Working Paper

Contagion and Loss Redistribution in Crypto Asset Markets: Modelling the Intersection of DeFi and CeFi

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Abstract: This paper addresses the growing concern of shock propagation in crypto asset markets. The integration of conventional financial institutions (CeFi) and decentralized financial protocols (DeFi) has introduced a set of complexities that are not adequately accounted for in current models. We build on the well-established framework by Eisenberg and Noe (2001) and propose a generalized extension that can be applied to mixed DeFi/CeFi networks. Our model serves as a tool for comprehending potential contagion channels and loss redistribution resulting from the non-recourse nature of DeFi loans.

Keywords: Crypto Assets, Decentralized Finance, Non-Recourse Loans, Shock Propagation, Systemic Risk.

JEL: G01, G10, G29.

1 Introduction

The growing interconnectedness of crypto asset markets raises concerns among policymakers regarding the risks of shock propagation. They fear that contagion effects may have the potential to extend beyond the boundaries of crypto asset markets and impact traditional finance (see, e.g., Abad et al., 2022; FSB, 2023; García Ocampo et al., 2023; OECD, 2022).

In traditional financial markets, shock propagation and transmission channels are thoroughly researched (see, e.g., Acmoglu et al., 2015; Allen and Gale, 2000; Eisenberg and Noe, 2001; Freixas et al., 2000; Haldane and May, 2011). While these models are useful to study contagion risks from intermediaries such as centralized exchanges and crypto banks (Centralized Finance, or CeFi), they are inadequate to capture the unique characteristics introduced by decentralized financial protocols (DeFi). Specifically, the vast majority of loans in DeFi networks are overcollateralized and non-recourse, making them more robust but also allowing pseudonymous agents to abandon their collateral position and strategically default. Furthermore, DeFi protocols are bound by a deterministic rule set, requiring them to act in accordance with what is specified in their smart contract code. It is crucial to give careful consideration to these characteristics, as they have a substantial impact on the dynamics of shock propagation and the redistribution of losses.

Recent events (FTX, Celsius, Terra) highlighted the relevance of the intersection between centralized crypto asset service providers and DeFi protocols. While the systemic fragility in DeFi has been examined (see, e.g., Lehar and Parlour, 2022), the intersection between DeFi and CeFi has received limited attention by economic scholars.

This paper works toward closing this gap by expanding upon the framework established by Eisenberg and Noe (2001). We propose a multi-step model that incorporates DeFi-specific characteristics to enhance our understanding of shock propagation within mixed DeFi/CeFi networks. We demonstrate how shocks can propagate throughout this network and how they
impact various actors, including traditional institutions, other DeFi protocols, savers, and borrowers.

2 Model

DeFi loans typically adhere to a non-recourse structure and require overcollateralization. The collateral is locked on a neutral infrastructure and governed by code (smart contracts). If a price shock causes a loan’s collateralization to fall below a predefined threshold value, the position is liquidated. This setup results in a robust system capable of withstanding price shocks, with the degree of overcollateralization determining its resilience. However, in the event of a significant collateral price shock, positions may still become undercollateralized. Borrowers then have the option to strategically default, leaving the protocol with bad debt.

To incorporate these considerations into Eisenberg and Noe (2001), we split the total nominal liabilities into non-recourse and recourse liabilities. We further introduce a second asset to model the relative price shock. Without loss of generality, we assume a preference for asset a as collateral and for asset b as non-recourse debt.

A wide range of factors can trigger an initial default. Most of these scenarios can be modeled as a price shock that results in undercollateralized non-recourse loans. To address this, our multi-step model incorporates three stages. First, an aggregate shock is applied to the relative price of the collateral asset. Second, the liquidation process is conducted for undercollateralized loans. Third, the network is cleared using a modification of the fictitious default algorithm proposed by Eisenberg and Noe (2001).

2.1 Aggregate Price Shock

The total nominal liability matrix $L$, with $L_{ij}$ representing a liability from node $i$ to node $j$, is defined as $L = R + N$, where the $R$ matrix contains the liabilities with recourse and the $N$ matrix those without. Taking the respective asset prices into account, these matrices are defined as:

$$
R = \rho_a \cdot R_a + \rho_b \cdot R_b
$$

$$
N = \rho_a \cdot N_a + \rho_b \cdot N_b,
$$

where the subscripts $a$ and $b$ distinguish between the assets and $\rho$ stands for the current price. We model a systemic risk event by applying a negative price shock to the preferred collateral asset $a$ in terms of the price of asset $b$. The post shock price is denoted by $\hat{\rho}_a$, where $\frac{\rho_a}{\hat{\rho}_a} > \frac{\rho_b}{\hat{\rho}_b}$. The post-shock liability matrices are denoted by $\hat{R}$ and $\hat{N}$ respectively.

$$
\hat{R} = \hat{\rho}_a \cdot R_a + \hat{\rho}_b \cdot R_b
$$

$$
\hat{N} = \hat{\rho}_a \cdot N_a + \hat{\rho}_b \cdot N_b.
$$

2.2 Liquidations of Non-Recourse Loans

The vector $\vec{\varphi}$, all of whose components are greater than 1, contains the minimum collateralization re-
requirements set by each node in the network. The post-shock collateralization ratio of each liability in the system is expressed as

\[ \phi_{ij} = \begin{cases} \frac{R_{ji}}{N_{ij}}, & \text{if } N_{ij} > 0 \\ \infty, & \text{otherwise.} \end{cases} \]

Any position with \( \phi_{ij} < \varphi_j \) is closed by a third party outside our system through liquidation. Figure 2 shows how positions with \( \phi_{ij} < 1 \) accrue bad debt for the lending node \( j \). We assume liquidations to be frictionless, i.e., immediate and at no cost for the liquidated party.

![Diagram](https://ssrn.com/abstract=4499113)

**Figure 2**: Payoffs considering strategic default

Liquidations are accounted for via a contraction matrix \( \Omega \), which represents the liabilities leaving the system as a consequence.

\[ \Omega_{ij} = \begin{cases} -\hat{R}_{ij}, & \text{if } \phi_{ij} < 1 \\ -\hat{N}_{ij}, & \text{if } 1 \leq \phi_{ij} < \varphi_j \\ 0, & \text{otherwise} \end{cases} \]

The post-liquidation recourse and non-recourse liabilities are then calculated using \( \Omega \).

\[ \hat{R} = \hat{R} + \Omega \]
\[ \hat{N} = \hat{N} + \Omega \]

### 2.3 Clearing of the Network

Eisenberg and Noe (2001) propose a fictitious default algorithm to identify the unique payments that clear a network of \( n \) financial entities connected through unsecured loans. Under mild regularity conditions, the result satisfies three generalized bankruptcy conditions: *Absolute priority*, which ensures that no node retains any cash flow unless all liabilities are paid in full; *limited liability*, which sets total cash flow of a node as its upper bound for the payment it can make to clear the network; and *proportionality*, which mandates that if a node defaults due to insufficient cash flow, all its creditors get the same proportional reduction in payment. The network itself is defined by a nominal liability matrix denoted as \( L \) and a non-negative operating cash flow vector represented as \( e \). The node’s total cash flow is determined by combining its operating cash flow with incoming payments from liabilities. It can be interpreted as the node’s additional available funds, such as profit from outside the system, reserves, or excess liquidity in the case of a DeFi protocol.

In our adaptation of the algorithm, we pair our post-liquidation liabilities, denoted as \( \hat{R} \) and \( \hat{N} \) respectively, with a given \( e \). We then account for the distinct nature of non-recourse liabilities, employing a two-step approach in each iteration of the algorithm. The algorithm starts with \( \Gamma = \mathbb{I} \), where \( \Gamma_{ii} \) represents the current presumed proportional payment of node \( i \) on its recourse liabilities. The first step of each iteration accounts for *strategic defaults* between nodes with a non-recourse relationship. In the absence of additional costs, the rational choice for agent \( i \) is to opt for a strategic default if \( \hat{N}_{ij} > \Gamma_{jj} \cdot \hat{R}_{ij} \).

Before network clearing, we account for these strategic defaults by eliminating them from both the debt and associated collateral liability, using the iteration-specific \( D \):

\[ D_{ij} = \begin{cases} -\Gamma_{ii} \cdot \hat{R}_{ij}, & \text{if } \hat{N}_{ii} > \Gamma_{jj} \cdot \hat{R}_{ij} \\ -\hat{N}_{ij}, & \text{if } \hat{N}_{ij} > \Gamma_{jj} \cdot \hat{R}_{ij} \\ 0, & \text{otherwise} \end{cases} \]

Utilizing \( D \), we compute the adjusted total liability matrix:

\[ \hat{L}' = \Gamma \cdot \hat{R} + \hat{N} + D \]
Combining $\tilde{L}$ with $e$, we attempt to clear the network following Eisenberg and Noe and identify nodes in solvency default. For this, we check whether the total cash flow of each node is sufficient to cover all its outgoing liabilities to other nodes:

$$\sum_{j=1}^{n} \tilde{L}_{ij} + e_i \geq \sum_{j=1}^{n} L_{ij}$$

Should an iteration result in one or more nodes newly defaulting because of insufficient cash flow, the outgoing payments on the recourse liabilities of all defaulting nodes are adjusted through $\Gamma$ in line with bankruptcy conditions. In case of circular dependencies, the adjustment of $\Gamma$ is solved numerically, as described by Eisenberg and Noe (2001). Utilizing the updated $\Gamma$, the next iteration starts by re-determining $D$ and recomputing $\tilde{L}$. The algorithm continues in this manner until no further solvency defaults arise and then concludes. The $\tilde{L}$ of the final iteration constitutes the post clearing liability matrix $L^*$ and represents the unique matrix of nominal payments exchanged among nodes in the cleared network. This assumes that all nodes adhere to the general bankruptcy laws for their recourse liabilities and rationally choose strategic defaults whenever beneficial for their non-recourse obligations.

3 Loss Distribution

Whenever a node defaults due to solvency or strategic reasons, all its lending nodes incur proportional losses adding up to the the defaulting node’s failed payments. These losses can further propagate in the network if additional defaults occur. Observing the equity $\tau$ of each node, we can track how these losses are redistributed in the network. In the base model, a node’s equity is defined as its total cash flow minus payments to creditors:

$$\tau_i = \sum_{j=1}^{n} (L_{ji} - L_{ij}) + e_i.$$  

A node’s equity post-liquidation $\bar{\tau}$ and after clearing $\tau^*$ can be computed by substituting the liability matrix with $\tilde{L}$ and $L^*$ respectively. Note that the clearing process is value-conservative in a sense that no equity leaves the system. Furthermore, the limited liability condition for $L^*$ ensures that $\tau^*$ is non-negative.

Under these assumptions, we measure the effect of shock propagation on each node individually by comparing $\bar{\tau}$ to $\tau^*$. Defaults in the initial clearing iteration show $\bar{\tau} < 0$ with clearing establishing $\tau^* = 0$ again. All nodes in the system that are affected by loss redistribution and absorb these losses will exhibit a decrease in equity ($\bar{\tau}_i > \tau^*_i$). Cases where nodes have positive $\bar{\tau}$, which then is reduced to zero during the clearing process ($\tau^* = 0$), suggest default contagion caused by loss redistribution.

Given that we differentiate between various node types, it is possible to examine the effects of loss redistribution on distinct groups in the network. The ability to observe loss redistribution in such detail allows us to model the individual and collective effects on specific node types, such as saver nodes.

4 Discussion

Our model allows us to study systemic shocks at the intersection of traditional and decentralized financial markets. These shocks may originate from a large variety of sources, in both DeFi and CeFi, and propagate through the mixed network connected by recourse and non-recourse liabilities in two assets. Shock sources may include the default of a financial intermediary, a stablecoin issuer’s inability or unwillingness to maintain the peg, or the failure of a smart contract-based protocol. All of these events can be modeled as price shocks on the collateral asset, or as idiosyncratic shocks on a node’s reserves.

We adapt the clearing algorithm proposed by Eisenberg and Noe (2001) to observe how these shocks unfold and are propagated throughout the network. Under normal circumstances, overcollateralization provides a cushion that can prevent defaults and losses to the lender and other nodes. However, in the event of a substantial shock or network congestion, there is a risk that liquidations are not executed promptly, resulting in undercollateralized positions.

Our model suggests that the prevalent non-recourse nature of DeFi loans has a profound effect on loss re-
distribution. More specifically, borrowers can strategically default by intentionally abandoning their position without repaying the loan. This strategic choice creates a discrepancy between the outstanding value of the loan and the collateral’s value, resulting in a loss that is not borne by the defaulting borrower, but instead redistributed to other actors.

In a non-recourse context, the extent to which shocks are propagated depends on how interconnected the protocols (institution nodes) are. At present, there exist very few liabilities between DeFi lending protocols, indicating that shocks and resulting defaults are more likely to be contained within the user base of a particular protocol. This inherent resilience in DeFi, which limits shock propagation, aligns with observations made during periods of heightened price volatility. However, it is essential to closely monitor DeFi data, as future emergence of liabilities between DeFi protocols could pose significant risks to the system. Similarly, if connections between DeFi and CeFi are established, for example through centralized intermediaries that use protocols for yield generation or via stablecoin issuers, shocks can propagate and potentially affect all parts of the mixed network.

Our research can help economic scholars, policymakers, and protocol developers in comprehending, monitoring, and evaluating propagation and stability risks arising from inter-protocol liabilities. It also addresses liabilities to the traditional financial system, such as those introduced by flat-backed stablecoins. Increased interconnectedness will amplify the contagion risk and ultimately increase the risk for investors.

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