The Root Cause of Japan’s 30-year Stagnation: 
Implications for the U.S., Europe, and East Asia

by

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ABSTRACT

After decades of spectacular growth, Japan has been stagnant for three decades since 1990. We argue that this reversal of fortune is what one would expect for any country experiencing a population aging as rapid as Japan’s. The argument is based on the finding of a negative association, across countries, between growth and aging. We also show that one can exploit this association to predict growth far into the future for a number of countries. China’s GDP relative to U.S. GDP for 2050 will be higher than it currently is, but not by much.

JEL Classification Codes: J11, O11

Keywords: Japan’s stagnation, population aging, demographic transition, China

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1. Introduction and Summary

After decades of spectacular growth, Japan has been stagnant for three decades since 1990. This paper is an attempt to identify its cause and to explore how the same cause affects other countries, notably the U.S., China, and other East Asian countries.

There is now a body of academic literature on why Japan stopped growing. An early contribution is Hayashi and Prescott (2002), which finds that a slowdown in TFP (total factor productivity) growth can explain both the growth slowdown and an increase in the capital intensity. A symposium issue Ariga et al. (2006) contains explanations ranging from demand deficiency to a lack of firm turnover. Caballero et al. (2008) find that “zombies”, unproductive firms propped up by banks, depress industry productivity.

However, these explanations, certainly the one by Hayashi and Prescott (2002), are “proximate” or not “fundamental”, in the sense of Chapter 4 of Acemoglu (2008). We claim that our explanation is “fundamental”. Another difference from most existing explanations is that we address the issue in an international comparative context.

We argue that the culprit is population aging. For a perspective, start out with the relatively recent papers by Acemoglu and Restrepo (2017) and Eggertsson et al. (2019). Using the international cross-section data from the Penn World Table, Acemoglu and Restrepo (2017) find that percapita growth is not related to the change in a standard measure of aging, namely the ratio of the population of the old (those 50 years or over) to that of the young. Eggertsson et al. (2019) argue that the change in the ratio is significant for post-Lehman growth. In contrast, in an overlooked paper, An and Jeon (2006) find that the level of the old-to-young ratio is significantly related to growth but the relationship is hump-shaped.

The first contribution of the paper is a finding that the level of the old-to-young ratio explains Japan’s reversal of fortune. This point can be seen from Figure 1, which plots percapita GDP growth over 1960-1990 or 1990-2020 against the level of the old-to-young ratio (for now, ignore the gray downward-sloping dashed line). The peak of the hump is at 0.4. For those countries whose old-to-young ratio is above 0.4, there is a fairly strong negative correlation between growth and the level of aging. Japan has changed from being the second youngest OECD country to the oldest in the world. The negative
association suggests that a country experiencing this rapid aging must suffer a great deal in growth. We will argue that the negative association can be interpreted as a causation, namely that the rapid aging caused Japan to stagnate.

Figure 1: Percapita GDP Growth against the Ratio, 1960-1990 and 1990-2020 Combined, OECD + Asian Tigers+CHN+IND

Note: The Y axis measures real per capita GDP growth over 1960-1990 or 1990-2020. Let the old-to-young ratio for each year be the ratio of the number of people aged 50-79 to those aged 20-49. The X axis measures the geometric average within the period. For 1960-1990, it is the geometric average of the ratio over 1960, 1961,..., 1989; for 1990-2020 it is the geometric average over 1990, 1991,..., 2019. The OECD countries for 1960-1990 are as of 1990. The 1990-2020 OECD countries are as of 2020. Percapita GDP is from PWT (Penn World Table) 10.0. The GDP measure from PWT is \( RGDP^{na} \) (real GDP at constant national prices obtained from national accounts data from each country). See Feenstra et al. (2015) for more details. The last year of PWT’s GDP series is 2019. GDP growth from 2019 to 2020 is assumed to be the same as that from the World Bank’s PPP GDP estimate. Demographic information is from the United Nations estimate released in July 2022, more specifically, from the Excel file WPP2022_POP_F02_1_POPULATION_5YEAR_AGE_GROUPS_BOTH_SEXES.xlsx. We divide GDP by UN’s population estimate to arrive at per capita GDP.

This demographic explanation naturally lends itself to growth projection. The paper’s second contribution is to forecast GDP for 2050 for a number of countries including the U.S., Europe, China, Japan, and other East Asian countries. We forecast per capita
growth during 2020-2050 based on the ratio for 2020-2050 calculated from the recent population projection by the United Nations. The projected per capita growth can then be converted to the level of GDP for 2050.

To be clear, our GDP projection is different from what is practiced in the popular press. GDP equals per capita GDP multiplied by the country’s population. The extensive margin through which aging affects GDP is the population, while the intensive margin is per capita GDP. The popular press only looks at the extensive margin.

A section-by-section summary of the rest of the paper is as follows. Section 2 interprets the negative association between growth and the old-to-young ratio by the growth equation. The growth equation posits that per capita growth is composed of the rate of technical progress (to be referred to as efficiency growth) and the convergence effect tied to the initial per capita output. The primal source of the negative correlation is the negative age composition effect that efficiency growth depends negatively on the level of the old-to-young ratio. The effect is captured in the downward-sloping dashed line in Figure 1. To bring out the strength of the convergence effect, we supplement the graphical analysis by an OLS (ordinary least squares) estimation of the so-called Barro regression.

Section 3 shows that the negative age composition effect can be estimated by combining the growth equation for 1960-1990 with one for 1990-2020.

Section 4 implements the out-of-sample projection to 2050. The in-sample regression is the Barro regression for the 1990-2020 growth. Its OLS estimate is then utilized for the out-of-sample forecasting of the 2020-2050 growth. For 2020, according to the Penn World Table, the ratio of China’s GDP to the U.S. GDP on the PPP (purchasing power parity) basis is 1.05. For 2050, our baseline point estimate puts the China/U.S. GDP ratio at around 1.19. We provide the associated confidence interval computed by the formula derived in Appendix 1.

Section 5 argues that the negative association of the old-to-young ratio with growth is a causal relation because the ratio is predetermined by the fertility rate over many preceding years. This is particularly true for Japan. Japan’s demographic transition, namely a drop of its fertility rate from well above the reproduction rate of 2.1 to below it, took place in ten short years between 1947 and 1957.

Section 6 provides two models that deliver the growth equation with the negative
age composition effect. One is the expanding-variety semi-endogenous growth model of Jones (1995) embedded in a closed-economy two-period OLG framework. Appendix 2 is a detailed exposition of the model. The other is its open-economy version with an admittedly ad hoc partial adjustment mechanism for the foreign interest rate.

Section 7 concludes the paper.

Percapita GDP rather than GDP per working-age population is the predominantly popular measure of output intensity. We have obliged by entertaining the former, even though the growth equation is about the latter. For the most part, use of this more theoretically appropriate measure does not alter our results. The figures collected in Appendix 3 verify this. One notable difference is regarding the baseline point estimate of the China/U.S. GDP ratio for 2050. As mentioned above, the estimated ratio is 1.19 when the underlying output growth is percapita GDP growth. It is lower at 1.14 with GDP per working-age population.

2. Interpreting the Negative Correlation

As we noticed for Figuer 1, the correlation between percapita growth and the old-to-young ratio (the ratio of those aged 50-79 to those aged 20-49) is negative for those OECD countries whose old-to-young ratio is greater than 0.4. We interpret this negative association by assuming that the standard growth equation is applicable to those countries. We will refer to countries to which the growth equation applies as major leaguers.

The growth equation posits that percapita growth is the sum of efficiency growth (the growth rate of labor-augmenting technical progress) and the convergence effect (the component that is proportional to the gap between the steady-state value of percapita GDP and its initial value). As will be formally estimated in the next section from data on major leaguers, the efficiency growth component depends negatively on the old-to-young ratio. This negative age composition effect is described by the gray dashed line in the figure. The vertical distance between the circle (whose coordinates are the ratio and the percapita growth rate for the country) and the gray dashed line is the convergence effect. The negative correlation between growth and the ratio comes about because of the negative age composition effect and a negative correlation, found in the data, between the old-to-young ratio and the initial percapita output. It is notable that, consistent with the growth
equation, the major leaguers lie above the dashed line, even though the dashed line was estimated without such a constraint.

Turning to the countries to the left of 0.4, we notice that the dashed line separates the Asian Tigers as of 1960-1990 from the others such as TUR (Turkey). One interpretation is that the Asian Tigers in the 1960-90 period were already major leaguers. None of our conclusions, however, will depend on this interpretation, although including them in the major league will make our estimates somewhat sharper.

The separation between major and minor leaguers in the left-of-0.4 region is much clearer when all available countries are included, as seen in Figure 2 for the 1960-90 period and Figure 3 for 1990-2020. The gray dashed line is the same dashed line in Figure 1 that is estimated exclusively from data on major leaguers. It looks very much like a separating hyperplane. Comparing between the two figures, we notice that CHN (China) and perhaps VNM (Viet Nam) have transitioned to the major league, with IND (India) following suit.

**Figure 2: Percapita GDP Growth against the Ratio, 1960-1990, All Countries**

Note: A sample of 101 countries for which data are available. Excludes non-OECD countries whose 1990 population is less than 1 million. See the note to Figure 1 for data sources and for how the variables in the X and Y axes are defined.
In order to bring out the convergence effect, which is obscured in two-dimensional plots such as Figures 1-3, we turn to the so-called Barro regression. We focus on the 1990-2020 growth because for 1960-1990, perhaps due to small sample sizes and to a fairly high correlation between the initial percapita output and the old-to-young ratio, the age effect is dominated by the convergence effect. Table 1 collects OLS estimates on various samples. Start with regression (C) with 34 OECD countries (as of 2020) whose old-to-young ratio is greater than 0.4. The difference from the Barro regression considered in Acemoglu and Restrepo (2017) is that we include the level as well as the change in the old-to-young ratio. The level measure is highly significant while the change is not. The convergence effect, represented by the negative coefficient in the initial output level, too, is highly significant.
Table 1: Barro Regressions, 1990-2020

<table>
<thead>
<tr>
<th></th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sample</td>
<td>all countries, ratio &gt; 0.4, BIH excluded</td>
<td>UN more developed countries, BIH excluded</td>
<td>OECD(2020), ratio &gt; 0.4</td>
<td>OECD(2020) + Tigers and China, ratio &gt; 0.4</td>
</tr>
<tr>
<td>sample size</td>
<td>56</td>
<td>41</td>
<td>34</td>
<td>38</td>
</tr>
<tr>
<td>Log of initial per capita GDP</td>
<td>$-0.66^{***}$</td>
<td>$-0.57^*$</td>
<td>$-1.01^{***}$</td>
<td>$-1.06^{*****}$</td>
</tr>
<tr>
<td>Log of ave. old-to-young ratio</td>
<td>$-2.04^{**}$</td>
<td>$-2.35^*$</td>
<td>$-2.73^{***}$</td>
<td>$-2.94^{*****}$</td>
</tr>
<tr>
<td>Log change in old-to-young ratio</td>
<td>$1.35^*$</td>
<td>0.93</td>
<td>0.11</td>
<td>0.28</td>
</tr>
<tr>
<td>$R$-squared</td>
<td>0.579</td>
<td>0.267</td>
<td>0.591</td>
<td>0.756</td>
</tr>
</tbody>
</table>

Note: $^{*****}$ = significant at 0.001%, $^{****}$ = significant at 0.01%, $^{***}$ = significant at 0.1%, $^{**}$ = significant at 1%, $^*$ = significant at 5%. The dependent variable is the annualized per capita GDP growth in percent over 1990-2020. The parameter estimates by OLS (Ordinary Least Squares). The intercept term is not reported here. The initial per capita GDP is for 1990. The old-to-young ratio is the ratio of the number of those aged 50-79 to those aged 20-49. The log change in the ratio is the difference between the log of the ratio for 2020 and that for 1990. The average old-to-young ratio is the geometric average of the ratio over 1990, 1991, ..., 2019. For regressions (A) and (B), BIH (Bosnia-Herzegovina) is excluded from the sample. It is a clear outlier, most likely due to the Bosnian War of 1992-1995. There are 37 OECD countries as of 2020. Of those, countries whose average old-to-young ratio is less than 0.4 are: TUR (Turkey), MEX (Mexico), and COL (Columbia). The UN’s definition of “more developed countries”, 42 in number, are OECD countries excluding TUR, KOR (Korea), ISR (Israel), MEX (Mexico), CHL (Chile), COL (Columbia) but includes all non-OECD East European countries, Belarus, Ukraine, and Russia.

Regressions (A) and (B) show that both the convergence effect and the level age effect remain statistically significant with more numerous countries. Regression (B)’s sample is a subset of regression (A)’s, because none of the countries deemed “more developed” by the UN have the old-to-young ratio less than 0.4. Regression (D) shows, relative to regression (C), that expanding the sample to 38 countries by treating the Asian Tigers and China as major leaguers for 1990-2020 results in similar point estimates with more precision.
3. Identifying the Age Composition Effect in the Growth Equation

In this section, we describe how the gray dashed line in Figures 1-3 is estimated. We do so after quickly reviewing the growth equation.

3.1. A Refresher on the Growth Equation

For country $i$ at year $t$, let $Y_{it}$ be aggregate output, $L_{it}$ the labor input, and $A_{it}$ the level of labor-augmenting technology. Per capita output is $y_{it} \equiv Y_{it}/L_{it}$. Output per efficiency unit of labor is $y^{E}_{it} \equiv Y_{it}/(A_{it}L_{it}) = y_{it}/A_{it}$. The growth rate of $A_{it}$ is what we have referred to as efficiency growth. The convergence hypothesis underlying the growth equation is that output growth per efficiency unit is proportional to the gap between its steady state value and its initial value. For any given time interval $[t, s]$, the convergence hypothesis states:

$$\log(y^{E}_{is}) - \log(y^{E}_{it}) = \left(1 - e^{-(s-t)\kappa}\right)\left[\log(y^{E}_{i,\infty}) - \log(y^{E}_{it})\right], \quad \kappa = \text{"speed of adjustment"}$$  (1)

where $y^{E}_{i,\infty}$ is the steady-state value of output per efficiency unit. This key equation can be derived — with linear approximation — from prototype growth models such as the Solow model and the Cass-Koopmans model. Crucially for the analysis below, the speed of adjustment $\kappa$ is assumed to be the same across countries.

This equation can be converted to an equivalent version of the growth equation. Let $\gamma_{i,[t,s]}$ and $g_{i,[t,s]}$ be the average per capita output growth and efficiency growth over the interval $[t, s]$. That is,

$$\gamma_{i,[t,s]} \equiv \frac{1}{s-t} \left[ \log (y_{is}) - \log (y_{it}) \right], \quad g_{i,[t,s]} \equiv \frac{1}{s-t} \left[ \log (A_{is}) - \log (A_{it}) \right].$$

Rewrite (1) as

$$\gamma_{i,[t,s]} = g_{i,[t,s]} + \beta \left[ \log(y^{E}_{i,\infty}) - \log \left( \frac{y_{it}}{A_{it}} \right) \right], \quad \beta \equiv \frac{1}{s-t} \left(1 - e^{-(s-t)\kappa}\right),$$

or

$$\gamma_{i,[t,s]} = g_{i,[t,s]} + \left[ -\beta \log(y_{it}) + \beta \log(y^{E}_{i,\infty}) + \beta \log(A_{it}) \right]$$  (2)

We note in passing that the OLS estimates of the Barro regression, such as those

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1This is equation (1) in the handbook survey by Durlauf et al. (2005).
displayed in Table 1, are hard to interpret due to regression endogeneity. In the conditional convergence literature, efficiency growth $g_{i,[t,s]}$ is the same across countries, the log steady-state output per efficiency unit $\log(y_{i,\infty}^E)$ is related to time-invariant country characteristics such as the distance from the equator, and the log initial efficiency $\log(A_{it})$ is part of the error term. So the error term is likely to be correlated with the log initial per-capita output $\log(y_{it})$. If we enter time-variant country characteristics such as the level or the change in the old-to-young ratio as regressors, the implicit assumption is that they affect efficiency growth. But then the log initial efficiency $\log(A_{it})$ would be correlated with those time-varying characteristics as well.

### 3.2. Linking Per-capita Output Growth to Efficiency Growth

The growth equation is good for any $(t, s)$ ($t < s$). Set $t = 1960$ and $s = 1990$ in (2) (so $s - t = 30$). Then $\beta = \frac{1}{30} (1 - e^{-30\kappa})$ and

$$
\gamma_{i,[1960,1990]} = g_{i,[1960,1990]} + \beta \left[ y_{i,\infty}^E - \log \left( \frac{y_{i,1960}}{A_{i,1960}} \right) \right].
$$

(3)

Set $t = 1990$ and $s = 2020$ in (2) (so $s - t = 30$ again) to obtain

$$
\gamma_{i,[1990,2020]} = g_{i,[1990,2020]} + \beta \left[ y_{i,\infty}^E - \log \left( \frac{y_{i,1990}}{A_{i,1990}} \right) \right].
$$

(4)

Taking the difference between these two equations and recalling the definitions above of $\gamma_{i,[t,s]}$ and $g_{i,[t,s]}$, we obtain

$$
\gamma_{i,[1990,2020]} - e^{-30\kappa} \gamma_{i,[1960,1990]} = g_{i,[1990,2020]} - e^{-30\kappa} g_{i,[1960,1990]}.
$$

(5)

That is, the only source of growth, when properly quasi-differenced, is efficiency growth.

To capture the negative age composition effect, consider the least-squares projection of efficiency growth on demographics for each of the two time intervals:

$$
\begin{align*}
  g_{i,[1960,1990]} &= \pi_0 + \pi_1 \log(z_{i,[1960,1990]}) + \nu_{i1}, \\
  g_{i,[1990,2020]} &= \pi_0 + \pi_1 \log(z_{i,[1990,2020]}) + \nu_{i2}.
\end{align*}
$$

(6)

The demographics measures, $z_{i,[1960,1990]}$ and $z_{i,[1990,2020]}$, are the ones we entertained in the previous plots. That is, $z_{i,[1960,1990]}$ is the (geometric) average of the old-to-young ratio for $t$, $z_{it}$, over $t = 1960, ..., 1989$ and $z_{i,[1990,2020]}$ is the same for $t = 1990, ..., 2019$. By
construction, in each least squares projection, the demographics variable and the error are uncorrelated. We will assume that the “cross” correlations between $z_{i,1960,1990}$ and $v_{i2}$ and between $z_{i,1990,2020}$ and $v_{i1}$, too, are zero:

**Assumption:** $\text{Cov}(z_{i,1960,1990}, v_{i2}) = 0$, $\text{Cov}(z_{i,1990,2020}, v_{i1}) = 0$.

Substitute (6) into (5) to obtain

$$
\gamma_{i,1990,2020} = e^{-30\kappa} \gamma_{i,1960,1990} + \pi_0(1 - e^{-30\kappa}) + \pi_1 \left[ \log(z_{i,1990,2020}) - e^{-30\kappa} \log(z_{i,1960,1990}) \right] + \left( v_{i2} - e^{-30\kappa} v_{i1} \right). 
$$

We will estimate this equation with the last term $(v_{i2} - e^{-30\kappa} v_{i1})$ as the error term. The assumption laid out above implies that the demographic variables, $\log(z_{i,1960,1990})$ and $\log(z_{i,1990,2020})$, are uncorrelated with the error term. Because the error term forms a component of efficiency growth, it can be correlated with lagged output growth $\gamma_{i,1960,1990}$. So lagged output growth is treated as endogenous. It is instrumented by a dummy for World War II victors (to be specified in a moment). The estimation equation (7) is nonlinear in the parameters, $(\kappa, \pi_0, \pi_1)$. We estimate the equation by nonlinear GMM with conditional homoskedasticity.

The derivation of the estimation equation assumes that the country is a major leaguer both in the 1960-1990 and 1990-2020 periods. Therefore the sample consists of the OECD countries as of 1990 for which the average old-to-young ratio is above 0.4 for both periods. There are 23 such countries, whose identities can be seen from Figure 4 below. Of the 23 countries in the sample, the World War II victors are USA, ISL, CHE, CAN, NZL, and AUS. The parameter estimates by GMM are ($t$-values in brackets):$^2$

- sample size = 23, $\hat{\pi}_0 = -1.48$, $\hat{\pi}_1 = -5.12$, $\hat{\kappa} = 0.031$. So $e^{-30\kappa} = 0.40$.

Figure 4 plots the quasi-differenced growth $\gamma_{i,1990,2020} - e^{-30\kappa} \gamma_{i,1960,1990}$ against

$^2$The parameter estimates are very similar if the Asian Tigers are included:

- sample size = 27, $\hat{\pi}_0 = -1.46$, $\hat{\pi}_1 = -5.15$, $\hat{\kappa} = 0.031$. So $e^{-30\kappa} = 0.40$. 


\[ \log(z_{i,1990,2020}) - e^{-30\hat{\kappa}} \log(z_{i,1960,1990}). \] The slope of the dashed line in the figure is \( \hat{\pi}_1 \) and the intercept is \( \hat{\pi}_0(1 - e^{-30\hat{\kappa}}) \).

**Figure 4: Per capita Growth and the Old-to-Young Ratio in Quasi Differences**

\[ \text{Note: Plot of } \gamma_{i,[1990-2020]} - e^{-30\hat{\kappa}} \gamma_{i,[1960-1990]} \text{ against } \log(z_{i,1990,2020}) - e^{-30\hat{\kappa}} \log(z_{i,1960,1990}), \]

\[ \text{where } \gamma \text{ is the average annualized growth for the country over the indicated 30-year period,} \]

\[ z_{i,1960,1990} \text{ is the average old-to-young ratio for 1960-1990, and} \]

\[ z_{i,1990,2020} \text{ is the average old-to-young ratio for 1990-2020.} \]

\[ e^{-30\hat{\kappa}} \text{ is about 0.40. The sample consists of 23 countries. They are the} \]

\[ \text{subset of OECD countries (as of 1990) for which } z_{i,1960,1990} > 0.4 \text{ and } z_{i,1990,2020} > 0.4. \]

Going back to Figures 1-3, with \( z_i \) either \( z_{i,1990,2020} \) (the average old-to-young ratio for 1960-1990) or \( z_{i,1990,2020} \) (the average ratio for 1990-2020) in the X axis in log scale and \( \gamma_i = \gamma_{i,1960,1990} \) or \( \gamma_i = \gamma_{i,1990,2020} \) in the Y axis, the downward-sloping dashed line is the graph of \( \hat{\pi}_0 + \hat{\pi}_1 \log(z_i) \). The vertical distance between the circle representing \( (\log(z_i), \gamma_i) \) and the dashed line is an estimate of the convergence effect. It is a noisy estimate because the vertical distance includes \( v_i (= v_{i1} \text{ or } v_{i2}), \) the non-demographic determinant of efficiency growth.

4. **Projection to 2050**

To forecast per capita output growth over 2020-2050, \( \gamma_{i,[2020,2050]} \), we follow the textbook out-of-sample forecasting procedure. It consists of the following two steps:

(i) Estimate the least squares projection of \( \gamma_{i,[1990,2020]} \) on a set of variables \( x_i \) by OLS.
The in-sample regression is thus \( \gamma_{i,[1990,2020]} = x_i \delta + \varepsilon_i \) \((i = 1, 2, ..., n)\). Let \( \hat{\delta} \) be the OLS estimate of the least squares projection coefficients \( \delta \).

(ii) The projected value \( \gamma_{i,[2020,2050]} \) is given by \( \hat{x}_i' \hat{\delta} \), where \( \hat{x}_i \) is a value of \( x_i \) projected for 2020-2050. The underlying out-of-sample regression is \( \gamma_{i,[2020,2050]} = \hat{x}_i \delta + \eta_i \) \((i = 1, 2, ..., n)\).

The set of countries in (i) does not have to be the same as those in (ii), but the baseline projection below is for the same set of countries.

We choose \( x_i \) to be the regressors in the Barro regression in Table 1.\(^3\) Thus (with \( y_{it} = \) percapita output for country \( i \) for year \( t \), \( z_{it} = \) the old-to-young ratio for country \( i \) for year \( t \), and \( z_{i,[1990,2020]} = \) the ratio averaged over 1990-2020)

\[
x_i = (1, \log(y_{i,1990}), \log(z_{i,[1990,2020]}), \log(z_{i,2020}) - \log(z_{i,1990}))' .
\]

For \( \hat{x}_i \), it is given by

\[
\hat{x}_i = (1, \log(y_{i,2020}), \log(z_{i,[2020,2050]}), \log(z_{i,2050}) - \log(z_{i,2020}))' .
\]

We take \( z_{i,2050} \) and \( z_{i,[2020,2050]} \) from the United Nations population projection released in 2022.

The sample for the in-sample regression in step (i) above should consist of major leaguers for the 1990-2020 period. In addition to the OECD countries as of 2020 whose old-to-young ratio is above 0.4, we include the non-OECD Asian Tigers (HKG, SGP, and TWN) and CHN (China). That is, the in-sample regression in step (i) is regression (D) in Table 1 on a sample of size 38. If the in-sample regression is regression (C), the projection will be similar, with a wider confidence interval.

Figure 5 shows the plot of \( \gamma_{i,[2020,2050]} \) against \( z_{i,[2020,2050]} \). The figure’s title is “Secular Stagnation” because, unlike in Figures 1-3, the \( X \) and \( Y \) axes need to be rescaled to accommodate the substantially lower growth rate and the higher old-to-young ratio than in the preceding 30-year periods. For 1990-2020, Japan was by far the oldest country. For 2020-2050, Italy, Hong-Kong, and Korea will join Japan as the world’s oldest countries. Aging in western European countries will not be as rapid as in those countries.

\(^3\)Another possibility is to take \( x_i \) to be the right-hand-side variables in (7). The drawback is its inclusion of the lagged growth \( \gamma_{i,[1960,1990]} \), which limits the sample size. Former Soviet block countries that are admitted to the OECD after 1990 do not provide GDP data for 1960.

Electronic copy available at: https://ssrn.com/abstract=4553699
**Figure 5: Secular Stagnation**

*Note: Plot of per capita GDP growth projected for 2020-2050 against the ratio for 2020-2050. The sample of 38 countries, consisting of the OECD countries (as of 2020) whose old-to-young ratio is above 0.4, non-OECD Asian Tigers, and China.*

The downward-sloping dashed line in the figure is the linear extrapolation of the dashed line shown in Figures 1-3 to higher values of the old-to-young ratio. Notably, consistent with our growth equation-based interpretation of the data, the circles representing $(z_i, \gamma_i, 2020, 2050)$ lie above the dashed line. The vertical distance between them and the dashed line is an estimate of the convergence effect.

Of particular interest is CHN (China) as compared to USA. China’s fertility has declined since 1970 due to the one-child policy. The U.S. aging, on the other hand, is far slower thanks in large part to projected inflow of immigrants. As a consequence, China’s old-to-young ratio during 2020-2050 will be higher, taking the efficiency growth component down along the dashed line. Nevertheless, China’s annual per capita growth during 2020-2050 will be higher than that for the U.S. by about 1.1 percentage point. This is because the convergence effect for China, with the initial per capita output still relatively low, is far greater. For 2020, according to the PWT (Penn World Table), the ratio of China’s GDP to the U.S. GDP, namely $Y_{CHN,2020}/Y_{USA,2020}$, is about 1.05.\(^4\) If the population growth during

\(^4\)Both the World Bank PPP estimate and the IMF estimate put the ratio at 1.15 for 2020.
2020-2050 were the same for the two countries, the CHN/USA GDP ratio for 2050 would be 1.38 (≈ 1.05 + 0.011 × 30).

Figure 6 shows the GDP ratio $Y_{i,2050}/Y_{USA,2050}$ for the largest 20 of the 38 sample countries. The CHN/USA GDP ratio for 2050 is not as large as 1.38 because China’s projected annualized (continuously compounded) population growth of −0.27% is less than 0.37% for the U.S. The GDP ratio $Y_{CHN,2050}/Y_{USA,2050}$ is 1.189 (≈ 1.05 + 0.011 × 30 + (−0.0027 − 0.0037) × 30).

**Figure 6: GDP, 2050 (normalized to 1 for the U.S.)**

Note: The largest 20 countries in the universe of the 38 countries (the OECD countries (as of 2020) whose old-to-young ratio is above 0.4, non-OECD Asian Tigers, and China).

Thus, China’s GDP for 2050 is projected to be about 18.9% higher than the U.S. GDP. The associated projected log difference $\log(Y_{CHN,2050}/Y_{USA,2050})$ is 17.4%. The projection is uncertain for three reasons. First, it is based on the in-sample OLS estimate ($\hat{\delta}$ above) which has the sampling error. Second, even if the true coefficient $\delta$ were available so that the projection is $\hat{x}'\delta$ rather than $\hat{x}'\hat{\delta}$, the underlying out-of-sample regression involves the error term ($\eta_i$ in step (ii) above). Third, the population projection ($z_{i,2050}$ and $z_{i,2050}$ in $\hat{x}_i$ above) is uncertain. Appendix 1 shows how to construct the confidence
interval that takes either the first uncertainty alone or both the first and second 
uncertainties into account. The 5% confidence interval (whose coverage probability is 
95%) around the projection of 17.4% is [−11%, 45%]. If both the first and second 
uncertainties are taken into account, the confidence interval is [−42%, 76%].

5

5. Did Population Aging Cause Japan’s Stagnation?

We have identified the negative age composition effect, as reflected in the negative old-to-
young ratio coefficient in the least squares projection (6). In this section we argue that it 
can be interpreted as a causation.

Figure 7 shows the fertility rate over the last 141 years. Each country in the figure 
made a demographic transition in which the fertility rate went from well above the 
reproduction rate of 2.1 to below it. Japan’s transition was swift: it went from 4.5 to 2.1 in 
just ten years, from 1947 to 1957. By 1957, Japan was one of the lowest fertility-rate 
countries in the world only after Latvia, Estonia, and Servia. For KOR (Korea), TWN 
(Taiwan), and CHN, the transition was triggered decades after Japan’s transition. Their 
transition length is longer, a couple or more decades rather than just 10 years, but the 
magnitude of transition was larger.

The formula for the confidence interval is (A1.28). Drop “+2” in (A1.19) if only the first 
uncertainty is to be taken into account.

If the in-sample regression is regression (C) in Table 1 on the sample of 34 countries, the 
projected value of \( \log(Y_{CHN,2050}/Y_{USA,2050}) \) is 14.1%. The 5% confidence interval that takes 
only the first uncertainty (the sampling error) into account is [−18%, 47%]. If the log change 
in the old-to-young ratio is dropped from regression (D) but the sample is the same as for 
regression (D) with 38 observations, the projected value of \( \log(Y_{CHN,2050}/Y_{USA,2050}) \) is 13.0%. 
The 5% confidence interval that takes only the sampling error into account is [−8%, 33%].
To what extent does the demographic transition affect the old-to-young ratio? If immigration and emigration are non-existent, the time sequence of the old-to-young ratio is an exact function of the initial age distribution, the sequence of births implied by the fertility rate, and the sequence of age-dependent mortality rates. Figure 8 shows the sensitivity of the old-to-young ratio to the fertility rate for Japan. The gray line describes how the old-to-young ratio would behave if the mortality rates were to remain the same as in 1947. Therefore, the gap between the actual and simulated paths are due to advances in mortality-reducing medical technologies. The simulated ratio tracks the actual almost perfectly until year 2000, implying that mortality improvements occurred uniformly across age brackets. After 2000, the improvements are more concentrated in old ages. The figure shows that Japan’s old-to-young ratio is to a very large extent determined by the fertility rate, whose largest movement took place between 1947 and 1957.
Figure 8: Actual and Simulated Old-to-Young Ratio, Japan, 1947-2022

Note: The simulated ratio is calculated as follows. Take the 1947 actual age distribution as the initial condition and calculate the age distribution for subsequent years assuming that the age-dependent mortality rates remain the same as those for 1947. For each year, the age distribution at age 0 is the number of births implied by the fertility rate. The data on births, mortality rates, and the age distribution are from the Japanese Mortality Database.

6. Why does Efficiency Growth Depend on the Level of the Ratio?

Our interpretation of the observed growth-aging correlation has been couched in the growth equation that relates the efficiency component to the level of aging. This section provides two alternative derivations of the growth equation with this property. One is the expanding-variety semi-endogenous growth model of Jones (1995) embedded in a two-period OLG (overlapping generations) structure. It assumes a closed economy. The other imagines a world of small open economies, each facing different levels of exogenously-given interest rates.

6.1. The Semi-Endogenous OLG Model

In this subsection, we drop the country subscript \( i \) and focus on the country in question. The details of the model are in Appendix 2. Output per worker \( y_t \) evolves according to (see (A2.27) and (A2.37) of the appendix)

\[
y_{t+1} = B (z_{t+1})^{1-\alpha} A_{t+1}^{\alpha} y_t^{1-\alpha},
\]

where \( A_{t+1} \) is the level of labor-saving technology and \( z_{t+1} \) is the old-to-young ratio. Since there are \( N_{t+1} \) young individuals and \( N_t \) old ones in period \( t + 1 \), the ratio \( z_{t+1} \) equals
\(N_t/N_{t+1}\). Recall from Section 3 that \(y_t^E \equiv y_t/A_t\) is output per efficiency units. A routine manipulation of (8) yields

\[
\log(y_{t+1}^E) - \log(y_t^E) = \alpha \left\{ \frac{1 - \alpha}{\alpha} \left[ \log(B(z_{t+1})) - \log(A_{t+1}/A_t) \right] - \log(y_t^E) \right\}.
\]

(9)

If the term in brackets is evaluated at the steady state where \(A_{t+1}/A_t = 1\) (which, as seen in (14) below, implies \(z_{t+1} = 1\)), this equation becomes

\[
\log(y_{t+1}^E) - \log(y_t^E) \approx \alpha \left[ \frac{1 - \alpha}{\alpha} \log(B(1)) - \log(y_t^E) \right],
\]

which is the key convergence equation (1).

Main features of the semi-endogenous OLG model are the following.

• The production side is exactly the same as in Jones (1995). The aggregate production function for the final good, when consolidated with the intermediate goods sector, is

\[
Y_t = \left( A_t L_{Y_t} \right)^\alpha K_t^{1-\alpha},
\]

(11)

where \(L_{Y_t}\) here is labor input to final goods production, \(K_t\) is the capital stock, and \(A_t\) is the stock of knowledge produced by the R&D sector. This aggregate production function results from equations (A1) and (A8) of Jones (1995). As is well known, the knowledge stock \(A_t\) acts as the level of labor-augmenting technology.

• Regarding the R&D sector, the R&D equation in Jones (1995) is: \(\dot{A} = \delta (L_A)^\lambda A^\phi\) where \(L_A\) is labor input to R&D (this is his equation (7)). Its discrete-time version, if we assume 100% depreciation and if we focus on the benchmark case of \(\phi = 0\) considered by Jones (1995), is

\[
A_{t+1} = \delta (L_A)^\lambda.
\]

(12)

• Both generations work, but only talented young workers, which is a fraction \(\mu\) of \(N_t\) young workers in period \(t\), can work in the R&D sector. That is,

\[
L_{A_t} = \mu N_t.
\]

(13)

Substituting this into (12) and taking the log time difference, we obtain the age
composition effect:

\[
\log(A_{t+1}/A_t) = \lambda \log(N_t/N_{t-1}) = -\lambda \log(z_t).
\]  

(14)

The last equality holds because the old-to-young ratio \( z_t \) equals \( N_{t-1}/N_t \). Thus we have shown that efficiency growth, namely the growth rate of labor-augmenting technology, depends negatively on the level of aging as measured by \( z_t \). It is actually linear in \( \log(z_t) \), as assumed in (6).

- Regarding the household sector, generation \( t \)'s lifetime utility function, defined over the lifetime consumption profile \((c_1t, c_{2,t+1})\), is \((1 - \rho) \log(c_1t) + \rho \log(c_{2,t+1})\). The young individual chooses her holdings of the capital stock and the knowledge stock to maximize lifetime utility. The capital stock as well as the knowledge stock are assumed to fully depreciate in a generation.

6.2. Small Open Economies

Re-introduce the country subscript \( i \) and write country \( i \)'s aggregate production function for the final good as

\[
Y_{it} = \left(A_{it}L_{i,Yt}\right)^{\alpha}K_{it}^{1-\alpha}, \quad \text{or} \quad y_{it} \equiv \frac{Y_{it}}{L_{i,Yt}} = A_{it}^{\alpha} \left(\frac{K_{it}}{L_{i,Yt}}\right)^{1-\alpha}.
\]

(15)

The small country \( i \) has an unlimited access to foreign capital at the (gross) interest rate of \( R_{it} \). The capital stock adjusts instantaneously so that the marginal product of capital \( \frac{\partial Y_{it}}{\partial K_{it}} \) equals \( R_{it} \) for all \( t \). This pins down the capital-labor ratio at

\[
\frac{K_{it}}{L_{i,Yt}} = (1 - \alpha)^{\frac{1}{\alpha}} A_{it} R_{it}^{\frac{1}{\alpha}}.
\]

(16)

Substituting this into the aggregate production function and recalling that \( y_{it}^{E} \equiv y_{it}/A_{it} \), we obtain

\[
y_{it}^{E} = (1 - \alpha)^{\frac{1-\alpha}{\alpha}} R_{it}^{\frac{1-\alpha}{\alpha}}.
\]

(17)

Let \( R \) be the world (gross) interest rate which is assumed constant over time. If all countries had access to the common world interest rate, the convergence would be instantaneous. To capture the convergence effect, which is so prominent in the Barro regression, we imagine that the country-specific interest rate \( R_{it} \) adjusts only slowly. An
admittedly *ad hoc* formulation is the standard partial adjustment:

\[
\log(R_{i,t+1}) - \log(R_{it}) = \left(1 - e^{-\kappa}\right) \left[\log(R_{it}) - \log(R)\right], \quad \kappa = \text{“speed of adjustment”}. \tag{18}
\]

It is then a routine algebra to derive the key convergence equation (1) with

\[
y_{i,\infty}^E \equiv (1 - \alpha)^{\frac{1-\alpha}{\alpha}} R^{\frac{1-\alpha}{\alpha}}. \tag{19}
\]

Thanks to the small-country assumption, we have been able to derive the growth equation without ever specifying the household sector. We could have an annual model with a large number of generations as in standard annual OLG models. We could then define the old-to-young ratio \(z_t\) literally as in the preceding empirical sections. Furthermore, if we assume that only a fraction of those aged between 20 and 49 can work in the R&D sector, the same R&D accumulation equation (14) will obtain.

7. Conclusion

The paper’s main conclusions are the following.

- Japan was not exceptional during either 1960-1990 or 1990-2020. Its demise is what you would expect for any country experiencing a population aging as rapid as Japan’s.
- The ultimate cause of Japan’s stagnation during 1990-2020 is the swift demographic transformation that occurred during 1947-1957.
- The ratio of China’s GDP to the U.S. GDP, which is 1.05 for 2020 according to the Penn World Table, will not be much higher in 2050. The baseline point estimate of the GDP ratio for 2050 is 1.19.
Appendix 1. Calculation of Confidence Intervals

This appendix describes the derivation of the confidence intervals employed in the text.

As in the text, the annualized average growth rate of per capita output in percent over the period \([t, s]\) for country \(i\) is denoted by (assuming \(s - t = 30\))

\[
\gamma_{i,[s-t]} = \frac{100}{30} \left( \log(y_{is}) - \log(y_{it}) \right), \tag{A1.1}
\]

where \(y_{it}\) is country \(i\)’s per capita output for year \(t\). The in-sample regression is

\[
\gamma_{i,[1990,2020]} = x'_{it} \beta + \varepsilon_i \quad (i = 1, 2, \ldots, n). \tag{A1.2}
\]

Just to provide a refresher of the textbook finite-sample theory with normal errors, let \(b\) be the OLS estimate of \(\beta\). Its exact distribution is given by

\[
b - \beta \sim N \left( \begin{array}{c} 0 \\ \sigma^2 (X'X)^{-1} \end{array} \right), \tag{A1.3}
\]

where \(K\) is the number of regressors including the constant, and \(X_{(n \times K)}\) is the data matrix whose \(i\)-th row is \(x_i\). Let \(s^2\) be the OLS estimate of the error variance \(\sigma^2\). It is the sum of squared residuals divided by the degrees of freedom \(n - K\). The sampling error for the \(j\)-th coefficient, \(b_j - \beta_j\), divided by the standard error is the \(t\) ratio. It has the student \(t\) distribution with \(n - K\) degrees of freedom. That is, with \(s(b_j)\) denoting the standard error,

\[
\frac{b_j - \beta_j}{s(b_j)} \sim t_{n-K} \quad \text{with} \quad s(b_j) \equiv s^2 ((X'X)^{-1})_{jj}. \tag{A1.4}
\]

The associated confidence interval for \(\beta_j\) is

\[
\left[ b_j - c \times s(b_j), \ b_j + c \times s(b_j) \right]. \tag{A1.5}
\]

If the critical value \(c\) is chosen so that \(\text{Prob}(|t_{n-K}| > c) = \alpha\), the coverage probability, namely the probability that \(\beta_j\) is included in this confidence interval, equals \(1 - \alpha\).

Forecasting for the out-of-sample period 2020-2050 is based on the out-of-sample regression for the period 2020-2050:

\[
\gamma_{i,[2020,2050]} = \hat{x}'_i \beta + \eta_i \quad (i = 1, 2, \ldots, n), \tag{A1.6}
\]

where \(\hat{x}_i\) is the projected value to the period 2020-2050 of the regressors. By the very nature of out-of-sample forecasting, the out-of-sample regression coefficients in (A1.6) equal the in-sample counterpart \(\beta\) in (A1.2), and the variance of the out-of-sample error \(\eta_i\) equals that of the in-sample error \(\varepsilon_i\) (i.e., \(\text{Var}(\varepsilon_i) = \text{Var}(\eta_i) = \sigma^2\)). The out-of-sample forecast of
$\gamma_{i,[2020,2050]}$ is

$$\gamma_{i,[2020,2050]} = \hat{x}'_i b,$$  \hfill (A1.7)

where $b$ is the OLS estimate from the in-sample for the period 1990-2020. So the out-of-sample forecast error is

$$\bar{\gamma}_{i,[2020,2050]} - \gamma_{i,[2020,2050]} = \hat{x}'_i (b - \beta) - \eta_i.$$  \hfill (A1.8)

Assume that the normal out-of-sample error $\eta_i$ is independent of the in-sample errors $(\varepsilon_1, \ldots, \varepsilon_n)$. Then $\eta_i$ is independent of $b$. The exact distribution of the out-of-sample forecast error is given by

$$\bar{\gamma}_{i,[2020,2050]} - \gamma_{i,[2020,2050]} \sim \mathcal{N}(0, V_i)$$  \hfill (A1.9)

with

$$V_i = \hat{x}'_i \text{Var}(b) \hat{x}_i + \sigma^2 (\hat{x}'_i (X'X)^{-1}\hat{x}_i + 1).$$  \hfill (A1.10)

Consider the ratio for the out-of-sample forecast:

$$\frac{\bar{\gamma}_{i,[2020,2050]} - \gamma_{i,[2020,2050]} s_i}{\sigma^2 (\hat{x}'_i (X'X)^{-1}\hat{x}_i + 1)}$$  \hfill (A1.11)

This ratio can be rewritten as $A/\sqrt{B}$ with

$$A = \frac{\hat{x}'_i (b - \beta) - \eta_i}{\sqrt{\sigma^2 (\hat{x}'_i (X'X)^{-1}\hat{x}_i + 1)}}, \quad B = \frac{(n - K)s^2}{\sigma^2 n - K}.$$  \hfill (A1.12)

By (A1.9), we have $A \sim \mathcal{N}(0, 1)$. From the textbook finite-sample theory, we have

$$\frac{(n - K)s^2}{\sigma^2} \sim \chi^2_{n-K}. \quad \text{since} \quad \frac{(n - K)s^2}{\sigma^2} \text{ is independent of both } b \text{ and } \eta_i,$$

the ratio in (A1.11) is the $t$ ratio distributed as $t_{n-K}$. Thus $s_i$ is the standard error for the out-of-sample forecast error. The associated confidence interval for $\gamma_{i,[2020,2050]}$ is

$$\left[ \gamma_{i,[2020,2050]} - c \times s_i, \gamma_{i,[2020,2050]} + c \times s_i \right].$$  \hfill (A1.13)

Of particular interest is $h_{ij}$, the difference in the growth rate between a pair of countries $(i, j)$, and its out-of-sample forecast $\hat{h}_{ij}$. They are defined as

$$h_{ij} \equiv \gamma_{i,[2020,2050]} - \gamma_{j,[2020,2050]} \quad \text{and} \quad \hat{h}_{ij} \equiv \bar{\gamma}_{i,[2020,2050]} - \bar{\gamma}_{j,[2020,2050]}.$$  \hfill (A1.14)

Since

$$\hat{h}_{ij} - h_{ij} = (\gamma_{i,[2020,2050]} - \gamma_{j,[2020,2050]}) - (\bar{\gamma}_{j,[2020,2050]} - \bar{\gamma}_{j,[2020,2050]}),$$  \hfill (A1.15)
we have, by (A1.8),
\[ \hat{h}_{ij} - h_{ij} = (\hat{x}_i - \hat{x}_j)'(b - \beta) - (\eta_i - \eta_j). \]  
(A1.16)

The exact distribution of this out-of-sample forecast error is given by
\[ \hat{h}_{ij} - h_{ij} \sim N(0, V_{ij}) \]  
(A1.17)

with
\[ V_{ij} = (\hat{x}_i - \hat{x}_j)' \text{Var}(b)(\hat{x}_i - \hat{x}_j) + 2\sigma^2 \left( (\hat{x}_i - \hat{x}_j)'(X'X)^{-1}(\hat{x}_i - \hat{x}_j) + 2 \right). \]  
(A1.18)

The same argument that led us to the \( t \) ratio (A1.11) shows that
\[ \hat{h}_{ij} - h_{ij} \sim t_{n-K} \]  
with
\[ s_{ij} \equiv \sqrt{\frac{s^2 ((\hat{x}_i - \hat{x}_j)'(X'X)^{-1}(\hat{x}_i - \hat{x}_j) + 2)}{n-K}}. \]  
(A1.19)

Thus \( s_{ij} \) is the standard error. The associated confidence interval is
\[ \left[ \hat{h}_{ij} - c \times s_{ij}, \hat{h}_{ij} + c \times s_{ij} \right]. \]  
(A1.20)

We can also provide the confidence interval for the (log of) \( Y_{i,2050} \) where \( Y_{it} \) is output (GDP) for country \( j \) for year \( t \). Percapita output \( y_{i,2050} \) is related to the annualized average growth in percent \( \gamma_{i,[2020,2050]} \) by the identity
\[ \log(y_{i,2050}) = 30 \frac{100}{100} \times \gamma_{i,[2020,2050]} + \log(y_{i,2020}) \]  
(by (A1.1)).  
(A1.21)

If \( POP_{it} \) is country \( i \)'s population in year \( t \), we have \( Y_{it} = y_{it} \times POP_{it} \). Substituting this into (A1.21), we obtain
\[ \log(Y_{i,2050}) = \frac{30}{100} \times \gamma_{i,[2020,2050]} + \log(y_{i,2020}) + \log(POP_{i,2050}), \]  
(A1.22)

where \( POP_{i,2050} \) is a projected value of the population for 2050. The out-of-sample forecast of \( \log(Y_{i,2050}) \) is then given by
\[ \log(Y_{i,2050}) = \frac{30}{100} \times \gamma_{i,[2020,2050]} + \log(y_{i,2020}) + \log(POP_{i,2050}). \]  
(A1.23)

Taking the difference between (A1.23) and (A1.22), we obtain
\[ \log(Y_{i,2050}) - \log(Y_{i,2050}) = \frac{30}{100} \times (\gamma_{i,[2020,2050]} - \gamma_{i,[2020,2050]}). \]  
(A1.24)

Therefore, the confidence interval for \( \log(Y_{i,2050}) \) immediately follows from (A1.13) as
\[ \left[ \log(Y_{i,2050}) - c \times \frac{30}{100} s_i, \log(Y_{i,2050}) + c \times \frac{30}{100} s_i \right], \]  
(A1.25)
where the standard error $s_{ij}$ is defined in (A1.11).

Providing the confidence interval for the log ratio

$$r_{ij} \equiv \log(Y_{i,2050}/Y_{j,2050}) = \log(Y_{i,2050}) - \log(Y_{j,2050})$$

proceeds similarly. The out-of-sample forecast of $r_{ij}$ is

$$\hat{r}_{ij} \equiv \log(Y_{i,2050}) - \log(Y_{j,2050}). \quad (A1.26)$$

It then follows easily that

$$\hat{r}_{ij} - r_{ij} = \left( \log(Y_{i,2050}) - \log(Y_{j,2050}) \right) - \left( \log(Y_{i,2050}) - \log(Y_{j,2050}) \right)$$

(since $r_{ij} = \log(Y_{i,2050}) - \log(Y_{j,2050})$)

$$= \left( \log(Y_{i,2050}) - \log(Y_{i,2050}) \right) - \left( \log(Y_{j,2050}) - \log(Y_{j,2050}) \right)$$

$$= \frac{30}{100} \left\{ \left( \gamma_{i,[2020,2050]} - \gamma_{i,[2020,2050]} \right) - \left( \gamma_{i,[2020,2050]} - \gamma_{i,[2020,2050]} \right) \right\} \quad \text{(by (A1.24))}$$

$$= \frac{30}{100} \left( \hat{h}_{ij} - h_{ij} \right). \quad \text{(by (A1.15))} \quad (A1.27)$$

The confidence interval for $r_{ij}$ immediately follows from (A1.20) as

$$\left[ \hat{r}_{ij} - c \times \frac{30}{100} s_{ij}, \hat{r}_{ij} + c \times \frac{30}{100} s_{ij} \right], \quad (A1.28)$$

where the standard error $s_{ij}$ is defined in (A1.19).
Appendix 2. The Semi-Endogenous OLG Growth Model

This appendix presents the model from which the growth equation is derived. The model is an OLG (overlapping generations) rendition of the expanding-variety model of Jones (1995). Previous OLG renditions include Strulik et al. (2013) and Hashimoto and Tabata (2016). The production side of the model consists of the final goods, the intermediate goods, and the R&D sectors. The difference here is that only young talented individuals can work in the R&D sector.

The Final Goods Sector

Final goods output in period \( t \), \( Y_t \), is produced according to

\[
Y_t = L_Y^a t \int_0^{A_t} x_{it}^{1-a} \, di, \tag{A2.1}
\]

where \( L_Y t \) is labor, \( x_{it} \) is intermediate input of variety \( i \), and \( A_t \) measures the available variety of intermediate inputs. Firms in this sector act competitively. The FOCs (the first-order conditions) are

\[
(w_t \text{ with respect to } L_Y t) \quad w_Y t = \alpha L_Y^{-1} t \int_0^{A_t} x_{it}^{1-a} \, di, \tag{A2.2}
\]

where \( w_Y t \) is the wage rate in terms of the final good, and

\[
(p_{it} \text{ with respect to } x_{it}) \quad p_{it} = (1-\alpha)L_Y^{a} t x_{it}^{-\alpha}, \tag{A2.3}
\]

where \( p_{it} \) is the price of the intermediate good in terms of the final good. Thanks to constant returns to scale, we have the output exhaustion by factor inputs:

\[
Y_t = w_Y t L_Y t + \int_0^{A_t} p_{it} x_{it} \, di. \tag{A2.4}
\]

The Intermediate Goods Sector

The intermediate goods sector is composed of a continuum of firms indexed by \( i \in [0, A_t] \). Each firm has a technology that converts one unit of capital services into the same unit of the firm-specific intermediate product. Each firm is a monopolist. Its decision problem is

\[
\text{(the monopolist’s decision problem)} \quad \max_{x} p_t(x) x - r_t x, \tag{A2.5}
\]

where \( r_t \) is the rental rate of capital services and \( p_t(x) \) is the inverse demand curve derived from the FOC (A2.3), namely \( p_t(x) \equiv (1-\alpha)L_Y^{a} t x^{-\alpha} \). The solution for this monopolist’s decision problem (A2.5) is

\[
x_t = (1-\alpha)^{-\frac{1}{a}} r_t^{-\frac{1}{a}} L_Y t, \tag{A2.6}
\]
which does not depend on \( i \). In what follows, we drop the subscript \( i \) from \( x_{it} \). Define \( \pi_t \) as

\[
\pi_t \equiv p_t(x_t)x_t - r_tx_t.
\]  

(A2.7)

The Final Goods and Intermediate Goods Sectors Combined

By simply setting \( x_{it} = x_t \) in (A2.1) and (A2.2), we obtain

(production function) \( Y_t = L_t^\alpha A_t x_t^{1-\alpha} \), \hfill (A2.1')

(FOC with respect to \( L_{Y_t} \)) \( w_{Y_t} = \alpha L_t^{-1} A_t x_t^{1-\alpha} \). \hfill (A2.2')

Setting \( x_{it} = x_t \) and \( p_{it} = p_t(x_t) \) in (A2.4) and using (A2.7), we obtain output exhaustion for the final and intermediate goods sectors combined:

(output exhaustion) \( Y_t = w_{Y_t} L_{Y_t} + A_t r_t x_t + A_t \pi_t \). \hfill (A2.4')

Whatever is left after paying for labor and intermediate inputs will be distributed to the household sector as dividends. The amount is \( A_t \pi_t \).

Substituting (A2.6) into (A2.1'), (A2.2'), and (A2.7) (with \( p_t(x) \equiv (1 - \alpha)L_t^\phi x^{-\alpha} \)), we obtain

\[
Y_t = (1 - \alpha) 2^{(1-\alpha)} \frac{2}{\alpha} A_t L_{Y_t} r_t^{1-\alpha}.
\]  

(A2.8)

\[
w_{Y_t} = \alpha (1 - \alpha) 2^{(1-\alpha)} \frac{2}{\alpha} A_t r_t^{1-\alpha}.
\]  

(A2.9)

\[
\pi_t = \alpha (1 - \alpha) 2^{(1-\alpha)} L_{Y_t} r_t^{1-\alpha}.
\]  

(A2.10)

The R&D Sector

The R&D sector is the discrete-time version of that in Jones (1995). That is, \( A_{t+1} \), the number of new designs produced by a typical R&D firm in period \( t \) to be utilized in period \( t+1 \), is produced according to the linear technology

\[
A_{t+1} = \delta L_{At},
\]  

(A2.11)

where \( L_{At} \) is labor. The left-hand-side is \( A_{t+1} \) rather than \( A_{t+1} - A_t \). Thus the depreciation rate is assumed to be 100%. The coefficient \( \delta \) is given by

\[
\delta = \delta \phi^{\lambda-1} A_t^\phi, \quad \phi < 1.
\]  

(A2.12)

Here, \( \ell_{At} \) is the average of \( L_{At} \) over all R&D firms. In equilibrium, we have \( \ell_{At} = L_{At} \). This \( \ell_{At} \) captures an externality due to duplication of ideas. We have \( \lambda = 1 \) if there is no such
externality. The restriction $\phi < 1$ is what makes the model the “semi-endogenous” growth model.

Since each representative firm takes $\ell_{At}$ as given, the FOC is

$$w_{At} = \delta p_{At}, \quad (A2.13)$$

where $w_{At}$ and $p_{At}$ are the wage rate and the price of a design, both in terms of the final good. The output exhaustion follows from (A2.11) and (A2.13):

(output exhaustion, R&D sector) \ \ \ p_{At}A_{t+1} = w_{At}L_{At}. \quad (A2.14)

Substituting (A2.12) into (A2.11) and (A2.13) and setting $\ell_{At} = L_{At}$, we obtain

(production function, R&D sector) \ \ \ A_{t+1} = \delta L_{At}^{\lambda} A_{t}^{\phi}, \quad (A2.15)

(FOC, R&D sector) \ \ \ w_{At} = \delta p_{At} L_{At}^{\lambda-1} A_{t}^{\phi}. \quad (A2.16)

### Households

Individuals live for two periods. Generation $t$ lives in periods $t$ and $t+1$. There are two types of individuals, either “talented” or “ordinary”. For each generation, fraction $\mu$ are talented. If $N_t$ is the population of generation $t$, there are $\mu N_t$ talented individuals and $(1 - \mu)N_t$ ordinary individuals. In period $t$, talented young individuals from generation $t$ work in the R&D sector for wage $w_{At}$, while ordinary young ones work in the final goods sector along with the old for wage $w_{Yt}$. We assume individuals are insured against the type, so that consumption and saving decisions do not depend on the type.

The decision problem for the representative individual from generation $t$ is

$$\max_{c_{1t}, c_{2,t+1}, \tilde{k}_{t+1}, a_{t+1}} (1 - \rho) \log(c_{1t}) + \rho \log(c_{2,t+1}) \quad (A2.17)$$

subject to

(budget constraint when young) \ \ \ c_{1t} + \tilde{k}_{t+1} + p_{At}a_{t+1} = \mu w_{At} + (1 - \mu)w_{Yt}, \quad (A2.18)

(budget constraint when old) \ \ \ c_{2,t+1} = r_{t+1}\tilde{k}_{t+1} + \pi_{t+1}a_{t+1} + w_{Y,t+1}. \quad (A2.19)

Here, $(c_{1t}, c_{2,t+1})$ is the individual’s consumption profile over periods $(t, t+1)$, $\tilde{k}_{t+1}$ is her investment in capital, $a_{t+1}$ is her investment in designs. For both vehicles of saving, capital and designs, to be positive, their rates of return must be the same:

(no arbitrage) \ \ \ r_{t+1} = \frac{\pi_{t+1}}{p_{At}}. \quad (A2.20)

Under the linear logarithmic utility function, saving when young, $\tilde{k}_{t+1} + p_{At}a_{t+1}$, is
given by

\[
\tilde{k}_{t+1} + p_{At}a_{t+1} = [\mu w_{At} + (1 - \mu)w_{Yt}] - (1 - \rho) \left\{ [\mu w_{At} + (1 - \mu)w_{Yt}] + r_{t+1}^{-1}w_{Y,t+1} \right\} \\
= \rho [\mu w_{At} + (1 - \mu)w_{Yt}] - (1 - \rho)r_{t+1}^{-1}w_{Y,t+1}.
\]  

\hspace{1cm} (A2.21)

**Model Parameters**

The following table lists the parameters of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>First appeared in</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>labor share</td>
<td>(A2.1)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>efficiency in R&amp;D</td>
<td>(A2.12)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>degree of externality in R&amp;D (no externality if ( \lambda = 1 ))</td>
<td>(A2.12)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>effect of R&amp;D stock on arrival of new ideas</td>
<td>(A2.12)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>time preference parameter</td>
<td>(A2.17)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>fraction of the young who are talented</td>
<td>(A2.18)</td>
</tr>
</tbody>
</table>

**Market-Clearing Conditions**

There are five markets. The associated market-clearing conditions (the demand-equals-supply conditions) for period \( t \) are

\[
\begin{align*}
\text{(labor for R&D)} & \quad L_{At} = \mu N_t, \\
\text{(labor for final goods production)} & \quad L_{Yt} = N_{t-1} + (1 - \mu)N_t, \\
\text{(designs)} & \quad a_{t+1}N_t = A_{t+1}, \\
\text{(capital services)} & \quad A_{tx_t} = \tilde{k}_tN_{t-1}, \\
\text{(final goods)} & \quad c_{1t}N_t + c_{2t}N_{t-1} + \tilde{k}_{t+1}N_t = Y_t.
\end{align*}
\]  

\hspace{1cm} (A2.22) \hspace{2cm} (A2.23) \hspace{2cm} (A2.24) \hspace{2cm} (A2.25) \hspace{2cm} (A2.26)

Thus the labor market is segmented in that only the talented young can work in the R&D sector while others work in the final goods sector, as in (A2.22) and (A2.23). Alternatively, we could assume that the talented young can choose between the two sectors. They choose the R&D sector because the model parameters are such that \( w_{At} > w_{Yt} \) in equilibrium.

By Walras’s law, only four of the above five equilibrium conditions are independent. One can, for example, derive the final goods equilibrium condition (A2.26) from the rest by combining them with the budget constraints and the output exhaustion conditions.

**Definition of Equilibrium**

An equilibrium given \( \tilde{k}_1 > 0 \) and \( A_1 > 0 \) is a sequence of prices \( \{w_{At}, w_{Yt}, p_{At}, r_t\}_{t=1}^{\infty} \) and associated quantities \( \{c_{1t}, c_{2t}, \tilde{k}_{t+1}, a_{t+1}, L_{Yt}, x_t, A_{t+1}, L_{At}\}_{t=1}^{\infty} \) such that
\(x_t\) solves the monopolist’s decision problem (A2.5) given \(p_t(x) \equiv (1 - \alpha)L_{Y,t}^a x^{-\alpha}\).

(iii) The no-arbitrage condition (A2.20) holds, where \(\pi_t \equiv p_t(x_t)x_t - r_t x_t\).

(iv) \(c_{2t}\) for \(t = 1\) (consumption by the initial old) equals \(r_1 \tilde{k}_1 + \pi_1 a_1 + w_{Y_1}\).

(v) \((c_1t, c_{2,t+1}, \tilde{k}_{t+1}, a_{t+1})\) solves generation \(t\)’s decision problem.

(vi) All markets clear.

Dynamics

From the equations shown so far, we can derive the following dynamics for per capita output \(y_t \equiv Y_t/L_{Y,t}\):

\[
y_{t+1} = B_t^{1-\alpha} A_{t+1}^\alpha y_t^{1-\alpha},
\]

where

\[
B_t \equiv \frac{(1 - \mu)\alpha}{1 + (1 - \rho)\alpha(1-\alpha)^2} \left[ \frac{N_t}{L_{Y,t+1}} + (1 - \alpha) \right].
\]

(A2.27)

Here is a sketch of the derivation.

- Define aggregate capital stock \(K_t\) as \(K_t \equiv \tilde{k}_t N_{t-1}\), and the capital/labor ratio \(k_t\) as \(k_t \equiv K_t / L_{Y,t}\). The market-clearing condition for capital services (A2.25) can then be written as \(A_t x_t = K_t\). Substituting (A2.6) into this and solving for \(r_t\) yields

\[
r_t = (1 - \alpha)^2 A_t^\alpha k_t^{-\alpha}.
\]

(A2.29)

Use of \(A_t x_t = K_t\) on (A2.1’) yields

\[
y_t (\equiv Y_t/L_{Y,t}) = A_t^\alpha k_t^{1-\alpha}.
\]

(A2.30)

- Imposing the market-clearing condition (A2.22) on (A2.15) and (A2.16), we obtain

\[
A_{t+1} = \delta \mu^\lambda N_{t+1}^\lambda A_t^\phi.
\]

(A2.31)

\[
w_{At} = \delta \mu^{\lambda-1} N_{t}^{\lambda-1} p_{At} A_t^\phi.
\]

(A2.32)

- Combine (A2.10) and (A2.20) to obtain

\[
p_{At} = \alpha (1 - \alpha)^{\frac{2-\alpha}{\alpha}} \left[ L_{Y,t+1} r_{t+1}^\frac{-1}{\alpha} \right]
\]

(A2.33)

- Using the relation \(\tilde{k}_{t+1} = K_{t+1}/N_t = k_{t+1} L_{Y,t+1}/N_t\) and (A2.24) on (A2.21), we obtain

\[
k_{t+1} \left[ L_{Y,t+1} N_t \right] + p_{At} \frac{A_{t+1}}{N_t} = \rho [\mu w_{At} + (1 - \mu)w_{Yt}] - (1 - \rho)r_{t+1} w_{Y,t+1}
\]

(A2.34)
• Use (A2.33), (A2.32), and (A2.9) on (A2.34) to write $p_{At}$, $w_{At}$, $w_{Yt}$, and $w_{Yt+1}$ as dependent on $r_t$ and $r_{t+1}$. Then use (A2.29) to replace $r_t$ and $r_{t+1}$ by $k_t$ and $k_{t+1}$. This produces, with the help of (A2.31), the difference equation for $k$:

$$k_{t+1} = B_t A_t^{\alpha} k_t^{1-\alpha}.$$  \hspace{1cm} (A2.35)

• Finally, use (A2.30) to derive (A2.27).

In period $t$, there are $N_{t-1}$ old individuals and $N_t$ young individuals. Define

$$z_t \equiv \frac{N_{t-1}}{N_t}.$$  \hspace{1cm} (A2.36)

Since $L_{Yt} = N_{t-1} + (1 - \mu)N_t$ by the equilibrium condition (A2.23) for labor for final goods production, the factor $B_t$ in (A2.27) can be written as

$$B_t = B(z_{t+1}) \equiv \frac{(1 - \mu)\alpha \frac{z_{t+1}}{z_{t+1} + (1 - \mu)}}{1 + (1 - \rho)\alpha(1 - \alpha)^{-2} \left[ \frac{z_{t+1}}{z_{t+1} + (1 - \mu)} + (1 - \alpha) \right]}.$$  \hspace{1cm} (A2.37)
Appendix 3. Figures with GDP per Working-Age Population

This appendix displays Figures 1, 4, 5, ad 6 when the output intensity measure is GDP per working-age population rather than per capita GDP.

Figure A.1: Growth of GDP per Working-Age Population against the Ratio, 1960-1990 and 1990-2020 Combined, OECD + Asian Tigers+CHN+IND

Note: The Y axis measures real growth rate of GDP per working-age population over 1960-1990 or 1990-2020. See the note to the corresponding figure in the text for other details.
Figure A.4: Growth and the Old-to-Young Ratio in Quasi Differences

Note: The output intensity measure is GDP per working-age population, not GDP per capita. See the note to the corresponding figure in the text for other details.

Figure A.5: Secular Stagnation

Note: The output intensity measure is GDP per working-age population, not GDP per capita. See the note to the corresponding figure in the text for other details.
Note: The largest 20 countries in the universe of the 38 countries (the OECD countries (as of 2020) whose old-to-young ratio is above 0.4, non-OECD Asian Tigers, and China).
References


