banks should therefore have experience in setting margins for embedded options. The specific arrangements here are different, but the guiding principles are the same.

As a first approximation, banks can analyse the repayment option in the same way they would analyse a traditional loan to a BAU farm. The interest rate will be set at a level that exceeds the risk-free rate by a margin that reflects the estimated default probability and the loss given default. A complicating factor is that farmers will be more willing to trigger a “synthetic” default by repayment compared to actual default because they get to retain ownership of the farm, live on the land, and avoid the stigma associated with bankruptcy. Thus, banks should not use their usual default probabilities when setting interest rates for these loans. This will require estimating new probability models, but the problem should not be insurmountable. In fact, theoretical models might be more useful in predicting synthetic default than they are in predicting actual default, because the former is a purely financial decision, without the human factors associated with the latter. Offsetting this, banks should use lower loss rates, reflecting the synthetic nature of the default event and the resulting absence of the usual bankruptcy costs.\footnote{These two differences are also relevant for calculating the credit risk weights attached to farm debt for the purposes of banking regulation. These weights convert the actual size of an exposure into a risk-weighted asset that determines how much capital banks must hold.}
6.4 Would these loans create any unwelcome incentives?

The analysis in Section 4 assumed that farmers cannot exercise the repayment option before they exercise the conversion option, primarily for modelling convenience. As was evident in Figure 2, this gives farmers an incentive to change land-use inefficiently early—in some situations, they gain more from repaying loans on favourable terms than they lose from changing land use too soon.

There is another, more subtle, potential problem. In the theoretical analysis, the change in land use is unambiguous. That is, there are clear “before” and “after” states. The bank and the farmer will agree on when land use has changed (even if they disagree on when it should have changed). However, there can be much more ambiguity in practice. For example, if a farmer reduces their herd size by ten percent in order to plant trees for carbon farming, will the bank agree that this reduction is large enough to constitute a change in land use for the farm as a whole and thereby activate the repayment option? If not, would a 20 percent reduction be large enough? Banks will have a strong incentive to argue that the change in land-use has not risen to the threshold needed to activate the repayment option. This potential source of conflict needs to be minimised if the proposed loans are to be viable.

Three possibilities should be considered. One approach is to include in the loan contract an enforceable definition of a change in land use needed to activate the repayment option. For example, a dairy farmer might need to reduce their herd size below a specified limit for a specified period of time. However, there could still be bank–farmer conflict. For example, the dairy farmer might temporarily reduce stock numbers and then resume full-intensity dairy farming after the loan has been repaid. A second approach allows the farmer to repay part of the loan following partial changes in land use, with the percentage of the loan being able to be repaid equalling the percentage of land that has been converted. Objective measures of the scale of land-use change will not be possible for all types of farming, but when they are possible, this approach would weaken the farmer’s incentive to engage in premature land-use change. Finally, if neither of these approaches is feasible, the requirement that the conversion option must be exercised before the repayment option could be abandoned. This would make early repayment of the loan more likely, but would eliminate the potential for bank–farmer conflict over the definition of land-use change.

7 Conclusion

Many farmers will need to change the way they use their land if they are to remain financially viable in the face of growing pressure to reduce their environmental impact. This paper shows that highly indebted farmers will find this especially difficult due to the debt overhang problem, which can create situations where profitable land-use changes are not actually in farmers’ own best interests. The paper proposes a new form of farm debt that features an early repayment provision designed to alleviate the debt overhang problem by allowing farmers to retain a greater share of the benefits of land-use changes. Banks can also benefit from this arrangement if the introduction of the repayment provision is accompanied by a suitable increase in the interest rate charged by the bank. The economic surplus generated by the improved timing of land-use changes and lower default probabilities ensures that farmers and banks can both gain from the proposed arrangement. The status quo, after all, is likely to see highly indebted farmers not changing the way they use their land and struggling on until an inevitable loan default and farm sale. The new owners may change land use, but only after a delay and only after banks (and farmers) have incurred significant costs.

The disincentive to transform a firm’s business operations can arise in any situation in which a decision-maker has high debt levels. In principle, the alternative form of debt proposed here could also mitigate the effects of debt overhang in these situations. However, this form of debt seems much better suited to farming applications. The biggest implementation challenge is cal-
culating the level of repayment, which requires a reasonably transparent process for estimating
the value of the borrower if they continued to operate in their original line of business. This is
relatively straightforward for most farms, which have reasonably clearly defined lines of business.
In contrast, many non-farming firms—especially the large ones that are the focus of much cor-
porate finance research—have diverse operations, which makes the valuation process much more
complicated. Thus, although the loan product here could be used outside the agricultural sector,
it is probably only viable for the subset of firms with relatively focussed business operations.

A    Proofs

A.1    Proof of Lemma 1

If \( y \leq \hat{y}_d \) then the farmer defaults immediately and \( f_{\text{post}}(y) = 0 \). Suppose, instead, that \( y > \hat{y}_d \). The farmer continues to service the debt during the next increment of time, so that the value
of her ownership stake satisfies the differential equation

\[
0 = \frac{1}{2} \sigma_y^2 f_{\text{post}}''(y) + (\mu_y - \lambda_y) y f_{\text{post}}'(y) - r f_{\text{post}}(y) + y - c.
\]

The usual value-matching condition at the default boundary implies that \( f_{\text{post}}(\hat{y}_d) = 0 \). The
value of the default option vanishes as \( y \to \infty \), so that

\[
\lim_{y \to \infty} f_{\text{post}}(y) = \frac{y}{r + \lambda_y - \mu_y} - \frac{c}{r}.
\]

Therefore

\[
f_{\text{post}}(y) = \frac{y}{r + \lambda_y - \mu_y} - \frac{c}{r} - \left( \frac{\hat{y}_d}{r + \lambda_y - \mu_y} - \frac{c}{r} \right) \left( \frac{y}{\hat{y}_d} \right)^\gamma,
\]

where \( \gamma \) is the negative root of

\[
0 = \frac{1}{2} \sigma_y^2 \gamma (\gamma - 1) + (\mu_y - \lambda_y) \gamma - r.
\]

The value function is maximised by setting \( \hat{y}_d \) equal to the quantity in equation (1).

A.2    Proof of Lemma 2

The farmer has three possible actions. If she defaults immediately, then her ownership stake is
worthless. As this option is always available, \( f_{\text{pre}}(x, y) \) must always be greater than or equal to
zero. On the other hand, if she changes land use immediately, then she pays \( k \) and her ownership
stake’s value changes to \( f_{\text{post}}(y) \). As this option is always available, \( f_{\text{pre}}(x, y) \) must always be
greater than or equal to \( f_{\text{post}}(y) - k \). Finally, if she services the debt during the next increment
of time and then reevaluates her decision, her ownership stake is worth

\[
(x - c)dt + e^{-r dt} E^*[f_{\text{pre}}(x + dx, y + dy)] + o(dt)
= (x - c)dt + (1 - r dt) (f_{\text{pre}}(x, y) + E^*[df_{\text{pre}}(x, y)]) + o(dt)
= f_{\text{pre}}(x, y) + (x - c + \mathcal{D} f_{\text{pre}}(x, y) - r f_{\text{pre}}(x, y)) dt + o(dt)
\]
as \( dt \to 0 \), where \( E^* \) denotes that the expected value calculation uses the risk-neutral process for
\( x \) and \( y \). The farmer’s ownership stake must always be worth at least this much, which implies that

\[
0 \geq x - c + \mathcal{D} f_{\text{pre}}(x, y) - r f_{\text{pre}}(x, y).
\]

The function \( f \) satisfies all three inequalities, with one of them—corresponding to the optimal
action—holding with equality.
A.3 Proof of Lemma 3

The farmer has three options: to default; to repay the loan; and to continue servicing the loan during the next increment of time. If the farmer defaults immediately, then her ownership stake is worthless, so that $f_{\text{post}}(x, y) \geq 0$. If she repays the loan immediately, then she pays a lump sum of $x/(r + \lambda - \mu_x)$ and ends up owning a debt-free farm worth $y/(r + \lambda_y - \mu_y)$.\footnote{This holds whether the refinancing is funded by an equity injection or a new loan. In the former case, the farmer contributes cash of $x/(r + \lambda_x - \mu_x)$. In the latter case, in order to raise the funds needed to repay the original loan, the farmer will have to commit to a repayment schedule on the new loan with present value equal to $x/(r + \lambda_x - \mu_x)$.} Therefore

$$f_{\text{post}}(x, y) \geq \frac{y}{r + \lambda_y - \mu_y} - \frac{x}{r + \lambda_x - \mu_x}.$$ 

Finally, if the farmer services the debt during the next increment of time and then reevaluates her decision, her ownership stake is worth

$$\left( y - c \right) dt + e^{-r dt} E^*[f_{\text{post}}(x + dx, y + dy)] + o(dt)$$

$$= \left( y - c \right) dt + (1 - r dt)(f_{\text{post}}(x, y) + E^*[df_{\text{post}}(x, y)]) + o(dt)$$

$$= f_{\text{post}}(x, y) + (y - c + Df_{\text{post}}(x, y) - rf_{\text{post}}(x, y)) dt + o(dt)$$

as $dt \to 0$. The farmer’s ownership stake must always be worth at least this much, which implies that

$$0 \geq y - c + Df_{\text{post}}(x, y) - rf_{\text{post}}(x, y).$$

The function $f_{\text{post}}$ satisfies all three inequalities. The one corresponding to the optimal action will hold with equality.

B Numerical solution method

This appendix briefly outlines how the model can be solved on a grid in $(x, y)$-space using repeated application of the successive over-relaxation (SOR) method. This involves solving a sequence of problems involving pre- and post-conversion states and farm debt and equity. I describe the various problems in the order in which they are solved.

B.1 Farm equity, post-conversion

If the farm has traditional debt, I use the exact solution in Lemma 1 to calculate the present value of equity. If the farm has the alternative form of debt, I use the approach here to solve the farmer’s problem on a grid in $(x, y)$-space. The initial estimate of the present value of farm equity corresponds to now-or-never default and repayment. That is, I begin by setting

$$f_{\text{post}}(x, y; c) = \max \left\{ 0, \frac{y}{r + \lambda_y - \mu_y} - \frac{x}{r + \lambda_x - \mu_x}, \frac{y}{r + \lambda_y - \mu_y} - \frac{c}{r} \right\}.$$ 

I update this estimate using the projected SOR approach, with default and repayment payoffs equal to 0 and $y/(r + \lambda_y - \mu_y) - x/(r + \lambda_x - \mu_x)$, respectively. I impose simple numerical conditions $f_{\text{post,xx}} = 0$ along the two $x$ boundaries and $f_{\text{post,yy}} = 0$ along the two $y$ boundaries.

B.2 Farm equity, pre-conversion

The approach described here works for traditional and alternative forms of debt. The initial estimate of the present value of farm equity corresponds to now-or-never default and conversion. That is, I begin by setting

$$f_{\text{pre}}(x, y; c) = \max \left\{ 0, f_{\text{post}}(x, y; c) - k, \frac{x}{r + \lambda_x - \mu_x} - \frac{c}{r} \right\}.$$
The precise value of $f_{\text{post}}(x, y; c)$ depends on whether or not the repayment option is present. I update this estimate using the projected SOR approach, with default and conversion payoffs equal to 0 and $f_{\text{post}}(x, y; c) - k$, respectively. I impose simple numerical conditions $f_{\text{pre},xx} = 0$ along the two $x$ boundaries and $f_{\text{pre},yy} = 0$ along the two $y$ boundaries.

### B.3 Debt-free farm, pre-conversion

After conversion, a debt-free farm is worth $f_{\text{post}}(x, y; 0) = y/(r + \lambda_y - \mu_y)$. I calculate the pre-conversion value of this farm using the approach for an indebted farm, modified by deleting the (irrelevant) default option. In particular, I set the initial estimate equal to

$$f_{\text{pre}}(x, y; 0) = \max \left\{ f_{\text{post}}(x, y; 0) - k, \frac{x}{r + \lambda_x - \mu_x} \right\}$$

and update it using the projected SOR approach, with the conversion payoff $f_{\text{post}}(x, y; 0) - k$. I use the same boundary condition as for the indebted farm.

### B.4 Loan, post-conversion

If the farm has traditional debt, I use the exact solution for the present value of farm debt analogous to the expression for the value of equity in Lemma 1, which is

$$\ell_{\text{post}}(y; c) = \begin{cases} (1 - \theta)y_{\text{d}} & \text{if } y \leq \hat{y}_{\text{d}}, \\ \frac{x}{r} - \left( \frac{c}{r} - \frac{(1 - \theta)\hat{y}_{\text{d}}}{r + \lambda_y - \mu_y} \right) \left( \frac{y}{\hat{y}_{\text{d}}} \right)^{\gamma} & \text{if } y > \hat{y}_{\text{d}}. \end{cases}$$

For the alternative form of debt, the starting value for the present value of the farm loan is

$$\ell_{\text{post}}(x, y; c) = \min \left\{ f_{\text{post}}((1 - \theta)x, (1 - \theta)y; 0), \frac{x}{r + \lambda_x - \mu_x}, \frac{c}{r} \right\}.$$  

I update this estimate using the SOR approach to solve the system of equations, with the equation at grid point $(x, y)$ determined by the indebted farmer’s optimal action at that grid point. I impose simple numerical conditions $\ell_{\text{post},xx} = 0$ along the two $x$ boundaries and $\ell_{\text{post},yy} = 0$ along the two $y$ boundaries.

### B.5 Loan, pre-conversion

The approach described here works for traditional and alternative forms of debt. The initial estimate of the present value of the farm loan is

$$\ell_{\text{pre}}(x, y; c) = \min \left\{ f_{\text{pre}}((1 - \theta)x, (1 - \theta)y; 0), \ell_{\text{post}}(x, y; c), \frac{c}{r} \right\}.$$  

I update this estimate using the SOR approach to solve the system of equations, with the equation at grid point $(x, y)$ determined by the indebted farmer’s optimal action at that grid point. I impose simple numerical conditions $\ell_{\text{pre},xx} = 0$ along the two $x$ boundaries and $\ell_{\text{pre},yy} = 0$ along the two $y$ boundaries.

### C Post-conversion default–repayment policies

Figure 5 shows the optimal default–repayment policy for a farmer after the land-use change, with each graph corresponding to a different amount of debt. If the combination of $(x_t, y_t)$ lies in the dark-grey region, then the farmer should default on the loan immediately. If it lies in the light-grey region, then the farmer should repay the loan immediately. Otherwise, the farmer should wait until $(x_t, y_t)$ moves into one of the shaded regions and then act accordingly.
Figure 5: Optimal post-conversion policy for different levels of alternative debt

Notes. Each graph shows the farmer’s optimal policy after the land-use change. If the combination of \((x_t, y_t)\) lies in the dark-grey region, then the farmer should default on the loan immediately. If it lies in the light-grey region, then the farmer should repay the loan immediately. Otherwise, the farmer should wait until \((x_t, y_t)\) moves into one of the shaded regions and then act accordingly.
Assuming $y_t$ is so large that the farmer will not default (and lose ownership of the post-conversion farm), the farmer will repay the loan at the same time that she would default on a traditional loan with the same coupon if there were no conversion option. That is, unless $y_t$ is low, the farmer’s repayment-timing problem is equivalent to the default-timing problem of a farmer who cannot change land use. In both cases, the farmer waits until $x_t$ is sufficiently low before exercising the option. For the repayment-timing problem, a farmer considering delaying repayment weighs the costs of continuing to service the debt against the benefit of being able to achieve an even smaller repayment amount in the future. For the default-timing problem, a farmer considering delaying default weighs the costs of continuing to incur losses against the benefit of being able to return to profitability in the future.

References


