Joint or Independent? Integrated Sourcing and Inspection Decisions in Emerging Economies

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Abstract

In the presence of supplier quality risk, offering suppliers a higher procurement price or increasing inspection effort are two approaches to motivate a supplier to invest in higher quality. In this paper, we consider three sourcing and inspection strategies for a brand with two branch shops facing supplier quality risk: joint, mixed, and independent strategies. Under the joint strategy, the brand jointly sources for its two shops by offering a uniform procurement price and inspects for both shops in a centralized way (joint sourcing and joint inspection). Under the mixed strategy, the brand jointly sources for its two shops by offering a procurement price and then each shop decides its own inspection frequency (joint sourcing and independent inspection). Under the independent strategy, each shop decides its own procurement price and inspection frequency (independent sourcing and independent inspection). We find that among the three strategies, the joint strategy mostly relies on the inspection tool to motivate supplier’s high-quality effort, and the mixed strategy mainly utilizes the procurement price tool by offering the highest price for supplier’s high quality. We further find that, no matter the brand is purely profit driven or cares about the quantity of low-quality products sold to the market, the mixed strategy is always preferred to independent strategy. The preference between mixed and joint strategies is jointly determined by the fixed cost incurred in joint sourcing and inspection accuracy. For intermediate fixed cost, if the inspection accuracy is sufficiently high, the mixed strategy is preferred to joint strategy; otherwise, the joint strategy is preferred to the mixed strategy. When a brand becomes more socially responsible (i.e., it cares more about the quantity of low-quality products sold to the market), it has a greater incentive of using the mixed strategy. Interestingly, a brand’s utility is always higher when it is more socially responsible.

Keywords: Sourcing, inspection, free riding, socially responsible brand, supply chain

1. Introduction

Halal beef is a certification product prepared in accordance with Islamic law. In January 2021, fake halal beef was found in Malaysia as kangaroo and horse meat were mixed with and sold as halal beef. According to Daniele (2021), a cartel sold “beef that was not slaughtered according to Islamic customs or sourced from approved stakeholders”, and “fraudulently certified dubious or low-grade meat products mixed with halal-certified meat and repacked with fake halal logos.” As the
downstream distributors of this exposed cartel, “major retail stores are reeling from losses. Aeon Retail Malaysia’s chief human resource officer and corporate communications director, Kasuma Satria, told The Malaysian Insight that it had suffered a 40 percent drop in red meat sales as a result of the scandal.

In addition to the meat adulteration in emerging economies, counterfeit alcohol was found in Dominican Republic causing the death of tourists in 2019. According to McLaughlin (2019), counterfeit alcohol was produced in a hurry to make easy money. To speed up the process, some producers might cut distillation processes short by mixing water with indigestible alcohol compounds, like methanol, instead of the traditional ethanol.

Earlier in 2016, counterfeit alcohol was also identified in India. According to Kazmin (2016), “12 people died after drinking illegally produced alcohol in northern India, with dozens more in hospitals. The victims, all from the same district in Uttar Pradesh state, fell ill after drinking the toxic alcohol on the night of July 15 and were taken to hospital suffering from severe stomachache, vomiting and blurred vision.” In many states of India, only a few retailers were allowed by regional governments to distribute the alcohol to consumers. A close investigation showed that their suppliers were responsible for lethal drinking tragedies. As noted by Kazmin (2016), “spurious low-grade alcohol was made using opaque production techniques and often adulterated with dangerous chemicals, masquerading as more costly Indian-made foreign spirits or upmarket imported brands by using second-hand bottles of high-end liquor brands and fake labels.”

Moreover, there used to be milk adulteration scandals: In 2008, milk and infant formula along with other food materials and components for Yili and Mengniu were adulterated with melamine (Babich and Tang 2012, Zhang and Xue 2016). A close investigation shows that their suppliers are responsible for this adulteration. Likewise, on September 8, 2023, lamb meat adulteration was found by consumers (with DNA testing) in the shop selling hot pot food under the brand Megaland: Its high-calcium lamb rolls were detected with ingredients of duck meat.1 On September 9, with all its shops closed for rectification, Megaland issued a public statement that its selected supplier Senang was responsible for this adulteration.2

In the above cases, the buyer purchases from a risky supplier which may adulterate. In most practical situations, buyers also have alternative options to source from a reliable supplier with no adulteration. Compared to the reliable backup supplier, the risky supplier, though maybe irresponsible, has cost advantage. For example, in Malaysia, the halal beef is distributed from the

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2 http://hb.dzwww.com/p/pdaJ6XqS1G9.html
supplying cartel to consumers through supermarkets such as Aeon Retail Malaysia (Ariff et al., 2015). Supermarkets can choose to source from a tight-standard supplier with transparency and accountability or a risky supplier. The tight-standard supplier has a long and well-established history of supplying high-quality products; however, sourcing from such a reliable supplier is costly. In contrast, the product of the risky supplier suffers from quality risk but with cost advantage. A primary concern for buyers is when enjoying cost advantage of the local risky suppliers, how to motivate suppliers’ high effort and reduce suppliers’ adulteration. Not only in food and beverage industries, motivating suppliers to exert a high effort of providing high-quality products is also a typical problem in other industries. For example, in the automobile industry, Ford reported that most of its quality problems stem from its first-tier suppliers (Chao et al., 2009). So did many other manufacturers like Toyota and General Motors. One objective of this paper is to understand how to motivate the suppliers’ high efforts so as to reduce the quantity of low-quality products.

Some critics believe that buyers’ profit squeeze results in a low profit margin for suppliers, which backfires by leading suppliers to retreat to low-quality solutions (Wharton, 2009). A high procurement price, though reducing the buyers’ profits, is credited to be an economic incentive to motivate suppliers’ high-quality efforts. In addition to this economic incentive, buyers can leverage inspection to identify the suppliers’ adulteration, and reject the suppliers’ products once adulteration is identified. However, inspection can be inaccurate and costly. Both tools, the economic incentive of setting a high procurement price and the inspection for suppliers’ products, can be jointly used when sourcing from a risky supplier. In the presence of these two tools for risky suppliers, one research question this paper tackles is whether to choose the risky supplier with quality risk or the backup reliable supplier.

This sourcing problem is complicated for a brand with multiple shops (or branches), as there are three sourcing strategies. (1) Independent strategy \(i\), in which each shop sources independently/separately, i.e., each shop decides its own procurement price offered to the supplier, and independently operates the purchasing delivery, including inspecting products through a sampling procedure and being responsible for the logistics of products from the supplier to the shop. In this strategy, the shops conduct sampling inspection separately and share the inspection result with each other. That is, if one shop identifies adulterated products, and the other shop does not identify any, the adulteration is known to all the shops owned by the brand because of information sharing. (2) Joint strategy \(j\), in which the brand sources for its shops by setting a uniform procurement price and inspects for its shops. Typically, products are shipped to a common site, and then transported to each shop after passing the brand’s sampling inspection. (3) Mixed strategy \(m\)
(joint sourcing and independent inspection), in which the brand sources for its shops by offering a uniform procurement price to the supplier, but each shop independently operates the purchasing delivery, including inspecting products through a sampling procedure and being responsible for the logistics of products.

On one hand, compared with joint inspection in strategy $j$, independent inspection in strategy $i$ or $m$ results in a positive externality between shops, because even if one shop does not identify adulterated products from the risky supplier, this shop may still benefit from adulteration identification by another shop. However, given other shops’ inspection efforts, such a positive externality lures the free-riding behavior of a shop in the form of lowering its own sampling frequency for products from the risky supplier. Such misaligned interests among shops disappear in joint inspection. Thus, when dealing with a risky supplier, joint inspection seems to benefit the buying brand by eliminating the negative free-riding behavior of lowering inspection effort in independent inspection.

On the other hand, to conduct joint inspection the products need to be shipped to a common site, and then delivered to each shop once passing the inspection. Accordingly, the brand incurs a fixed cost in strategy $j$ particularly in emerging economies with less developed logistics infrastructure (Zhang and Swaminathan, 2020). In the aforementioned Megaland case, the sourcing and inspection for all the shops were centralized, and products were delivered by Megaland to shops at a lumpsum cost. Other examples include Yang Guofu, a Chinese brand selling spicy hot pot food, whose meat materials for all shops are centrally purchased and inspected by the brand’s supply chain centers and then are distributed nationwide at lumpsum costs.\(^3\)

Under each shop’s independent sourcing in strategy $i$, the procurement price to the supplier is quoted independently by each shop. In contrast, in strategy $j$ and strategy $m$, the brand quotes a uniform price to the supplier for all shops under centralized sourcing. Joint sourcing seems more attractive than independent sourcing for the buying brand, because under joint sourcing the objective of the brand is maximized in the form of optimizing the sum of all shops’ objectives, whereas under independent sourcing the objective of each shop is maximized separately. As such, joint sourcing may lower the procurement price due to the quantity discount (or volume discount, i.e., a reduction in price paid by a buyer when buying a large quantity of items or goods). However, such price squeeze under joint sourcing can backfire, since a low profit margin may lead suppliers to retreat to low-quality solutions (Wharton 2009).

\(^3\)https://ishare.ifeng.com/c/a/7rAbK8yUYZd

Considering the pros and cons of decentralized inspection and centralized sourcing, it is less
clear which strategy out of the independent, joint, and mixed strategies is the most desirable for the brand. This paper aims to address the following questions: What is the optimal strategy for the brand? How do different strategies affect the buyer’s selection between the risky supplier and the backup reliable supplier? How does the buyer’s endogenous strategy choice affect the quantity of low-quality products sold to the market?

To address these questions, we consider a socially responsible brand with two branches/shops. By being socially responsible, we mean the brand not only cares about its profit, but also has a disutility when selling low-quality products to the market. We compare the three strategies in the context with the supplier’s moral hazard in choosing product quality.

Although both a higher procurement price and a higher inspection sampling frequency are credited to motivate the supplier’s high-quality effort, they are substitutes, as a higher procurement price drives the supplier’s high effort, and accordingly reduces the need of exerting a high inspection effort. That is, a higher procurement price can result in a lower inspection sampling frequency, and vice versa. After examining the impact of the different sourcing and inspection types upon these two levers, we uncover the interaction between the inspection effort from the buyer and the supplier’s quality effort. The main findings are summarized below.

First, compared to strategy \( m \), strategy \( i \) requires a higher inspection accuracy for sourcing from the risky supplier to be the optimal choice. However, it is possible that the risky supplier is more or less likely to be chosen in strategy \( j \) than in either strategy \( i \) or \( m \). For example, when the fixed cost of joint inspection is low enough, the risky supplier is most likely to be chosen in strategy \( j \) than in strategy \( i/m \). In this case, with intermediate inspection accuracy, the quantity of low-quality products in strategy \( j \) can be higher than in strategy \( i \) or \( m \), because the risky supplier is used in strategy \( j \) while the backup supplier is used in strategy \( i/m \).

Moreover, when the inspection accuracy is high enough, the risky supplier is used in each strategy, and the procurement price in strategy \( j/m \) is the lowest/highest. That is, given independent inspection, the procurement price in joint sourcing (i.e., strategy \( m \)) is higher than in independent sourcing (i.e., strategy \( i \)). Interestingly, this implies quantity discount due to joint sourcing over independent sourcing may not hold under independent inspection. This is because in strategy \( i \), both shops set their procurement prices separately, and one shop’s offer of high procurement price that leads to high supplier’s quality effort also benefits the other shop. This spillover effect due to decentralized procurement price decisions discourages each shop’s incentive to offer a high procurement price. However, joint sourcing in strategy \( m \) does not entail this spillover problem, and the brand is willing to set a high procurement price to motivate supplier’s high-quality effort.
This explains why the procurement price in strategy $m$ is higher than in strategy $i$.

Accordingly, in this case of high enough inspection accuracy (in which the risky supplier is used in each strategy), the inspection sampling frequency of the buying side in strategy $m$ can be lower than in strategy $i$, due to the substitution between procurement price lever and the inspection lever. Moreover, the inspection sampling frequency in strategy $m$ can be the lowest, as it is also lower than that in strategy $j$. This is because joint inspection in strategy $j$ can avoid the free-riding incentive in decentralized inspection in strategies $m$ and $i$, and as a result the inspection sampling frequency is the highest in strategy $j$.

Second, the brand prefers strategy $m$ to strategy $i$, because though both strategies leverage independent inspection, in strategy $m$ the procurement price is set to maximize the brand’s utility instead of each shop’s in strategy $i$. In terms of the choice between strategies $j$ and $m$, if the fixed cost incurred in strategy $j$ is low enough, the brand always prefers strategy $j$; otherwise, the brand prefers strategy $m$ to $j$ when the inspection accuracy is sufficiently high. To understand this, note that when the inspection accuracy is very high, the risky supplier is used in both strategies $j$ and $m$, and (with such an effective inspection lever) the difference of the inspection frequencies and procurement prices between strategies $j$ and $m$ is small; thus, the advantage of joint inspection over independent inspection is less pronounced under high inspection accuracy, and accordingly strategy $j$ is less favorable due to the fixed cost incurred for joint inspection.

Third, as a socially responsible brand, the more the brand cares about low-quality products sold to the market, the more likely strategy $m$ is preferred. To reveal the logic behind, we show that as the brand cares more about low-quality products sold to the market, it will leverage more the inspection lever but less the procurement price lever in strategies $j$ and $m$ (since the inspection lever reduces low-quality products sold to the market more directly than the procurement price lever); as a result, the difference of inspection levers between strategies $m$ and $j$ becomes smaller, and the advantage of joint inspection over independent inspection is less significant, which leads strategy $j$ to be less favorable to the brand compared to strategy $m$. However, we have shown that when the risky supplier is used in both strategies $j$ and $m$, strategy $m$ leads to a larger quantity of low-quality products sold to the market. This raises a paradox that as the brand cares more about low-quality products sold to the market, its strategy can switch from $j$ to $m$, which leads to more low-quality products sold to the market.

Our results help explain the growing practice of mixed mode (i.e., strategy $m$ of joint sourcing and independent inspection) in reality along with the rapid development of inspection technology. Furthermore, provided that strategy $m$ is preferred by the brand to strategy $j$ under high inspection accuracy,
accuracy, our model reveals the conflict between the brand and the government aiming to reduce the adulterated products in the market: With the improvement in inspection accuracy, the brand switches from strategy $j$ to strategy $m$. Such a mixed strategy adoption increases the brand’s utility, but meanwhile also increases the quantity of low-quality products in the market. Similarly, with a greater goodwill loss of the brand caused by product adulteration, the brand may also switch from strategy $j$ to strategy $m$. Such strategy switch benefits the brand, but hurts the society. In such situations, in order to de-risk adulteration, it is necessary for the government to conduct more spot checks.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model setup. Section 4 characterizes the equilibrium of each strategy and Section 5 compares the strategies. Section 6 concludes the paper. All proofs are provided in the E-Companion.

2. Literature Review

This paper is related to the stream of literature on quality management in supply chain settings. In our paper, the upstream supplier sets its effort level which affects product quality level, and the downstream buyer endogenously decides its inspection policy affecting the consequent penalty if the product fails to pass the inspection. As the event that the supplier’s product fails in the buyer’s inspection before delivering to consumers is termed as the internal failure by the quality management literature, the penalty dependent upon the inspection results is referred to as the internal failure penalty. Because the quality management literature is broad, we focus our review on the literature where the downstream buyer leverages the inspection and the consequent internal failure penalty to motivate the supplier’s high effort.

In the dyad with one downstream retailer and one upstream supplier, Hwang et al. (2006) compare the effectiveness of internal failure penalty and supplier certification, and find that even if the supplier’s inspection is effective, the buyer prefers certification. Babich and Tang (2012) compare the effectiveness of internal failure penalty and deferred payment contingent upon the product failure when used by consumers. They show that when inspection is accurate and less costly, the buyer prefers internal failure penalty to deferred payment. Rui and Lai (2015) extend Babich and Tang (2012) to consider an endogenous procurement quantity and general defect discovery process. Lee and Li (2018) examine internal failure penalty when the buyer’s effort is either contractible or non-contractible, and the efforts of both players are either complementary or sub-
stitutable. Lim (2001) is the only paper in this stream that assumes asymmetric information about the supplier’s status quo process quality. The author studies how different suppliers’ process quality can be screened by the internal failure penalty and penalty contingent upon the product failure when used by consumers. All the above papers consider a vertical supply chain with one supplier and one buyer. Similar to inspection in the quality management literature, auditing is leveraged to identify the supplier’s unethical or irresponsible effort (Chen and Lee, 2017). For example, Caro et al. (2018) examine different audit-penalty mechanisms in a supply chain comprising two buyers and one supplier. In a supply chain with a supplier, a buyer and a third-party auditor, uncover the effect of supplier-auditor collusion on the buyers auditing and contracting strategy in responsible sourcing.

In contrast, we consider a different supply chain structure with one brand owning two shops and two types of suppliers (risky supplier and reliable backup supplier). We capture the externality of independent inspection/sourcing between shops. That is, under independent inspection, if one shop does not identify adulterated products from the risky supplier, this shop may still benefit from adulteration identification by another shop. However, given other shops’ inspection efforts, such a positive externality lures the free-riding behavior of a shop in the form of lowering its own sampling frequency for products from the risky supplier. Such misaligned interests among shops disappear in joint inspection. Moreover, under independent sourcing, we realize that if one shop offers a high procurement price to the supplier, it can motivate the risky supplier’s high-quality effort; but this effect spills over to the other shop. Therefore, each shop hopes to free ride on the other’s offer of high procurement price. This free-riding incentive is absent in joint sourcing.

In addition, there are growing studies by operations and supply chain management community on economically motivated adulteration, and particularly on how to motivate the supplier’s high-quality effort to reduce food adulteration. Mu et al. (2014) and Mu et al. (2016) investigate the buyers’ inspection and farmers’ quality incentives of milk adulteration with different supply structures. Products from different farmers can be mixed together for testing. Such a mixture can increase the farmers’ free-riding incentive if without proper mechanism design. They prove that in a particular supply chain structure the quality of milk critically depends on the quality test mechanism design. Levi et al. (2020) focus on characterizing the suppliers’ endogenous preemptive and reactive strategies for economically motivated adulteration. They caution that investing in quality without enhancing testing capabilities may inadvertently increase the suppliers’ adulteration risk. Dong et al. (2022) compare two types of food safety auditing structures: decentralized and centralized audits. Decentralized food safety auditing systems are common in developing
economies; that is, different tiers of a food supply chain are audited by separate government agencies. Such an auditing structure has been widely criticized as ineffective in mitigating food safety risks. Some developing countries began implementing centralized auditing systems as a remedy for the mis-coordination across local auditing agencies. They show that changing the auditing structure from decentralization to centralization may fail to improve food safety.

Another related literature compares centralized and decentralized procurement/sourcing. Typically, product adulteration issue is not considered in this literature. For example, Arya et al. (2015) investigate decentralized decision making in procurement for a multinational brand with multiple divisions. They do not consider quality issue, and their main focus is on procurement timing and the consequent mitigation of double marginalization.

3. Model Setup

We consider a buyer/brand with two shops (or branches) indexed by 1 and 2. For analytical tractability, we consider symmetric shops: Each shop’s total demand/order size is normalized to 1. Shop \( z = 1, 2 \) can obtain a revenue of \( r \) by selling one unit of product to consumers. There are two types of suppliers, a risky supplier and a backup reliable supplier. The product quality of the backup supplier is reliable. By sourcing from the backup supplier, shop \( z \) can guarantee a profit of \( b \). Sourcing from the risky supplier may generate a higher profit. However, the risky supplier may make low effort in quality and hence there will be quality risk by sourcing from the risky supplier.

**Risky Supplier’s Quality Effort.** In the following, when we mention supplier without particular specification, we refer to the risky supplier. We consider the supplier’s binary quality effort \( e_s \): high effort \( (e_s = H) \) and low effort \( (e_s = L) \). Activities such as adulteration by mixing the expired meat with the fresh meat or producing defective products correspond to low effort; no adulteration is considered as high effort. Suppose the supplier exerts a high effort with probability \( y \) and a low effort with probability \( 1 - y \). When the high and low efforts are chosen with such a mixed strategy, the supplier’s production cost is \( c_y \) for each unit of product. If later the product sold to consumers is identified to be made by irresponsible adulteration actions (i.e., low efforts), the supplier incurs a reputation loss of \( g \). Suppose the adulterated product sold to the market will be finally identified by consumers with probability \( \lambda \). Thus, the adulterated product sold to consumers will cause an expected reputation loss of \( \lambda g \) for the supplier.

When an adulterated product is identified by consumers, it does not only cause a reputation loss for the supplier, but also a reputation loss of \( d \) for shop \( z = 1, 2 \) that sold the product. There-
Table 1: Notation

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<tr>
<th>Superscripts</th>
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<tr>
<td>$j$</td>
<td>joint inspection after joint sourcing</td>
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<tr>
<td>$i$</td>
<td>independent inspection after independent sourcing</td>
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<td>$m$</td>
<td>mixed mode of independent inspection after joint sourcing</td>
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<th>Subscripts</th>
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<td>$s$</td>
<td>supplier</td>
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<td>$b$</td>
<td>brand</td>
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<td>$z$</td>
<td>shop $z = 1, 2$</td>
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<th>Parameters</th>
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<td>$b$</td>
<td>shop’s profit from the backup option</td>
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<td>$c$</td>
<td>cost of the supplier’s high effort</td>
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<td>$d$</td>
<td>shop’s reputation loss of a low-quality product</td>
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<tr>
<td>$e$</td>
<td>quantity of low-quality products sold to the market</td>
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<tr>
<td>$g$</td>
<td>supplier’s reputation loss of a low-quality product</td>
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<td>$k$</td>
<td>inspection cost of the shop</td>
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<td>$r$</td>
<td>retail price of the shop</td>
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<td>$h$</td>
<td>buyer’s degree of caring the low-quality product sold in the market</td>
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<td>$F$</td>
<td>fixed cost incurred for joint inspection in strategy $j$</td>
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<td>$U$</td>
<td>utility of the buyer</td>
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<tr>
<td>$\pi$</td>
<td>profit of the player</td>
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<td>$\eta$</td>
<td>accuracy of the inspection</td>
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<td>$\lambda$</td>
<td>probability of finding a low-quality product in the market</td>
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<th>Decision Variables</th>
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<tr>
<td>$y$</td>
<td>probability of the supplier’s high effort</td>
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<td>$x$</td>
<td>probability of conducting an inspection</td>
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<td>$w$</td>
<td>procurement price paid to the supplier</td>
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fore, the brand and its shops have incentives to motivate the supplier’s high-quality effort. One approach is to conduct inspection to identify the supplier’s adulterated product before selling it to consumers. We consider two inspection mechanisms, independent inspection and joint inspection.

**Independent versus Joint Inspection.** Under independent inspection, each shop separately decides its own inspection effort for its product from the supplier. Shop $z$ may inspect the supplier’s product, and reject the incoming lot if the supplier’s product is identified adulterated. Suppose shop $z$ inspects the product with probability $x_z$, which incurs an inspection cost $kx_z$ for shop $z$. This $x_z$ can be viewed as shop $z$’s sampling frequency. The inspection is not perfect, as it can only detect the supplier’s adulteration with probability $h$. Thus, once the supplier does adulterate, under the sampling frequency of $x_z$, the probability with which shop $z$ identifies the supplier’s adulteration is $hx_z$. Moreover, as both shops are under the same brand, both shops share the inspection results. Hence, once the supplier does adulterate, the probability with which neither shop identifies the supplier’s adulteration is $(1-hx_1)(1-hx_2)$.

Under joint inspection, it is the brand that may inspect the supplier’s product, and reject the incoming lot if the supplier’s product does not pass the inspection. Given the normalized one unit of product demand for each shop, the brand’s total demand size is 2. Suppose the brand conducts an inspection for each unit of the product with probability $x$ (i.e., the sampling frequency is $x$). Then the inspection cost is $2kx$. As the inspection is not perfect, once the supplier adulterates, the brand can only identify the supplier’s adulteration with probability $\eta x$ (or fails to identify the supplier’s adulteration with probability $1-\eta x$) for each unit of the product. As such, under the sampling frequency $x$, once the supplier does adulterate, the probability with which the brand fails to identify the supplier’s adulteration is $(1-\eta x)^2$.

Note that if the brand conducts joint inspection, the products need to be first shipped to a centralized warehouse for inspection, and then after inspection shipped to two shops that are geographically spread out. Thus, there is a fixed cost $F$ incurred for joint inspection, such as the cost of building the warehouse for centralized inspection and the additional shipping cost. This fixed cost is not incurred under independent inspection in strategy $m$ or $i$. Furthermore, sourcing from the reliable backup supplier will not incur this fixed cost either, since there is no need for inspection with the backup supplier.

No matter under joint or independent inspection, when inspection identifies the supplier’s adulteration (i.e., low effort), both shops refuse to buy the product and do not pay procurement prices, and meanwhile avoid the reputation loss of $d$ by using the backup supplier to earn a profit.
On top of inspection, offering suppliers a higher wholesale price can better motivate the supplier’s effort, because the revenue from the brand/shops will be lost once low quality is identified by inspection. In the following, we discuss two sourcing modes: independent sourcing and joint sourcing.

**Independent versus Joint Sourcing.** Under independent sourcing, the unit procurement price from shop $z$ to the supplier, $w_z$, is set by shop $z$; the two shops separately decide the prices. Note that, under independent sourcing, the products are separately shipped to the two shops, and hence there is no option of joint inspection, as only independent inspection is feasible. Since both shops under the same brand share their inspection results, once there is a shop identifying the adulteration, both shops reject the products and do not pay procurement prices $w_z$. In this case, the backup supplier will be used. On the other hand, if no low-quality product is identified by both shops’ inspections, shop $z$ accepts the product for sales and pays the supplier the procurement price $w_z$.

Under joint sourcing, the brand decides a uniform per unit procurement price, $w$, from shops to the supplier. There are two possible inspection modes in the case of joint sourcing, independent or joint inspection. If independent inspection is adopted, the inspections are conducted by each shop separately, and the payment arrangements are the same as above. That is, if no low-quality product is identified by both shops’ inspections, each shop accepts the product for sales and pays the supplier the procurement price $w$; otherwise, each shop rejects the product without paying $w$ and uses the backup supplier for a profit of $b$. If joint inspection is adopted, the inspection is conducted by the brand rather than its shops. If no low-quality product is identified by the brand’s inspection, each shop accepts the product for sales and pays the supplier the procurement price $w$. Otherwise, the brand rejects the products and the shops use the backup supplier.

In contrast to inspection, there is no fixed-cost difference between joint sourcing and independent sourcing, because under joint sourcing the brand just offers a uniform procurement price and the products do not need to be physically shipped to a centralized warehouse.

**Socially Responsible Brand.** Out of the brand’s choice over the sourcing mode and inspection mode, there are three strategies: joint strategy (i.e., joint sourcing and joint inspection), mixed strategy (joint sourcing and independent inspection), and independent strategy (independent sourcing and independent inspection). These three strategies are also referred to as strategy $j$, $m$, and $i$, respectively. Which strategy should the brand adopt? The brand may be purely profit driven or socially responsible. In this paper, we consider a socially responsible brand that cares...
profit and meanwhile may also care the quantity of low-quality products sold to the market.

The brand’s utility is its profit net the disutility of selling low-quality products. For simplicity, we model the disutility of selling low-quality products as a linear function of the quantity of low-quality products sold to consumers. Denote by \( \pi_z \) the profit of shop \( z \), and \( \pi_b = \pi_1 + \pi_2 \) the brand’s profit (i.e., the total profit of two shops). Denote by \( e_z \) the quantity of low-quality products sold by shop \( z \) (which is referred to as “the external failure” in quality management literature), and \( e = e_1 + e_2 \) the total quantity of low-quality products in the market. In strategy \( i \), the decisions are made by each shop separately; shop \( z \) decides the procurement price \( w_z \) and the inspection sampling frequency \( x_z \) to maximize its utility \( U_z = \pi_z - he_z \), where \( he_z \) measures shop \( z \)’s disutility of selling low-quality products. In particular, \( h = 0 \) refers to a profit-driven brand. In strategy \( m \), the brand decides the procurement price \( w \) offered to the supplier to maximize its utility \( U_b = U_1 + U_2 \), and each shop \( z \) decides the inspection sampling frequency \( x_z \) to maximize \( U_z = \pi_z - he_z \). In strategy \( j \), the brand decides the procurement price \( w \) and the inspection sampling frequency \( x \) to maximize its utility \( U_b \).

To rule out uninteresting cases, throughout this paper we make the following assumptions.

\[ c \geq \lambda g. \]  
\[ \text{(C1)} \]

Recall that \( c \) is the supplier’s marginal high-effort cost. Without (C1), the risky supplier’s high-effort cost is always lower than its expected reputation loss due to low-quality product, in which case the risky supplier always exerts a high effort, and hence the brand will source from the risky supplier and set the procurement price as low as possible. As such, all three strategies (i.e., \( i/j/m \)) lead to the same equilibrium. Condition (C1) rules out this uninteresting case.

\[ r - c \geq b \geq r - h - \lambda d. \]  
\[ \text{(C2)} \]

If the first inequality of (C2), i.e., \( r - c \geq b \), does not hold, the risky supplier is never used by the brand, as with \( b > r - c \) it is more profitable to work with the backup supplier. The second inequality of (C2), i.e., \( r - h - \lambda d \leq b \), ensures that once the risky supplier exerts a low effort for sure, the buyer obtains a utility lower than working with the backup supplier if the buyer does not conduct any inspection.

\[ F \leq 2(r - c - b). \]  
\[ \text{(C3)} \]

If (C3) does not hold, the risky supplier is never used by the brand in strategy \( j \) due to the high
fixed cost, that is, with $2(r - c) - F < 2b$ the brand prefers the backup supplier to the risky supplier. This is because with this risky supplier the maximum utility for the brand is $2(r - c) - F$, while with the backup supplier the brand’s utility is $2b$.

4. Equilibrium Analysis for Each Strategy

To derive the brand’s optimal choice among strategy $i, j,$ and $m$, we first need to derive the brand’s utility under each strategy. The equilibrium for each strategy is solved backward by first analyzing the inspection game and then the procurement price decision.

For expositional simplicity, we define two terms:

$$ a(w) = \frac{w - c}{w - \lambda g}, $$

$$ \beta(w) = \frac{k}{\eta(w + b + h + \lambda d - r)}, $$

where $a(w)$ measures the supplier’s incentive of providing a high-quality product, and $\beta(w)$ measures the buyer’s reluctance of exerting inspection effort. To understand $a(w)$, note that $w - c$ is the supplier’s profit of providing a high-quality product, and $w - \lambda g$ is the supplier’s profit of providing a low-quality product ($\lambda$ is the probability that the low-quality product sold to the market is identified by consumers, and this external failure causes a goodwill loss $g$ for the supplier).

To understand $\beta(w)$, note that for a low-quality product from the risky supplier, if the buyer fails to identify it and sells it to the market, its marginal utility is $r - w - \lambda d - h$ (among which a profit of $r - w - \lambda d$ is earned and a disutility of $h$ is incurred), while if the buyer identifies it, it earns a profit of $b$ from the backup supplier. Thus, $b - (r - w - \lambda d - h) = w + b + h + \lambda d - r$ is the buyer’s benefit of identifying a low-quality product. This benefit can be captured through the buyer’s inspection effort. Given inspection accuracy $\eta$, $\eta(w + b + h + \lambda d - r)$ (the denominator of $\beta(w)$) is the buyer’s marginal benefit of increasing the inspection frequency, while $k$ (the numerator of $\beta(w)$) is the buyer’s marginal cost of increasing the inspection frequency. As such, $\beta(w)$ measures the buyer’s reluctance of relying on the inspection lever to motivate the supplier’s high-quality effort.

Both a higher procurement price and a higher inspection sampling frequency are credited to motivate the supplier’s high effort. However, they are substitutes: A higher $w$ leads to a higher supplier’s quality effort, i.e., a greater $y$; anticipating the increase of $y$, the buyer may exert a lower inspection effort, i.e., a smaller $x$. Since the quantity of low-quality products sold to the market is
jointly determined by the supplier’s quality effort and the buyer’s inspection effort, it is not clear whether a higher procurement price \( w \) may lead to a smaller quantity of low-quality products sold to the market. In the following, we first solve the equilibrium outcomes under each strategy, and then discuss which strategy relies the most on price/inspection to motivate the supplier’s high-quality effort, and which strategy is preferred by the brand.

4.1 Strategy \( j \): Joint Sourcing and Joint Inspection

In this subsection, we consider strategy \( j \) in which the brand conducts a joint inspection after joint sourcing. The sequence of events is described as follows. First, in the supplier choice stage, each shop decides whether to source from the risky supplier or the backup supplier. If sourcing from the backup supplier, each shop obtains a profit of \( b \) and the game ends; otherwise, go to the next stage. Second, in the procurement price decision stage, the brand offers the procurement price \( w \) to the risky supplier to maximize its utility. Once the risky supplier accepts the price \( w \), it produces the products by choosing its own quality effort (i.e., exerting a high effort with probability \( y \)). Third, in the inspection stage, the brand decides the sampling frequency \( x \) for the products. If the brand identifies the adulteration through inspection, both shops reject the product, and the backup option is exercised. If no adulteration is identified, the delivered product is sold to consumers. Fourth, in the consumption stage, the adulteration (if any) is identified by the government or consumers with probability \( l \), and then the brand and supplier suffer from the reputation loss.

We first derive the brand’s utility of sourcing from the risky supplier by backward induction. When the risky supplier exerts a low effort, if the brand’s sampling frequency is \( x \), the brand fails to identify low quality with probability \((1 - h x)^2\). As such, the quantity of low-quality products sold to the market is \( e = 2(1 - y)(1 - \eta x)^2 \). The brand’s profit is

\[
\pi_b(x, y, w) = 2[y(r - w) + (1 - y)((1 - (1 - \eta x)^2)b + (1 - \eta x)^2((1 - \lambda)(r - w) + \lambda(r - w - d)))] - kx - F
\]

With \( e(x, y, w) = 2(1 - y)(1 - \eta x)^2 \) units of low-quality products sold to consumers, the brand’s problem when working with the risky supplier is to choose the inspection sampling frequency \( x \) to maximize its utility:

\[
\max_x U_b(x|y, w) = \pi_b(x, y, w) - he(x, y, w), \quad (2)
\]
where $-he$ is the brand’s disutility of selling low-quality products to the market, $w$ is the given wholesale price, and $y$ is the risky supplier’s quality effort.

The risky supplier’s problem is to choose the quality effort to maximize its profit given $w$.

$$\max_y \pi_s(y|x, w) = 2[y(w - c) + (1 - y)(1 - \eta x)^2((1 - \lambda)w + \lambda(w - g))].$$ \hspace{1cm} (3)

By sequentially solving the risky supplier’s and the brand’s problems in (2) and (3), we derive the brand’s optimal utility of working with the risky supplier (given wholesale price $w$).

**Lemma 1.** In strategy $j$, the equilibrium results given $w$ are summarized as follows.

(a) If $(1 - \eta)^2 \leq \alpha(w) \leq 1$ and $0 < \beta(w) \leq 2\sqrt{\alpha(w)}$, the brand will source from the risky supplier. The supplier makes a high-quality effort with probability $y^j = 1 - \frac{\beta(w)}{2\sqrt{\alpha(w)}}$, and the brand will conduct an inspection with probability $x^j = \frac{1 - \sqrt{\alpha(w)}}{\eta}$. Accordingly, the expected quantity of low-quality products sold to the market is $e^j(w) = \sqrt{\alpha(w)}\beta(w)$. The supplier will earn $\pi^j_s = 2(w - c)$, and the brand’s utility is $U^j_b(w) = 2(r - w) + \frac{k(1 - \sqrt{\alpha(w)})^2}{\eta \sqrt{\alpha(w)}} - \frac{\beta(w)\lambda d}{\sqrt{\alpha(w)}} - F$ with a profit of $\pi^j_b(w) = U^j_b(w) + he^j(w)$.

(b) Otherwise, the brand sources from the backup supplier.

Note that if offered a higher procurement price $w$, the supplier’s opportunity cost of losing the business is higher when its product is identified as low quality in inspection. Therefore, in the case of $(1 - \eta)^2 \leq \alpha(w) \leq 1$ and $0 < \beta(w) \leq 2\sqrt{\alpha(w)}$, a higher procurement price $w$ leads to higher supplier’s quality effort, i.e., $y^j = 1 - \frac{\beta(w)}{2\sqrt{\alpha(w)}}$ increases in $w$. Due to the substitution between the procurement price lever and the inspection lever, the brand’s inspection effort $x^j = \frac{1 - \sqrt{\alpha(w)}}{\eta}$ decreases in the procurement price $w$. That is, a higher procurement price $w$ can motivate the supplier’s high-quality effort, even though the brand lowers the inspection effort. As the quantity of low-quality products sold to consumers, $e^j = 2(1 - y^j)(1 - \eta x^j)^2 = \sqrt{\alpha(w)}\beta(w)$, is jointly governed by both the supplier’s and the brand’s efforts, an increase in the procurement price $w$ has two contrasting effects on $e^j$ (i.e., a smaller $1 - y^j$ and a greater $1 - \eta x^j$). Overall, we find that the quantity of low-quality products in the market, $e^j$, decreases in the procurement price $w$.

By comparing the brand’s utility of working with the backup supplier and that of sourcing from the risky supplier, we have the following result: According to the definition in (1), $(1 - \eta)^2 \leq \alpha(w) \leq 1$ refers to high supplier’s quality incentive, and $0 < \beta(w) \leq 2\sqrt{\alpha(w)}$ refers to the brand’s high inspection incentive. Therefore, under these two conditions, it is not difficult to understand that in strategy $j$, the brand chooses to work with the risky supplier. However,
when either the supplier does not have enough incentive to make high-quality investment, or the brand’s inspection incentive is not high, the brand will choose the backup supplier.

Next, we further solve the brand’s utility optimization problem over the procurement price $w$ when using the risky supplier, i.e., $\max_w U'_b(w)$, and then derive the brand’s choice between the risky and the backup supplier as follows.

**Lemma 2.** In strategy $j$, there exists a threshold $\eta^j$ such that (a) if $\eta \geq \eta^j$ (which results in $(1 - \eta)^2 \leq \alpha(w^j) \leq 1$ and $0 < \beta(w^j) \leq 2\sqrt{\alpha(w^j)}$), there exists a unique equilibrium in which the brand sources from the risky supplier, and offers a procurement price $w^j = \arg\max_w U'_b(w)$ that decreases in the inspection accuracy $\eta$ but increases in the inspection cost $k$; (b) otherwise, the brand sources from the backup supplier.

The inspection accuracy $\eta$ is important in determining the active supplier in equilibrium: The risky supplier is used as long as the inspection accuracy is high enough, and in this case higher inspection accuracy $\eta$ results in a lower profit for the supplier, since the procurement price $w^j$ decreases in $\eta$. Moreover, higher inspection accuracy $\eta$ increases the brand’s utility. Similarly, since the procurement price $w^j$ increases in the inspection cost $k$, a lower inspection cost $k$ leads to a lower profit for the supplier but a higher utility for the brand.

### 4.2 Strategy $m$: Joint Sourcing and Independent Inspection

In this subsection, we consider strategy $m$ in which the brand conducts joint sourcing and after which the shops conduct independent inspection and share the inspection results.

The sequence of events is as follows. The first stage of supplier selection, and the second stage of procurement price decision and the risky supplier’s quality effort decision follow the same procedure as in strategy $j$. In the third inspection stage, the products are delivered to shops. Once receiving the product, shop $z = 1, 2$ inspects it by a sampling frequency $x_z$ simultaneously. The inspection results are shared between the shops. If one shop identifies the adulteration through inspection, both shops reject the product, and the backup option is exercised. If no shop identifies the adulteration, the products are sold to the consumers. In the fourth consumption stage, the adulteration (if any) is identified by the government or consumers with probability $\lambda$, and then each shop and the supplier suffer from the reputation loss.

Again, we solve the equilibrium backward. We first solve each shop’s inspection effort decision when it works with the risky supplier. When the supplier exerts a low effort, if shop $z$’s
sampling frequency is \( x_z \), then shop \( z \) can identify it with probability \( \eta x_z \). Provided that with probability \( 1 - \eta x_z \) shop \( z \) fails to identify the supplier’s low effort and the inspection results are shared between shops, \((1 - \eta x_1)(1 - \eta x_2)\) is the probability that the inspection fails to detect the supplier’s low effort, in which case the expected quantity of low-quality products sold by shop \( z \) is \( e_z(x_z, x_{3-z}, y, w) = (1 - y)(1 - \eta x_1)(1 - \eta x_2) \).

The profit of shop \( z = 1, 2 \) when choosing inspection effort \( x_z \), given procurement price \( w \) and supplier’s quality effort \( y \) is

\[
\pi_z(x_z, x_{3-z}, y, w) = y(r - w) + (1 - y)\left\{(1 - \eta x_z)(1 - \eta x_{3-z})\right\}b + (1 - \eta x_z)(1 - \eta x_{3-z})((1 - \lambda)(r - w) + \lambda (r - w - d))\right\} - kx_z = y(r - w) + (1 - y)[b - (1 - \eta x_1)(1 - \eta x_2)(w + b + \lambda d - r)] - kx_z, \ z = 1, 2. \tag{4}
\]

The problem of shop \( z \) is to decide \( x_z \) to maximize its utility, formulated as

\[
\max_{x_z} U_z(x_z|x_{3-z}, y, w) = \pi_z(x_z, x_{3-z}, y, w) - h e_z(x_z, x_{3-z}, y, w). \tag{5}
\]

Alternatively, if buying from the backup supplier, there is no low-quality products in the market, and shop \( z \)’s utility is \( b \), the same as its profit.

Anticipating shops’ inspection efforts and given the procurement price, the risky supplier decides \( y \) to maximize its profit as follows.

\[
\max_y \pi_s(y|x_1, x_2, w) = 2y(w - c) + (1 - y)\sum_{z=1}^2 [(1 - \eta x_z)(1 - \eta x_{3-z})((1 - \lambda)w + \lambda (w - g))]
= 2[y(w - c) + (1 - y)(1 - \eta x_1)(1 - \eta x_2)(w - \lambda g)]. \tag{5}
\]

By solving problem (5), we derive Lemma 3 that characterizes the supplier’s quality decision and shop’s inspection decision in strategy \( m \).

**Lemma 3.** In strategy \( m \), the equilibrium results given \( w \) are summarized as follows.

(a) If \((1 - \eta)^2 \leq a(w) \leq 1 \) and \( 0 < \beta \leq \sqrt{a(w)} \), the brand will source from the risky supplier, the supplier will choose the high effort with probability \( y^m = 1 - \frac{\beta(w)}{\sqrt{a(w)}} \), and shop \( z = 1, 2 \) will conduct an inspection with probability \( x^m = \frac{1 - \sqrt{a(w)}}{\eta} \). Accordingly, the expected quantity of low-quality products sold to the market is \( e^m_z = \sqrt{a(w)} \beta(w) \). The supplier’s profit is \( \pi^m_s = 2(w - c) \) and shop \( z \)’s utility is \( U^m_z = r - w + \frac{(1 - \sqrt{a(w)})k}{\eta \sqrt{a(w)}} - \frac{\beta \lambda d \sqrt{a(w)}}{\sqrt{a(w)}} \) with a profit of \( \pi^m_z = U^m_z + h e^m_z \).

(b) Otherwise, the brand sources from the backup supplier.
Similar to the results in strategy $j$, only when the supplier’s quality incentive is high ($\left(1 - \eta\right)^2 \leq \alpha(w) \leq 1$) and the shops’ inspection incentive is also high ($0 < \beta(w) \leq \sqrt{\alpha(w)}$), the brand chooses to work with the risky supplier in strategy $m$. Otherwise, the brand chooses the backup supplier. By comparing the conditions to use the risky supplier in strategy $j$ and strategy $m$, we find that the condition required for $\alpha(w)$ is the same, but the required range for $\beta(w)$ is larger in strategy $j$ than in strategy $m$. This means that the brand in strategy $j$ is more tolerant for less attractive inspection lever to use the risky supplier; that is, for the same wholesale price $w$, the risky supplier is more likely chosen in strategy $j$ than in strategy $m$.

Next, we further solve the brand’s utility optimization problem over procurement price and supplier selection in strategy $m$. Comparing the brand’s utility of using the risky supplier with that of using the backup supplier, we derive the following lemma.

**Lemma 4.** In strategy $m$, there exists a threshold $\eta^m$ such that (a) if $\eta \geq \eta^m$ (which will result in $(1 - \eta)^2 \leq \alpha(w^m) \leq 1$ and $0 < \beta(w^m) \leq \sqrt{\alpha(w^m)}$), there exists a unique equilibrium in which the brand sources from the risky supplier and the procurement price is $w^m = \arg\max_w [U^m_1(w) + U^m_2(w)]$ that decreases in the inspection accuracy $\eta$ but increases in the inspection cost $k$; (b) otherwise, the brand sources from the backup supplier.

Similar to Lemma 2, in strategy $m$, the risky supplier is used as long as the inspection accuracy is high enough. In this case, the impact of $\eta$ and $k$ on the supplier’s profit and the brand’s utility is the same as in strategy $j$.

### 4.3 Strategy $i$: Independent Sourcing and Independent Inspection

In strategy $i$, both shops first conduct independent sourcing by offering their own wholesale prices and then conduct independent inspection and share the inspection results. The sequence of events is as follows. First, in the supplier-selection stage, each shop decides whether to source from the risky supplier or the backup supplier. If sourcing from the backup supplier, each shop obtains a profit of $b$ and the game ends; otherwise, go to the next stage. Second, in the stage of procurement price decision, shop $z = 1, 2$ offers the procurement price $w_z$ for the risky supplier. If the risky supplier accepts the price, it produces the product by choosing its own quality effort $y$. Third, in the inspection stage, the product is delivered separately to each shop and shop $z = 1, 2$ simultaneously decides its sampling frequency $x_z$. Both shops share the inspection results. If one shop
identifies the adulteration through inspection, both shops reject the product, and the backup supplier is used. If no shop identifies the adulteration, the delivered products are sold to consumers. Fourth, in the consumption stage, the adulteration (if any) is identified with probability \( \lambda \), and then each shop and the supplier suffer from the reputation loss.

Similar to the analysis in strategy \( m \), we start with solving the equilibrium when shop \( z = 1, 2 \) sources from the risky supplier. Given shop \( z \)'s sampling frequency \( x_z \), no shop identifies the supplier’s low effort with probability \( (1 - \eta x_1)(1 - \eta x_2) \). Because the inspection results are shared between shops within the brand, the expected quantity of low-quality products sold by shop \( z \) is \( e_z = (1 - \eta)(1 - \eta x_1)(1 - \eta x_2) \), given supplier’s quality effort \( \eta \). As such, the profit of shop \( z = 1, 2 \), given procurement price \( w_z \), is

\[
\pi_z = y(r - w_z) + (1 - \eta)[1 - (1 - \eta x_1)(1 - \eta x_2)]b + (1 - \eta x_1)(1 - \eta x_2)((1 - \lambda)(r - w_z) + \lambda(r - w_z - d)) - kx_z
\]

\[
= y(r - w_z) + (1 - \eta)[b - (1 - \eta x_1)(1 - \eta x_2)(w_z + b + \lambda d - r)] - kx_z, \quad z = 1, 2.
\]

Given the procurement prices \( w_1 \) and \( w_2 \), the problem of shop \( z \) in strategy \( i \) is to decide \( x_z \) to maximize utility \( U_z = \pi_z - \eta e_z \). Correspondingly, the supplier’s and each shop’s efforts are characterized by the following lemma.

**Lemma 5.** In strategy \( i \), given wholesale prices \( w_1 \) and \( w_2 \), the equilibrium results are summarized as follows.

\( (a) \) If \( (1 - \eta)^2 \leq \alpha(w_z) \leq 1 \) and \( 0 < \beta(w_z) \leq \sqrt{\alpha(w_z)} \) for \( z = 1, 2 \), the risky supplier is used. The supplier’s quality effort and the shops’ inspection efforts are

\[
y^i(w_1, w_2) = 1 - \frac{k}{\eta} \sqrt{\frac{w_1 + w_2 - 2c}{(w_1 + w_2 - 2c)(w_1 + b + h + \lambda d - r)(w_2 + b + h + \lambda d - r)}},
\]

\[
x_1^i(w_1, w_2) = x_2^i = 1 - \frac{1}{\eta} \sqrt{\frac{w_1 + w_2 - 2c}{w_1 + w_2 - 2\lambda g w_2 + b + h + \lambda d - r}} \quad (6)
\]

\( (b) \) Otherwise, shop \( z \) sources from the backup supplier.

Note that if \( w_1 = w_2 = w \), then (6) is the same as that in Lemma 3. That is, if the wholesale prices are the same, the equilibrium in strategy \( i \) is the same as in strategy \( m \). Anticipating the supplier’s effort and the shops’ inspection frequency in Lemma 5, both shops decide their procurement prices simultaneously.
Substituting (6) into shop z’s utility,  \( U_z = \pi_z - h e_z \), we can show that  \( U_z \) is concave in  \( w_z \), submodular in  \((w_1, w_2)\).

**Lemma 6.** In strategy i, there exists a threshold  \( \eta^i \) such that (a) if  \( \eta \geq \eta^i \) (which will result in  \((1 - \eta)^2 \leq \alpha(w_i) \leq 1 \) and  \( 0 < \beta(w_i) \leq \sqrt{\alpha(w_i)} \)), there exists a unique equilibrium in which each shop sources from the risky supplier and offers the same procurement price  \( w_i \), the shops’ inspection efforts are  \( x_i^1(w_i, w_i) = x_i^2(w_i, w_i) \), and the supplier’s quality effort is  \( y^i(w_i, w_i) \); (b) otherwise, each shop sources from the backup supplier.

Similar to Lemma 4, the inspection accuracy  \( \eta \) is important in determining the equilibrium in strategy i. Even though with exogenous procurement prices, the equilibrium in strategy i is the same as that in strategy m, the procurement prices in these two strategies are different, because the procurement price in strategy i is offered to maximize each shop’s utility instead of the brand’s in strategy m.

5. **Comparison Across the Three Strategies**

Recall that in each strategy, the risky supplier is chosen if and only if the inspection accuracy is higher than a threshold. The following proposition compares the three thresholds.

**Proposition 1.** (a) The risky supplier is more likely to be chosen in strategy m than in strategy i (i.e.,  \( \eta^m \leq \eta^i \)).

(b) There exist thresholds  \( F^\eta \) and  \( \bar{F}^\eta \) such that: When  \( F \leq F^\eta \),  \( \eta \leq \eta^m \leq \eta^i \); When  \( F^\eta < F \leq \bar{F}^\eta \),  \( \eta^m < \eta \leq \eta^i \); When  \( F > \bar{F}^\eta \),  \( \eta^m \leq \eta < \eta^i \).

Though Proposition 1 (a) implies that strategy i requires higher inspection accuracy to source from the risky supplier than strategy m (i.e.,  \( \eta^m \leq \eta^i \)), the relative magnitude between  \( \eta^i \) and the other two thresholds (i.e.,  \( \eta^m \) and  \( \eta^i \)) is unclear. In particular, when the fixed cost of joint inspection for dealing with the risky supplier in strategy j,  \( F \), is large enough (i.e.,  \( F > F^\eta \)), strategy j/m requires the maximum/minimum inspection accuracy to source from the risky supplier. Hence, for  \( \eta \in [\eta^m, \eta^i] \), the backup supplier is used in strategy j but the risky supplier is selected in strategy m; accordingly, strategy m leads to a positive quantity of low-quality products in the market while strategy j does not result in any low-quality products.
In contrast, in the case of low enough \( F \) (i.e., \( F \leq F^\eta \)), \( \eta^i \leq \eta^m \leq \eta^j \) implies that for \( \eta \in [\eta^j, \eta^m] \), strategy \( j \) leads to more low-quality products than strategy \( i \). This is because for relatively low inspection accuracy (i.e., \( \eta \in [\eta^j, \eta^m] \)), the risky supplier is used only in strategy \( j \), and hence only strategy \( j \) leads to low-quality products sold to the market. For relatively high inspection accuracy (i.e., \( \eta \in (\eta^m, \eta^i] \)), only strategy \( i \) uses the backup supplier and does not sell any low-quality products in the market.

5.1 Price Lever Versus Inspection Lever

In the following, we analyze when the inspection accuracy is high enough and the risky supplier is chosen in each strategy (i.e., \( \eta \geq \max\{\eta^i, \eta^j\} \)), which strategy relies the most on the procurement price lever or inspection lever to motivate the supplier’s high-quality effort.

**Proposition 2.** When the risky supplier is chosen in each strategy (i.e., \( \eta \geq \max\{\eta^i, \eta^j\} \)),

(a) \( w^m \geq w^i \) and \( x_z^m \leq x_z^i \), \( z = 1, 2 \); \( y^m \geq y^i \);

(b) There exists a threshold \( c^w \) such that for \( c \leq c^w \), \( w^i \geq w^j \) and \( x_z^i \leq x_z^j \), \( z = 1, 2 \).

Proposition 2 (b) shows the comparison results of procurement price and inspection effort for \( c \leq c^w \). The comparison for \( c > c^w \) is analytically intractable. In fact, we have numerically confirmed that the comparison results in Proposition 2 (b) also hold for \( c > c^w \). Then, Proposition 2, together with the numerical results, implies \( w^m \geq w^i \geq w^j \) and \( x_z^m \leq x_z^i \leq x_z^j \). That is, when the risky supplier is chosen, strategy \( m \) relies the most on the procurement price lever and the least on the inspection lever compared to the other two strategies; strategy \( j \) relies the most on the inspection lever and the least on the wholesale price lever.

First, we try to understand why \( w^m \geq w^i \). In strategy \( i \), the shops set their procurement prices separately; if one shop offers a high procurement price to the supplier which motivates the supplier’s high-quality effort, this quality improvement also benefits the other buyer. Due to this spillover effect, each shop’s incentive to offer a high procurement price is diminished. In contrast, in strategy \( m \), the procurement price is offered to maximize both shops’ total utility, and there is no such incentive misalignment issue. This explains \( w^m \geq w^i \).

For the rationale behind \( w^m \geq w^j \), note that strategies \( m \) and \( j \) only differ in the inspection stage; in strategy \( m \) each shop decides its own inspection effort (and has a free-riding incentive resulting from the positive spillover of the other shop’s inspection), while in strategy \( j \) the inspection effort of both shops is jointly optimized. As such, each shop’s incentive of exerting a
high inspection effort in strategy \( m \) is much lower than the brand’s incentive of exerting a high inspection effort in strategy \( j \). Accordingly, compared to strategy \( m \), strategy \( j \) relies more on the inspection lever (i.e., \( x^m \leq x^j \)) and hence less on the procurement price lever (i.e., \( w^m \geq w^j \)).

Furthermore, to understand why the inspection effort in strategy \( m \) is smaller than that in strategy \( i \) (i.e., \( x^m \leq x^i, z = 1, 2 \)), we note the substitution effect between the procurement price lever and the inspection lever. As explained before, strategy \( i \) relies less on the procurement price lever than strategy \( m \) (i.e., \( w^m \geq w^i \)); thus, as a remedy, the shops need to rely more on the inspection lever in strategy \( i \) compared to in strategy \( m \).

5.2 The Brand’s Optimal Strategy and the Risky Supplier’s Profit

In the following, we compare the brand’s utility in the three strategies for a general inspection accuracy \( \eta \).

Proposition 3. (a) (Strategy \( i \) vs. strategy \( m \)) The brand’s utility in strategy \( m \) is higher than that in strategy \( i \), i.e., \( U_b^i(w^i) \leq U_b^m(w^m) \).

(b) (Strategy \( j \) vs. strategy \( m \)) There exists a threshold \( F \leq F^h \) such that:

(b.1) If \( F < F^h \), the brand’s utility in strategy \( m \) is strictly lower than in strategy \( j \) when \( \eta > \eta^i \); when \( \eta \leq \eta^i \), the backup supplier is used, so the brand’s utilities in both strategies are the same.

(b.2) If \( F > F^h \), the brand’s utility in strategy \( m \) is strictly larger than in strategy \( j \) when \( \eta > \eta^m \); when \( \eta \leq \eta^m \), the backup supplier is used, so the brand’s utilities in both strategies are the same.

(b.3) Otherwise, there exists a threshold \( \bar{\eta} \geq \eta^m \) such that when \( \eta > \bar{\eta} \), the brand’s utility in strategy \( m \) is strictly larger than in strategy \( j \); when \( \eta \in (\eta^i, \bar{\eta}) \), the brand’s utility in strategy \( m \) is strictly smaller than in strategy \( j \); when \( \eta \leq \eta^i \), the backup supplier is used, so the brand’s utilities in both strategies are the same.

The result in Proposition 3(a) that strategy \( m \) leads to a higher brand’s utility than strategy \( i \) is relatively intuitive. This is because both strategies adopt independent inspection and hence given the same procurement price, the inspection decisions will be the same. Since the optimal procurement price in strategy \( i \) is feasible in strategy \( m \), strategy \( m \) leads to a higher brand’s utility as its objective is to maximize the brand’s utility in joint sourcing, overcoming the misaligned interest in strategy \( i \) that each shop maximizes its own utility and there is free-riding incentive in
procurement price decision. Specifically, if the inspection accuracy is high enough (i.e., \( \eta \geq \eta' \)), the risky supplier is used in both strategies \( m \) and \( i \), and the brand’s utility in strategy \( m \) is strictly higher than that in strategy \( i \). If the inspection accuracy is low enough (i.e., \( \eta < \eta'' \)), in both strategies, the reliable supplier is used and the brand’s utility is the same. With an intermediate inspection accuracy (i.e., \( \eta'' \leq \eta < \eta' \)), the risky supplier is used in strategy \( m \) while the reliable supplier is used in strategy \( i \). Provided that sourcing from the reliable supplier is a feasible choice in strategy \( m \), the optimal decision of buying from the risky supplier in strategy \( m \) implies that the brand’s utility in strategy \( m \) is higher than that in strategy \( i \). To summarize, if \( \eta > \eta'' \), the brand’s utility in strategy \( m \) is strictly higher than that in strategy \( i \).

Proposition 3(a) implies that strategy \( m \) dominates strategy \( i \) for the brand. As such, for the brand, it suffices to compare strategy \( j \) and strategy \( m \) hereafter. Proposition 3(b) derives the brand’s choice between strategy \( j \) and strategy \( m \), showing that if the fixed cost incurred in strategy \( j \) (for centralized inspection) is low enough (Figure 1(a)), the brand prefers strategy \( j \); if the fixed cost incurred in strategy \( j \) is high enough (Figure 1(b)), the brand prefers strategy \( m \); otherwise, the brand prefers strategy \( m \) for high inspection accuracy (Figure 1(c)).

![Figure 1](https://ssrn.com/abstract=4717627)

Figure 1: The brand’s utility comparison between strategies \( m \) and \( j \) (under \( b = 0.35, c = 0.5, g = 1.5, h = 0.025, k = 0.1, r = 1.5, \eta = 0.7 \) and \( \lambda = 0.125 \))

To understand the above result, note that when the inspection accuracy is high (i.e., \( \eta \geq \max\{\eta'', \eta'\} \)), the risky supplier is used in both strategies \( j \) and \( m \). Since the difference of the inspection frequency between strategies \( m \) and \( j \) decreases in \( \eta \), as the inspection accuracy increases, the difference of joint inspection in strategy \( j \) and independent inspection in strategy \( m \) becomes smaller, so as the procurement prices in the two strategies. This implies that there exists a threshold \( \tilde{\eta} \geq \max\{\eta'', \eta'\} \) such that when \( \eta > \tilde{\eta} \), the advantage of joint inspection in strategy
over independent inspection in strategy \( m \) becomes not pronounced, and the fixed cost incurred in strategy \( j \) makes it less favorable (i.e., \( U^i_b(w^i) \leq U^m_b(w^m) \)). In contrast, when \( \eta \) is slightly lower than \( \bar{\eta} \), joint inspection’s advantage is still sufficiently significant and strategy \( j \) is more attractive than strategy \( m \) for the brand.

On the other hand, when the risky supplier is chosen, its profit depends on its quality effort and whether its adulteration is identified. When considering and integrating all possible scenarios, our calculation shows that given the procurement price \( w \), its expected profit can be simplified as \( 2(w - c) \), which is equal to the profit of exerting a high-quality effort (in which case there is no adulteration). In fact, if exerting low effort, the risky supplier does not expend the cost \( c \), but there will be a chance of being identified adulteration and earning zero profit. Since the risky supplier adopts a mixed strategy of exerting high quality with probability \( y \), this supplier’s expected profit turns out to be the same as its profit of exerting high effort, \( 2(w - c) \). As such, the risky supplier’s profit increases in \( w \), and Proposition 2 together with some numerical analysis implies \( \pi^m_i \geq \pi^i_j \geq \pi^j_s \). That is, strategy \( m \) benefits the risky supplier the most.

5.3 Quantity of Low-Quality Products Sold to the Market

There will not be low-quality products sold to the market as long as the backup supplier is used by the brand. Once the risky supplier is chosen, the expected quantity of low-quality products sold to the market in strategy \( i \) is \( e^i(w^i) = 2\sqrt{a(w^i)b(w^i)} \), in strategy \( m \) is \( e^m(w^m) = 2\sqrt{a(w^m)b(w^m)} \), and in strategy \( j \) is \( e^j(w^j) = \sqrt{a(w^j)b(w^j)} \). Given the endogenous choice between the backup and risky suppliers implied by Proposition 1, the quantity of low-quality products sold to the market depends on the inspection accuracy as follows.

Proposition 4. Suppose \( F \leq F^i \).

(a) When \( \eta \geq \eta^i \), the risky supplier is chosen in each strategy. We have \( e^i(w^i) \geq e^m(w^m) \). Moreover, there exists a threshold \( c^* \) such that for \( c \leq c^* \), \( e^m(w^m) \geq e^i(w^i) \).

(b) When \( \eta^m \leq \eta < \eta^i \), the risky supplier is chosen in strategies \( j \) and \( m \), while the backup supplier is chosen in strategy \( i \). That is, the quantity of low-quality products sold to the market in strategy \( i \) is 0, smaller than in strategy \( j \) or \( m \).

(c) When \( \eta^i \leq \eta < \eta^m \), the risky supplier is chosen in strategy \( j \), while the backup supplier is chosen in strategies \( i \) and \( m \). That is, the quantity of low-quality products sold to the market in strategy \( i \) or \( m \) is
0, smaller than in strategy \( j \).

(d) When \( \eta < \eta^j \), the backup supplier is chosen in each strategy, resulting in no low-quality product sold to the market.

As implied by Proposition 1, \( \eta^i \leq \eta^m \leq \eta^j \) under \( F \leq F^\eta \). In this case, when the risky supplier is used in each strategy (i.e., \( \eta \geq \eta^i \)), Proposition 4 (a) compares the quantity of low-quality products sold to the market in the three strategies. Although we can only show analytically \( e^m(w^m) \geq e^j(w^j) \) for small cost of supplier’s quality effort \( c \leq c^e \), we have numerically confirmed that the result also holds for \( c > c^e \). That is, strategy \( i \) (\( j \)) leads to the largest (smallest) quantity of low-quality products sold to the market. Fortunately, we have shown in the previous subsection that from the brand’s perspective, strategy \( i \) is dominated by strategy \( m \). Hence, we only need to consider the quantity of low-quality products in strategies \( m \) and \( j \) as follows.

When the inspection accuracy is low (i.e., \( \eta < \eta^i \)), in both strategies \( j \) and \( m \), the backup supplier is chosen and no low-quality product is sold to the market. When the inspection accuracy is of an intermediate level, i.e., \( \eta^i \leq \eta < \eta^m \), provided that the risky supplier is chosen in strategy \( j \) while the backup supplier is used in strategy \( m \), the quantity of low-quality products sold to the market in strategy \( m \) turns out to be smaller than in strategy \( j \). When the inspection accuracy is high (i.e., \( \eta \geq \eta^m \)), the risky supplier is chosen in both strategies \( j \) and \( m \), and as stated above strategy \( j \) leads to a smaller quantity of low-quality products sold to the market compared to strategy \( m \), opposite to the case of intermediate inspection accuracy.

Moreover, the comparison in Proposition 4(a) also holds for a general \( F \) as long as the risky supplier is chosen in each strategy. Given the brand’s endogenous choice uncovered by Proposition 3, when \( \eta > \tilde{\eta} \), the brand prefers strategy \( m \), but strategy \( m \) leads to a larger quantity of low-quality products sold to the market than strategy \( j \). This reveals the conflict between the brand and the government aiming to reduce the adulterated products in the market. Thus, it is important for the government to spot check the adulterated product in the market especially when the inspection accuracy is high and the mixed strategy \( m \) is adopted by the brand.

5.4 The Effect of \( h \)

Next, we explore the effect of \( h \), the buyer’s degree of caring low-quality products sold in the market.

Electronic copy available at: https://ssrn.com/abstract=4717627
Proposition 5. (a) The brand’s utility in both strategies \( j \) and \( m \) increases in \( h \) (i.e., \( \frac{\partial U_j(w)}{\partial h} \geq 0 \) and \( \frac{\partial U_m(w^m)}{\partial h} \geq 0 \)).

(b) When the risky supplier is used in both strategies \( j \) and \( m \),

(b.1) the wholesale price decreases in \( h \) in both strategies (i.e., \( \frac{\partial w_j}{\partial h} \leq 0 \) and \( \frac{\partial w_m}{\partial h} \leq 0 \));

(b.2) the buyer’s inspection effort increases in \( h \) in both strategies (i.e., \( \frac{\partial x_j}{\partial h} \geq 0 \) and \( \frac{\partial x_m}{\partial h} \geq 0 \), \( z = 1, 2 \));

(b.3) the supplier’s quality effort increases in \( h \) in both strategies (i.e., \( \frac{\partial y_j}{\partial h} \geq 0 \) and \( \frac{\partial y_m}{\partial h} \geq 0 \)).

(c) When \( h \geq h_j \), with the risky supplier used in both strategies \( j \) and \( m \), there exist thresholds \( c^h \) and \( h^l \) such that when \( c \leq c^h \) and \( h \geq h^l \), the brand’s utility in strategy \( m \) is larger than in strategy \( j \), i.e., \( U_b^j(w) < U_b^m(w^m) \).

As shown in Figure 2(a), it is interesting to note that a brand’s utility is higher when it cares more about low-quality products sold to the market, though given all else being equal a higher \( h \) leads to a larger disutility term in the utility function (Proposition 5 (a)). This implies that when buyers care more about low-quality products, greater overall incentives are provided to the supplier to exert high effort, which leads to a smaller quantity of low-quality products sold to the market (Proposition 5 (b.3)).

However, which lever is more relied upon to provide greater incentives to the supplier? Proposition 5 (b.1) and (b.2) show that when caring more about low-quality products sold to the market, the inspection lever is more heavily utilized (since this inspection lever is more direct in reducing low-quality products sold to the market than the procurement price lever). Due to the substitution between the procurement price lever and inspection lever, the procurement price lever is even less relied upon. Moreover, greater utilization of the inspection lever in both strategies \( j \) and \( m \) implies a smaller difference between joint and independent inspection in the two strategies. This implies that, the more the brand cares about low-quality products sold to the market, the more likely strategy \( m \) is preferred. Hence, when \( h \) exceeds a certain level, strategy \( m \) is preferred to strategy \( j \) (Proposition 5 (c)).

Combining Propositions 3 and 5, we realize a paradox that as the brand cares more about low-quality products sold to the market, its strategy will switch from strategy \( j \) to strategy \( m \), resulting in a jump of low-quality products sold to the market because the quantity of low-quality products in strategy \( j \) is smaller than in strategy \( m \), as shown in Figure 2(b). This paradox arises because even though the brand cares more about low-quality products sold to the market, it is afterall a
utility maximizer: As the inspection efforts are very high in both strategies $m$ and $j$, the benefit of joint inspection in strategy $j$ compared to independent inspection in strategy $m$ becomes minimal, and the fixed cost for joint inspection in strategy $j$ makes it less attractive compared to strategy $m$. As the strategy switches from $j$ to $m$, the drop in fixed cost outweighs the slight increase in the low-quality products sold to the market, and overall the brand’s utility increases.

Figure 2: The impact of the disutility parameter $h$ (under $b = 0.35$, $c = 0.5$, $d = 10$, $F = 0.25$, $g = 1.5$, $k = 0.1$, $r = 1.5$, $\eta = 0.5$, and $\lambda = 0.125$)

5.5 The Effect of $\lambda$ and $d$

Recall that the parameter $\lambda$ refers to the probability of identifying adulterated products in the market. Figure 3(a) shows that the brand prefers strategy $m/j$ for large/small values of $\lambda$. The reason is that as $\lambda$ increases, the supplier’s reputation loss $\lambda g$ becomes larger, driving the supplier’s high effort. Moreover, a larger $\lambda$ leads to higher inspection effort in both strategies $j$ and $m$. As such, relying more on the inspection lever in strategies $j$ and $m$ not only implies a lower incentive of using the procurement price lever, but also reduces the difference between independent and joint inspection, which makes strategy $j$ less favorable due to its fixed cost. One implication from this fact is that when the government conducts more spot checks to identify adulterated products in the market (i.e., $\lambda$ increases), the more likely the mixed strategy $m$ will be adopted by the brand.

Moreover, Figure 3(b) shows that both the quantity of low-quality products in strategy $j$ (i.e., $e^j(w^j)$) and that in strategy $m$ (i.e., $e^m(w^m)$) decrease in $\lambda$. Thus, more spot checks conducted by
the government can decrease adulterated products in the market. However, if more spot checks lead the brand’s endogenous choice to switch from strategy $j$ to strategy $m$, the quantity of low-quality products sold to the market can arise. As such, more spot checks can play against the government’s intention of decreasing the quantity of low-quality products in the market.

A larger $\lambda$ increases the probability of identifying adulterated products in the market, and a larger $d$ leads to a higher reputation loss for the brand when adulterated products are identified. Since the brand’s reputation loss is $\lambda d$, the effects of $d$ on the brand’s optimal strategy choice and quantity of low-quality products sold to the market are similar to the effects of $\lambda$. That is, when the brand’s goodwill loss caused by the product adulteration is too large, the brand’s utility in strategy $m$ is larger than in strategy $j$, i.e., $U^j_b(w^j) \leq U^m_b(w^m)$. With a higher reputation loss due to low-quality products, the brand has a greater incentive of leveraging the inspection lever in both strategies $j$ and $m$, which leads to a smaller difference between joint and independent inspections, making strategy $j$ less favorable. As shown in Figure 4(c), both the quantity of low-quality products in strategy $j$ (i.e., $e^j(w^j)$) and strategy $m$ (i.e., $e^m(w^m)$) decreases in $d$; but as $d$ becomes larger so that there is a switch by the brand from strategy $j$ to strategy $m$, the quantity of low-quality products jumps.
Figure 4: The impact of the reputation loss parameter $d$ (under $b = 0.35$, $c = 0.5$, $F = 0.25$, $g = 1.5$, $h = 0.025$, $k = 0.1$, $r = 1.5$, $\eta = 0.7$ and $\lambda = 0.125$)

6. Conclusion

Three sourcing and inspection strategies are investigated for a brand with two shops: independent strategy ($i$), mixed strategy ($m$), and joint strategy ($j$). In strategy $j$, both procurement price and inspection decisions are made by the brand. In strategy $m$, the brand decides the procurement price and the shops make inspection decisions separately. In strategy $i$, each shop makes its own procurement price and inspection decisions.

Under independent sourcing, we realize that if one shop offers a high procurement price to the supplier, it can motivate the risky supplier’s high-quality effort; but this effect spills over to the other shop. Therefore, each shop hopes to free ride on the other’s offer of high procurement price. This free-riding incentive is absent in joint sourcing. As a result, given the same inspection mode, the wholesale price in joint sourcing ($m$) is even higher than that in independent sourcing ($i$). For independent versus joint inspection, the free-riding behavior of each shop in independent inspection results in under investment in the inspection sampling frequency compared with joint inspection. With these tradeoffs, this paper analytically shows that mixed strategy $m$ is preferred by the brand to independent strategy $i$. Moreover, between strategies $m$ and $j$, the fixed cost incurred in strategy $j$ and the inspection accuracy jointly influence the brand’s preference. For intermediate fixed cost, if the inspection accuracy is sufficiently high, strategy $m$ is preferred by
the brand to strategy \( j \); otherwise, strategy \( j \) is preferred by the brand.

As the brand or shops care more about the quantity of low-quality products sold to the market (i.e., \( h \) increases), interestingly the brand’s utility will increase regardless of a higher coefficient of disutility term. We also find that with a higher \( h \), in both strategies \( m \) and \( j \), the buyer relies more on the inspection lever and less on the procurement price lever to motivate the supplier’s higher quality effort. Under a given strategy, the quantity of low-quality products decreases as \( h \) increases. However, as \( h \) increases further, the brand’s choice will switch from strategy \( j \) to strategy \( m \), and there will be an upward jump in the quantity of low-quality products sold to the market, because for a given parameter setting, the quantity of low-quality products in strategy \( m \) is greater than that in strategy \( j \). This paradox that there can be more low-quality products in the market when the brand cares more about low-quality products arises because the brand is after all a utility maximizer. As \( h \) increases, the inspection lever is more heavily used in both strategies \( m \) and \( j \), in which case the advantage of joint inspection in strategy \( j \) becomes less pronounced compared to independent inspection in strategy \( m \). But due to the fixed cost incurred in strategy \( j \), the brand chooses strategy \( m \) when \( h \) exceeds a certain level.

Similar effects and driving forces exist as the probability of identifying adulterated products in the market (\( \lambda \)) increases or the reputation loss for the brand (\( d \)) increases. Thus, from the government’s perspective, adopting multifaceted strategies such as conducting more spot checks (i.e., increasing \( \lambda \)) and increasing the brand’s goodwill loss once adulteration is identified (i.e., increasing \( d \)) can decrease the quantity of low-quality products sold to the market given that the brand adopts a certain strategy. However, the government should note the caution that over utilizing such measures may induce the brand to change its strategy from \( j \) to \( m \), leading to more low-quality products sold to the market.

**References**


Proofs of the Lemmas and Theorems

Proof of Lemma 1: We start with solving the equilibrium when the brand sources from the risky supplier: As for the supplier’s best response, because \( \frac{d\pi_s(y)}{dy} = 2(w - c - (w - \lambda g)(1 - \eta x)^2) \), the supplier will set \( y = 0 \) if \( w - c - (w - \lambda g)(1 - \eta x)^2 < 0 \), \( y = 1 \) if \( w - c - (w - \lambda g)(1 - \eta x)^2 > 0 \), while the supplier is indifferent between any \( y \in [0, 1] \) if \( w - c - (w - \lambda g)(1 - \eta x)^2 = 0 \). On the other hand, as for the best response of the brand, because \( \frac{dU_b(x)}{dx} = 4(1 - y)(1 - \eta x)(w + b + h + \lambda d - r) - 2k \) (which implies that \( U_b(x) \) is concave in \( x \)), the brand will set \( x = 0 \) if \( 2(1 - y)(1 - \eta x)(w + b + h + \lambda d - r) - k < 0 \), \( x = 1 \) if \( 2(1 - y)(1 - \eta)(w + b + h + \lambda d - r) - k > 0 \), otherwise \( \text{(with } 2(1 - y)(1 - \eta)(w + b + h + \lambda d - r) \leq k \leq 2(1 - y)(w + b + h + \lambda d - r)) \) \( \text{the brand will set } x = \left[ -k/(2(1 - y)(w + b + h + \lambda d - r)) \right]/\eta. \) From checking the cross of the response curves (characterizing the response function of each player), we have Table 2 summarizing the equilibrium in the case of joint sourcing and joint inspection (for exposition we omit the argument \( w \) in \( \alpha(w) \) and \( \beta(w) \)). Assumption (C1) implies that it suffices to consider \( \alpha \leq 1 \). Specifically, in the case of \( \beta > 2\sqrt{\alpha} \), \( U_b = 2(r - h - w - \lambda d) - F \leq 2(r - h - \lambda d) - F \leq 2b - F \), where the last inequality is from Assumption (C2). Additionally, \( U_b = 2(b - k - (1 - \eta)^2 \frac{k}{\eta^2}) - F \leq 2b - F \) in the case of \( \alpha < (1 - \eta)^2 \) and \( \beta \leq 2\sqrt{\alpha}. \) The rest of this lemma follows easily.

Proof of Lemma 2: As implied by Lemma 1 and Table 2, when the risky supplier is used, the feasible equilibrium should satisfy \( (1 - \eta)^2 \leq \alpha(w) \leq 1 \) and \( 0 < \beta(w) \leq 2\sqrt{\alpha(w)} \). That is, once the risky supplier is used, the brand optimizes the procurement price \( w \) to maximize its utility \( U_b^1 = 2(r - w) + \frac{k(1 - \sqrt{a(w)})^2}{\eta \sqrt{a(w)}} - \frac{\beta(w)\lambda d}{\sqrt{a(w)}} - F \) subject to the constraints \( 0 < \beta(w) \leq 2\sqrt{\alpha(w)} \). To solve this brand’s utility optimization problem over the procurement price \( w \), we note that

\[
\frac{dU_b^1(w)}{dw} = -2 + \frac{k(c - \lambda g)}{2\eta[(w - \lambda g)(w - c)]^{3/2}} \left( \frac{\lambda d(w - \lambda g)}{w + b + h + \lambda d - r} - (c - \lambda g) \right) + \frac{k\lambda d}{\eta(w + b + h + \lambda d - r)^2} \frac{\sqrt{w - c}}{\sqrt{w - c}}.
\]

It suffices to check the domain of \( w \) satisfying \( \frac{\lambda d(w - \lambda g)}{w + b + h + \lambda d - r} - (c - \lambda g) \geq 0 \). Suppose \( \frac{\lambda d(w - \lambda g)}{w + b + h + \lambda d - r} - (c - \lambda g) > 0 \). This together with \( w - \lambda g \geq c - \lambda g \) implies that \( \frac{\lambda d}{w + b + h + \lambda d - r} \leq 1 \) (otherwise, \( \frac{\lambda d(w - \lambda g)}{w + b + h + \lambda d - r} - (c - \lambda g) = (w - \lambda g) - (c - \lambda g) \geq 0 \). As such, for the third term of \( \frac{dU_b^1(w)}{dw} \),

\[
\frac{\lambda d(w - \lambda g)}{w + b + h + \lambda d - r} \leq \frac{k\sqrt{w - c}}{\eta(w + b + h + \lambda d - r)^2} \frac{\sqrt{w - c}}{\sqrt{w - c}} = \frac{\beta(w)}{\sqrt{\alpha(w)}} \leq \frac{1}{2},
\]

where the last inequality is from the
constraint $\beta(w) \leq 2\sqrt{\alpha(w)}$. Thus,

$$\frac{dU_b^l(w)}{dw} \leq -2 + \frac{k(c - \lambda g)}{2\eta[(w - \lambda g)(w - c)]^{3/2}} \left[ \frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)} - (c - \lambda g) \right] + \frac{1}{2}$$

$$\leq -2 + \frac{1}{2} < 0,$$

where the second inequality is because the aforementioned condition $\frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)} - (c - \lambda g) \leq 0$ above implies that the second term of $\frac{dU_b^l(w)}{dw}$ is non-positive, i.e., $\frac{k(c - \lambda g)}{2\eta[(w - \lambda g)(w - c)]^{3/2}} \left[ \frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)} - (c - \lambda g) \right] \leq 0$. As a result, the brand will never choose a procurement price satisfying $\frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)} - (c - \lambda g) \leq 0$, so we focus on the domain of $\frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)} - (c - \lambda g) > 0$.

With regard to the second derivative $\frac{d^2U_b^l(w)}{dw^2} \leq 0$, we show that all three terms in $\frac{dU_b^l(w)}{dw}$ decreases in $w$. First note that in the third term of $\frac{dU_b^l(w)}{dw}$, $\frac{k\lambda d\sqrt{w - \lambda g}}{\eta(w + b + h + \lambda d - r)^2\sqrt{w - c}}$ increases in $w$, since both $(w + b + h + \lambda d - r)^2$ and $\sqrt{w - c}$ increase in $w$. Furthermore, for the second term of $\frac{dU_b^l(w)}{dw}$, $\frac{k(c - \lambda g)}{2\eta[(w - \lambda g)(w - c)]^{3/2}} \left[ \frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)(w - \lambda g)} - (c - \lambda g) \right] = \frac{k(c - \lambda g)}{2\eta} \left[ \frac{\lambda d}{(w + b + h + \lambda d - r)(w - \lambda g)} - (c - \lambda g) \right]$ are positive (in the domain of $\frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)} - (c - \lambda g) > 0$). Moreover, $(w - c)\sqrt{\frac{w - c}{w - \lambda g}}$ increases in $w \geq c$, resulting in $\frac{k(c - \lambda g)}{2\eta} \left[ \frac{\lambda d}{(w + b + h + \lambda d - r)(w - \lambda g)} - (c - \lambda g) \right]$ decreases in $w$. Additionally,

$$\frac{d}{dw} \left[ \frac{\lambda d}{(w + b + h + \lambda d - r)(w - \lambda g)} - (c - \lambda g) \right] = -\frac{\lambda d}{(w + b + h + \lambda d - r)^2(w - \lambda g)^2} - \frac{\lambda d}{(w + b + h + \lambda d - r)^2(w - \lambda g)^2} + \frac{c - \lambda g}{2(w - \lambda g)^3}$$

$$\leq -\frac{\lambda d}{(w + b + h + \lambda d - r)^2(w - \lambda g)^2} + \frac{c - \lambda g}{2(w - \lambda g)^3}$$

$$= \frac{\lambda d}{(w + b + h + \lambda d - r)^2(w - \lambda g)^2} + \frac{c - \lambda g}{2(w - \lambda g)^3}$$

$$\leq \frac{1}{(w - \lambda g)^2} \left[ \frac{\lambda d}{(w + b + h + \lambda d - r)} - (c - \lambda g) \right] \leq 0.$$

Therefore, the second term of $\frac{dU_b^l(w)}{dw}$ decreases in $w$.

Given the concavity of $U_b^l(w)$, the maximizer of $U_b^l(w)$ in the constrained problem, i.e., $w^l = \arg\max_w U_b^l(w)$ subject to $(1 - \eta)^2 \leq \alpha(w) \leq 1$ and $0 < \beta(w) \leq 2\sqrt{\alpha(w)}$, should exist. If this maximizer is not unique, we denote by the largest maximizer as $w^l$. Note that for the feasible domain of $\frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)} - (c - \lambda g) > 0$, since $\frac{w - \lambda g}{(w + b + h + \lambda d - r)}$ increases in $w$, denote by $w^l$ the minimum of $w$ satisfying $\frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)} - (c - \lambda g) \geq 0$, i.e., $w^l := \min\{w| \frac{\lambda d(w - \lambda g)}{(w + b + h + \lambda d - r)} - (c - \lambda g) \geq 0\}$. 35
With regard to the constraints \((1 - \eta)^2 \leq w - \frac{c}{\lambda g} \leq 1\) and \(0 < \beta(w) = \frac{k}{\eta(w + b + h + \lambda d - r)} \leq 2\sqrt{\alpha(w)} = 2\sqrt{\frac{w - c}{w - \lambda g}}\), because \(\alpha(w) \leq 1\) and \(\beta(w) > 0\) always hold, the effective constraints are \(\frac{w - c}{w - \lambda g} \geq (1 - \eta)^2\) and \(2\sqrt{\frac{w - c}{w - \lambda g}} \geq \frac{k}{\eta(w + b + h + \lambda d - r)}\). Note that both \(w^1 := \min\{w | \frac{w - c}{w - \lambda g} \geq (1 - \eta)^2\}\) and \(w^2 := \min\{w | 2\sqrt{\frac{w - c}{w - \lambda g}} \geq \frac{k}{\eta(w + b + h + \lambda d - r)}\}\) decrease in \(\eta\), while \(w^j\) is independent of \(\eta\). Thus, there exists a threshold \(\bar{\eta}^j\) such that when \(\eta \geq \bar{\eta}^j\), \(w^j \geq \max\{w^1, w^2\}\) and \(U_b^j(w) \geq b\). Then, when \(\eta \geq \bar{\eta}^j\), \(w^j\) is characterized by the interior solution \(\frac{\partial U_b^j(w)}{\partial w} |_{w = w^j} = 0\), i.e.,

\[
\frac{k(c - \lambda g)}{2\eta[(w^j - \lambda g)(w^j - c)]^{3/2}}\left[\frac{\lambda d(w^j - \lambda g)}{(w^j + b + h + \lambda d - r)} - (c - \lambda g)\right] + \frac{k\lambda d\sqrt{w^j - \lambda g}}{\eta(w^j + b + h + \lambda d - r)^2\sqrt{w^j - c}} = 1.
\]

(7)

In addition, given the concavity of \(U_b^j(w)\), in the interior solution case, the formulation of \(\frac{\partial U_b^j(w)}{\partial w} |_{w = w^j} \neq 0\) above implies that

\[
\frac{\partial U_b^j(w^j)}{\partial \eta} = -\frac{1}{\eta} \left[\frac{k(1 - \sqrt{\alpha(w^j)})^2}{\eta \sqrt{\alpha(w^j)}} - \frac{\beta(w^j)\lambda d}{\sqrt{\alpha(w^j)}}\right] \geq 0,
\]

where the second equality is because envelope theorem for the interior solution case, and the inequality is from \(\frac{k(1 - \sqrt{\alpha(w)})^2}{\eta \sqrt{\alpha(w)}} - \frac{\beta(w)\lambda d}{\sqrt{\alpha(w)}} = U_b^j(w) - 2(r - w) \leq 0\), as implied by the definition of \(U_b^j(w)\).

If \(w^j\) is not obtained from the interior solution, i.e., \(\frac{\partial U_b^j(w)}{\partial w} |_{w = w^j} \neq 0\), one of two effective constraints is binding in equilibrium, i.e., \(\frac{w - c}{w - \lambda g} |_{w = w^j} = (1 - \eta)^2\) (under \(w^j = w^1\)) or \(2\sqrt{\frac{w - c}{w - \lambda g}} - \frac{k}{\eta(w + b + h + \lambda d - r)} |_{w = w^2} = 0\) (under \(w^j = w^2\)). In either case, the concavity of \(U_b^j(w)\) implies that \(\frac{\partial U_b^j(w^j)}{\partial w} |_{w = w^j} < 0\), \(z = 1, 2\); because otherwise, the brand can increase \(w\) to improve its utility. This, together with the fact that both \(w^1\) and \(w^2\) decrease in \(\eta\), implies that under \(w^j = w^1\) or \(w^j = w^2\),

\[
\frac{\partial U_b^j(w^j)}{\partial \eta} = \frac{\partial U_b^j(w) |_{w = w^j}}{\partial w} \frac{\partial w^j}{\partial \eta} - \frac{1}{\eta} \left[\frac{k(1 - \sqrt{\alpha(w^j)})^2}{\eta \sqrt{\alpha(w^j)}} - \frac{\beta(w^j)\lambda d}{\sqrt{\alpha(w^j)}}\right] \geq -\frac{1}{\eta} \left[\frac{k(1 - \sqrt{\alpha(w^j)})^2}{\eta \sqrt{\alpha(w^j)}} - \frac{\beta(w^j)\lambda d}{\sqrt{\alpha(w^j)}}\right] \geq 0,
\]

where the last inequality holds due to the same reason as in the interior solution case.

Therefore, when the risky supplier is chosen, \(U_b^j(w^j)\) increases in \(\eta\). That is, \(U_b^j(w^j) - (2b - F)\) increases in \(\eta\). Accordingly, in strategy \(j\), there exists a threshold \(\bar{\eta}^j\) such that (a) if \(\eta \geq \bar{\eta}^j\) (which results in \((1 - \eta)^2 \leq \alpha(w^j) \leq 1\) and \(0 < \beta(w^j) \leq 2\sqrt{\alpha(w^j)}\)), there exists a unique equilibrium in which the brand sources from the risky supplier; (b) otherwise, the brand sources from the backup supplier.

\(\square\)
Proof of Lemma 3: We start with solving the equilibrium when each shop sources from the risky supplier: First, given the exogenous procurement price \( w \), both shops will choose the same inspection sampling frequency, i.e., \( x_1 = x_2 \), because if \( x_1 > x_2 \),

\[
\pi_1(x_1) = y(r-w) + (1-y)[b - (1-\eta x_1)(1-\eta x_2)(w+b+h+\lambda d-r)] - kx_1 \\
< y(r-w) + (1-y)[b - (1-\eta x_1)(1-\eta x_2)(w+b+h+\lambda d-r)] - kx_2 = \pi_1(x_2).
\]

As such, \( x_1 > x_2 \) cannot be an equilibrium. Similarly, \( x_1 < x_2 \) cannot be an equilibrium.

As for the supplier’s best response, because \( \frac{d\pi_1(y)}{dy} = 2(w-c-(w-\lambda g)(1-x_1)(1-x_2)) \), the supplier will set \( y = 0 \) if \( w-c-(w-\lambda g)(1-x_1)(1-x_2) < 0 \), \( y = 1 \) if \( w-c-(w-\lambda g)(1-x_1)(1-x_2) > 0 \), while the supplier is indifferent between any \( y \in [0,1] \) if \( w-c-(w-\lambda g)(1-x_1)(1-x_2) = 0 \). On the other hand, as for the best response of shop 1, because \( \frac{dU_1(x_1)}{dx_1} = (1-y)\eta(1-x_2)(w+b+h+\lambda d-r) - k \), shop 1 will set \( x_1 = 0 \) if \( (1-y)\eta(1-x_2)(w+b+h+\lambda d-r) - k < 0 \), \( x_1 = 1 \) if \( (1-y)\eta(1-x_2)(w+b+h+\lambda d-r) - k > 0 \), while shop 1 is indifferent between any \( x_1 \in [0,1] \) if \( (1-y)\eta(1-x_2)(w+b+h+\lambda d-r) - k = 0 \). Similarly, as for the best response of shop 2, because \( \frac{dU_2(x_2)}{dx_2} = (1-y)\eta(1-x_1)(w+b+h+\lambda d-r) - k \), shop 2 will set \( x_2 = 0 \) if \( (1-y)\eta(1-x_1)(w+b+h+\lambda d-r) - k < 0 \), \( x_2 = 1 \) if \( (1-y)\eta(1-x_1)(w+b+h+\lambda d-r) - k > 0 \), while shop 2 is indifferent between any \( x_2 \in [0,1] \) if \( (1-y)\eta(1-x_1)(w+b+h+\lambda d-r) - k = 0 \).

From checking the cross of the response curves (characterizing the response function of each player), we have Table 3 to summarize the equilibrium in the case of joint sourcing and independent inspection (for exposition we omit the argument \( w \) in \( \alpha(w) \) and \( \beta(w) \)). For example,

\[(1-\eta)^2 \leq \frac{w-c}{w-\lambda g} \leq 1 \quad \text{and} \quad \frac{k}{w+b+h+\lambda d-r} < \eta \sqrt{\frac{w-c}{w-\lambda g}} \]

the supplier’s response curve and each shop’s response curve cross at point \((x_1, x_2, y) = ((1-\sqrt{\frac{w-c}{w-\lambda g}})/\eta, (1-\sqrt{\frac{w-c}{w-\lambda g}})/\eta, 1-k/\eta(w+b+h+\lambda d-r)\)). This implies a mixed strategy for each player.

Assumption (C1) implies that it suffices to consider \( \alpha \leq 1 \). Specifically, in the case of \( \beta > 1 \), \( U_z = r-h-w-\lambda d \leq r-h-\lambda d \leq b \), where the last inequality is from Assumption (C2). Additionally, given \( b - \frac{k}{\eta} < b \) as well as \( b - k - (1-\eta)^2 \frac{k}{\eta} < b \), it is in the buyer’s interest to choose the backup supplier with no adulteration. The rest of this lemma follows easily.

\[ \square \]

Proof of Lemma 4: As implied by Lemma 3 and Table 3, when the risky supplier is used, the feasible equilibrium should satisfy \( (1-\eta)^2 \leq \alpha(w) \leq 1 \) and \( 0 < \beta(w) \leq \sqrt{\alpha(w)} \). That is, once the risky supplier is used, the brand optimizes the procurement price \( w \) to maximize its utility \( U_b^m = 2[r-w + \frac{(1-\sqrt{\alpha(w)})k-\beta(w)\eta d}{\eta \sqrt{\alpha(w)}}] \) subject to the constraints \( (1-\eta)^2 \leq \alpha(w) \leq 1 \) and \( 0 < \beta(w) \leq \sqrt{\alpha(w)} \). To solve this brand’s utility optimization problem over the procurement
### Table 2: Equilibrium in the case of joint sourcing and joint inspection

<table>
<thead>
<tr>
<th></th>
<th>$\alpha &lt; (1 - \eta)^2$</th>
<th>$(1 - \eta)^2 \leq \alpha \leq 1$</th>
<th>$\alpha &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta &gt; \frac{2\sqrt{\eta}}{\alpha}$</td>
<td>$y^i = 0, x^i = 0$</td>
<td>$y^i = 0, x^i = 0$</td>
<td>$y^i = 1, x^i = 0$</td>
</tr>
<tr>
<td></td>
<td>$\pi_s^i = 2(w - \lambda g)$</td>
<td>$\pi_s^i = 2(w - \lambda g)$</td>
<td>$\pi_s^i = 2(w - c)$</td>
</tr>
<tr>
<td></td>
<td>$U_b^i = 2(r - h - w - \lambda d) - F$</td>
<td>$U_b^i = 2(r - h - w - \lambda d) - F$</td>
<td>$U_b^i = 2(r - w) - F$</td>
</tr>
<tr>
<td>$\beta \leq \frac{2\sqrt{\eta}}{\alpha}$</td>
<td>$y^i = 0, x^i = 1$</td>
<td>$y^i = 1 - \frac{\beta}{2\sqrt{\eta}} x^i = \frac{1 - \sqrt{\pi}}{\eta}$</td>
<td>$\pi_s^i = 2(w - c)$</td>
</tr>
<tr>
<td></td>
<td>$\pi_s^i = 2(1 - 2\eta)(w - \lambda g)$</td>
<td>$\pi_s^i = 2(1 - 2\eta)(w - \lambda g)$</td>
<td>$\pi_s^i = 2(w - c)$</td>
</tr>
<tr>
<td></td>
<td>$U_b^i = 2[b - k - (1 - \eta)^2 \frac{k}{\eta}] - F$</td>
<td>$U_b^i = 2[r - w] + \frac{k(1 - \sqrt{\pi})^2}{\eta \sqrt{\alpha}} - \frac{\beta \lambda d}{\sqrt{\alpha}} - F$</td>
<td>$U_b^i = 2(r - w) - F$</td>
</tr>
</tbody>
</table>

### Table 3: Equilibrium with joint sourcing and independent inspection

<table>
<thead>
<tr>
<th></th>
<th>$\alpha &lt; (1 - \eta)^2$</th>
<th>$(1 - \eta)^2 \leq \alpha \leq 1$</th>
<th>$\alpha &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta &gt; 1$</td>
<td>$y^m = 0, x_z^m = 0$</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>$U_z^m = r - h - w - \lambda d$</td>
<td>$U_z^m = r - h - w - \lambda d$</td>
<td>$U_z^m = r - w$</td>
</tr>
<tr>
<td>$\max{(1 - \eta), \sqrt{\alpha}} \leq \beta \leq 1$</td>
<td>$y^m = 0, x_z^m = \frac{1 - \beta}{\eta}$</td>
<td>$y^m = 0, x_z^m = \frac{1 - \beta}{\eta}$</td>
<td>$y^m = 1 - \frac{\beta}{\sqrt{\eta}} x_z^m = \frac{1 - \sqrt{\pi}}{\eta}$</td>
</tr>
<tr>
<td></td>
<td>$\pi_s^m = 2(1 - \eta)^2 (w - \lambda g)$</td>
<td>$\pi_s^m = 2(1 - \eta)^2 (w - \lambda g)$</td>
<td>$\pi_s^m = 2(1 - \eta)^2 (w - \lambda g)$</td>
</tr>
<tr>
<td></td>
<td>$U_z^m = b - \frac{k}{\eta}$</td>
<td>$U_z^m = b - \frac{k}{\eta}$</td>
<td>$U_z^m = b - \frac{k}{\eta}$</td>
</tr>
<tr>
<td>$\beta &lt; \max{(1 - \eta), \sqrt{\alpha}}$</td>
<td>$y^m = 0, x_z^m = 1$</td>
<td>$y^m = 1 - \frac{\beta}{\sqrt{\eta}} x_z^m = \frac{1 - \sqrt{\pi}}{\eta}$</td>
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</tr>
<tr>
<td></td>
<td>$U_z^m = b - k - (1 - \eta)^2 \frac{k}{\eta \beta}$</td>
<td>$U_z^m = b - k - (1 - \eta)^2 \frac{k}{\eta \beta}$</td>
<td>$U_z^m = r - w + \frac{(1 - \sqrt{\pi}) k - \beta \lambda d}{\eta \sqrt{\alpha}}$</td>
</tr>
</tbody>
</table>
price \( w \), we note that
\[
\frac{dU^m_b(w)}{dw} = 2\left[ -1 + \frac{k(r - b - h - w)}{2\eta(w + b + h + \lambda d - r)(w - c)^{3/2}(w - \lambda g)^{3/2}} + \frac{k\lambda d}{\eta(w + b + h + \lambda d - r)^2} \frac{w - c}{w - \lambda g} \right].
\]

It suffices to check the domain of \( w \) satisfying \( w \leq r - b - h \): If \( r - b - h - w < 0 \),
\[
\frac{k(r - b - h - w)}{2\eta(w + b + h + \lambda d - r)(w - c)^{3/2}(w - \lambda g)^{3/2}} \leq 0
\]
implies \( \frac{dU^m_b(w)}{dw} < -1 + \frac{k\lambda d}{\eta(w + b + h + \lambda d - r)^2} \frac{w - c}{w - \lambda g} \). Moreover, \( b + h + w - r \geq 0 \) implies \( w + b + h + \lambda d - r \geq \lambda d \), or \( \frac{\lambda d}{w + b + h + \lambda d - r} \leq 1 \). This, together with \( \frac{k}{\eta(w + b + h + \lambda d - r)} \frac{w - c}{w - \lambda g} = \frac{\beta(w)}{\sqrt{\alpha(w)}} \leq 1 \), implies that \( -1 + \frac{k\lambda d}{\eta(w + b + h + \lambda d - r)^2} \leq 0 \). That is, \( \frac{dU^m_b(w)}{dw} < 0 \) when \( r - b - h - w < 0 \). As a result, the brand will never choose a procurement price satisfying \( r - b - h - w \leq 0 \), so we focus on the domain of \( w \leq r - b - h \).

With regard to the second derivative \( \frac{d^2U^m_b(w)}{dw^2} \), we show that all three terms in \( \frac{dU^m_b(w)}{dw} \) decreases in \( w \) in the feasible domain of \( w \leq r - b - h \). First note that in the third term of \( \frac{dU^m_b(w)}{dw} \),
\[
\frac{k\lambda d}{\eta(w + b + h + \lambda d - r)^2} \frac{w - c}{w - \lambda g} = \frac{k\lambda d}{\eta(w + b + h + \lambda d - r)^2} \frac{w - c}{w - \lambda g} \text{ decreases in } w, \text{ since both } (w + b + h + \lambda d - r)^2 \text{ and } \frac{w - c}{w - \lambda g} \text{ increase in } w. \text{ Furthermore, for the second term of } \frac{dU^m_b(w)}{dw}, \text{ i.e.,}
\]
\[
\frac{k(r - b - h - w)}{2\eta(w + b + h + \lambda d - r)(w - c)^{3/2}(w - \lambda g)^{3/2}} \frac{c - \lambda g}{w - \lambda g} = \frac{k(r - b - h - w)(c - \lambda g)}{2\eta(w + b + h + \lambda d - r)(w - c)^{3/2}(w - \lambda g)^{3/2}}, \text{ (given } w \leq r - b - h \text{) the denominator } 2(w + b + h + \lambda d - r)(w - c)^{1/2}(w - \lambda g)^{3/2} \text{ increases in } w, \text{ and the numerator } k(r - b - h - w)(c - \lambda g) \text{ decreases in } w. \text{ As both the denominator and numerator are non-negative in the domain of } w \leq r - b - h, \text{ the second term of } \frac{dU^m_b(w)}{dw}, \text{ i.e.,}
\]
\[
\frac{k(r - b - h - w)(c - \lambda g)}{2\eta(w + b + h + \lambda d - r)(w - c)^{3/2}(w - \lambda g)^{3/2}}, \text{ decreases in } w.
\]

Therefore, \( \frac{d^2U^m_b(w)}{dw^2} \leq 0 \). Given the concavity of \( U^m_b(w) \) with respect to \( w \), as in the proof of Lemma 2 above, we can easily show that the interior solution satisfies the following equation:
\[
-1 + \frac{k(r - b - h - w^m)}{2(w^m + b + \lambda d - r)(w^m - c)^{1/2}(w^m - \lambda g)^{3/2}} + \frac{k\lambda d}{\eta(w^m + b + h + \lambda d - r)^2} \frac{w^m - c}{w^m - \lambda g} = 0,
\]
and \( U^m_b(w) \) increases in \( \eta \). The rest of this lemma follows easily.

\[\square\]

**Proof of Lemma 5:** i. Similar to the analysis in the last section about strategy \( m \), in strategy \( i \), when the risky supplier is chosen, given the procurement prices \( w_1 \) and \( w_2 \),

- if \((1 - \eta)^2 \leq a(w_z) \leq 1 \) and \( 0 < \beta(w_z) \leq \sqrt{a(w_z)}, z = 1, 2 \), we have the interior solutions
about the supplier’s quality and each shop’s inspection efforts:

\[(1 - \eta x_1)(1 - \eta x_2) = \frac{w_1 + w_2 - 2c}{w_1 + w_2 - 2\lambda g}\]

\[\eta(1 - y)(1 - \eta x_2)(w_1 + b + h + \lambda d - r) = k\]

\[\eta(1 - y)(1 - \eta x_1)(w_2 + b + h + \lambda d - r) = k,\]

which leads to (6).

- otherwise, each shop’s utility is lower than \(b\).

**Proof of Lemma 6:** When the risky supplier is chosen, it suffices to check the case of \((1 - \eta)^2 \leq a(w_z) \leq 1\) and \(0 < \beta(w_z) \leq \sqrt{a(w_z)}, z = 1, 2\). Given \(y^i, x^i_1\) and \(x^i_2\) in (6),

\[
\frac{dx^i_2}{dw_1} = \frac{-\sqrt{(w_1 + b + h + \lambda d - r)(w_2 + b + h + \lambda d - r)(w_1 + w_2 - 2\lambda g)}}{2\eta \sqrt{(w_1 + w_2 - 2c)}}
\]

\[
\frac{2(c - \lambda g)}{(w_1 + b + h + \lambda d - r)(w_1 + w_2 - 2\lambda g)^2} - \frac{w_1 + w_2 - 2c}{(w_1 + w_2 - 2\lambda g)(w_1 + b + h + \lambda d - r)^2}
\]

\[
= \frac{2\eta \sqrt{(w_1 + w_2 - 2\lambda g)(w_1 + w_2 - 2c)(w_1 + b + h + \lambda d - r)}}{(w_1 + b + h + \lambda d - r)(w_1 + w_2 - 2\lambda g)(w_1 + b + h + \lambda d - r)^2}
\]

\[
\frac{2(c - \lambda g)}{(w_1 + w_2 - 2c)} - \frac{w_1 + w_2 - 2\lambda g}{(w_1 + b + h + \lambda d - r)}
\]

\[
\frac{k}{2\eta \sqrt{(w_1 + w_2 - 2\lambda g)(w_1 + w_2 - 2c)(w_1 + b + h + \lambda d - r)}}
\]

\[
\frac{2(c - \lambda g)}{(w_1 + w_2 - 2c)} + \frac{w_1 + w_2 - 2\lambda g}{(w_1 + b + h + \lambda d - r)}
\]

This together with (6) implies

\[
\frac{(k(w_1 + b + h + \lambda d - r))}{w_2 + b + h + \lambda d - r} \frac{dx^i_2}{dw_1} + (1 - \eta x^i_1)(1 - \eta x^i_2)(w_1 + b + h + \lambda d - r) \frac{dy^i}{dw_1}
\]

\[
= \frac{(k(w_1 + b + \lambda d - r))}{w_2 + b + h + \lambda d - r} \frac{dx^i_2}{dw_1} + \frac{(w_1 + w_2 - 2c)(w_1 + b + h + \lambda d - r)}{w_1 + w_2 - 2\lambda g} \frac{dy^i}{dw_1}
\]

\[
= \frac{k \sqrt{w_1 + b + h + \lambda d - r}}{2\eta \sqrt{(w_1 + w_2 - 2\lambda g)(w_1 + w_2 - 2c)(w_2 + b + h + \lambda d - r)}} \left( -\frac{2(c - \lambda g)}{w_1 + w_2 - 2\lambda g} + \frac{w_1 + w_2 - 2c}{w_1 + b + h + \lambda d - r} \right) +
\]

\[
\frac{k}{2\eta \sqrt{(w_1 + w_2 - 2\lambda g)(w_1 + w_2 - 2c)(w_1 + b + h + \lambda d - r)}} \left( \frac{2(c - \lambda g)}{w_1 + w_2 - 2c} + \frac{w_1 + w_2 - 2\lambda g}{w_1 + b + h + \lambda d - r} \right)
\]

\[
= \frac{k}{\eta} \sqrt{(w_1 + w_2 - 2\lambda g)(w_1 + b + h + \lambda d - r)(w_2 + b + h + \lambda d - r)}
\]
From (6) and
\[
U_i' = y'(r - w_1) + (1 - y')(b - (1 - \eta x_1')(1 - \eta x_2')(w_1 + b + h + \lambda d - r)) - kx_1',
\]
at the point characterized by (6),
\[
\frac{\partial U_i}{\partial w_1} = -y' - (1 - y')(1 - \eta x_1')(1 - \eta x_2'),
\]
\[
\frac{\partial U_i}{\partial y} = r - w_1 - b + (1 - \eta x_1')(1 - \eta x_2')(w_1 + b + h + \lambda d - r),
\]
\[
\frac{\partial U_i}{\partial x_1} = \eta(1 - y')(1 - \eta x_2')(w_1 + b + h + \lambda d - r) - k = 0,
\]
\[
\frac{\partial U_i}{\partial x_2} = \eta(1 - y')(1 - \eta x_1')(w_1 + b + h + \lambda d - r) = \frac{k(w_1 + b + h + \lambda d - r)}{w_2 + b + h + \lambda d - r},
\]
and so
\[
\frac{dU_i}{dw_1} = \frac{\partial U_i}{\partial w_1} + \frac{\partial U_i}{\partial y} \frac{dy}{dw_1} + \frac{\partial U_i}{\partial x_2} \frac{dx_2}{dw_1} + \frac{\partial U_i}{\partial x_1} \frac{dx_1}{dw_1}
\]
\[
= \frac{\partial U_i}{\partial w_1} + (r - w_1 - b + (1 - \eta x_1')(1 - \eta x_2')(w_1 + b + h + \lambda d - r)) \frac{dy}{dw_1}
\]
\[
= \frac{\partial U_i}{\partial w_1} + (r - w_1 - b) \frac{dy}{dw_1} + (1 - \eta x_1')(1 - \eta x_2')(w_1 + b + h + \lambda d - r) \frac{dy}{dw_1}
\]
\[
= \frac{\partial U_i}{\partial w_1} + (r - w_1 - b) \frac{dy}{dw_1} + \frac{k(w_1 + b + h + \lambda d - r)}{w_2 + b + h + \lambda d - r} \frac{dx_1}{dw_1}
\]
\[
= \frac{\partial U_i}{\partial w_1} + (r - w_1 - b) \frac{dy}{dw_1} + \frac{k}{\eta} \sqrt{\frac{(w_1 + w_2 - 2c)}{(w_1 + w_2 - 2\lambda g)(w_1 + b + h + \lambda d - r)(w_2 + b + h + \lambda d - r)}}
\]
\[
= (-y' - (1 - y')(1 - \eta x_1')(1 - \eta x_2')) + (r - w_1 - b) \frac{dy}{dw_1}
\]
\[
+ \frac{k}{\eta} \sqrt{\frac{(w_1 + w_2 - 2c)}{(w_1 + w_2 - 2\lambda g)(w_1 + b + h + \lambda d - r)(w_2 + b + h + \lambda d - r)}}
\]
\[
= -y' + \frac{(r - w_1 - b)(1 - y')}{2} \left(\frac{2(c - \lambda g)}{(w_1 + w_2 - 2c)(w_1 + w_2 - 2\lambda g)} + \frac{1}{w_1 + b + h + \lambda d - r}\right)
\]
Clearly, \(\frac{dU_i}{dw_1}\) decreases in \(w_2\) but increases in \(c\).

Similarly,
\[
\frac{dU_i}{dw_2} = -y' + \frac{(r - w_2 - b)(1 - y')}{2} \left(\frac{2(c - \lambda g)}{(w_1 + w_2 - 2c)(w_1 + w_2 - 2\lambda g)} + \frac{1}{w_2 + b + h + \lambda d - r}\right),
\]
which decreases in \(w_1\) but increases in \(c\).
Therefore, in strategy $i$, $U_i^z$, $z = 1, 2$, is supermodular in $(w_1, -w_2)$. According to the properties of the supermodular game, because the game is supermodular, the set of equilibria is a lattice and there exists one largest and one smallest equilibrium. As for the equilibrium that is (component-wise) largest among all equilibria, and preferred by all the shops, both $w_1$ and $w_2$ are increasing in $c$.

As for $f_z := \frac{dU_i^z}{dw_z}, z = 1, 2, \frac{\partial f_i}{\partial w_1} - \frac{\partial f_i}{\partial w_2} = -1-y' \left( \frac{2(c-\lambda g)}{(w_1+w_2-2c)(w_1+w_2-2\lambda g)} + \frac{1}{w_1+b+h+\lambda d-r} \right) - \frac{(r-w_1-b)(1-y')}{2} \frac{1}{w_1+b+h+\lambda d-r} = 0$. Because $U_i^z$ is concave in $w_z$, and submodular in $w_1$ and $w_2$, $\frac{\partial f_i}{\partial w_1} \leq 0$ and $\frac{\partial f_i}{\partial w_2} \leq 0$, so $\frac{\partial^2 f_i}{\partial w_1 \partial w_2} = -\frac{\partial f_i}{\partial w_1} \geq |\frac{\partial f_i}{\partial w_2}| = \frac{\partial f_i}{\partial w_2}$. By applying Theorem 5 and equation (2) in Cachon and Netessine (2004), the equilibrium $(w_1^i, w_2^i)$ is unique.

Then, for this unique equilibrium $(w_1^i, w_2^i) = (w_i^z, w_i^z)$, with

$$y' = 1 - \frac{k}{\eta} \sqrt{\frac{\sqrt{w_1^i + w_2^i - 2\lambda g}}{(w_1^i + w_2^i - 2c)(w_1^i + b + h + \lambda d - r)(w_2^i + b + h + \lambda d - r)}}$$

from (6),

$$\left. \frac{dU_i^1}{dw_1} \right|_{w_1 = w_1^i, w_2 = w_2^i} = -y' + \frac{(r-w_1^i-b)(1-y')}{2} \left( \frac{2(c-\lambda g)}{(w_1^i + w_2^i - 2c)(w_1^i + w_2^i - 2\lambda g)} + \frac{1}{w_1^i + b + h + \lambda d - r} \right) = 0,$$

$$\left. \frac{dU_i^2}{dw_2} \right|_{w_1 = w_1^i, w_2 = w_2^i} = -y' + \frac{(r-w_2^i-b)(1-y')}{2} \left( \frac{2(c-\lambda g)}{(w_1^i + w_2^i - 2c)(w_1^i + w_2^i - 2\lambda g)} + \frac{1}{w_2^i + b + h + \lambda d - r} \right) = 0.$$ 

Suppose $w_1^i > w_2^i$, the formulations above imply that $\left. \frac{dU_i^1}{dw_1} \right|_{w_1 = w_1^i, w_2 = w_2^i} = 0$ and $\left. \frac{dU_i^2}{dw_2} \right|_{w_1 = w_1^i, w_2 = w_2^i} = 0$ cannot co-exist. Similarly, we can prove that $w_1^i < w_2^i$ does not hold by contradiction. Thus, $w_1^i = w_2^i$.

As such, the brand sources from the risky supplier, and in the interior solution case the optimal procurement price and inspection efforts are

$$w_i = r - b - h - \lambda d + \frac{\sqrt{k\lambda d}}{\sqrt{\eta(1-\eta x_2^i)}},$$

$$x_1^i = x_2^i = \frac{1}{\eta} \left( 1 - \frac{w_i - c}{(w_i - \lambda g)(1-\eta x_2^i)} \right).$$

**Proof of Proposition 1:** (a). Regarding the choice between the risky and backup suppliers in three scenarios, the optimal solutions for strategy $i$ are feasible for strategy $m$ by letting $x_2^m = x_2^i, z = 1, 2$, and $w^m = w_1^i = w_2^i$, so $U_i^m - 2b \geq 2(U_i^z - b)$, resulting in $\eta^m \leq \eta^i$. 

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Next, for the comparison between strategies $j$ and $i/m$, because the optimal solutions for strategy $m$ are feasible for strategy $j$ by letting $x = x_1^m = x_2^m$ and $w = w^m$, which implies that for $F = 0$, in equilibrium $U_b^j - 2b \geq U_b^m - 2b$, that is, once the risky supplier is chosen in strategy $m$ (i.e., $U_b^m - 2b \geq 0$), it will also be chosen in strategy $j$. This implies $\eta^j \leq \eta^m$. That is, for $F = 0$, $\eta^j \leq \eta^m \leq \eta^i$.

Note that for a general $F$, $\eta^j$ increases in $F$, while both $\eta^i$ and $\eta^m$ are independent of $F$. Thus, there exist thresholds $F^\eta_i$ and $F^\eta_m$ such that: When $F \leq F^\eta_i$, $\eta^i \leq \eta^m \leq \eta^j$; When $F^\eta_i < F \leq F^\eta_m$, $\eta^m < \eta^i \leq \eta^j$; When $F > F^\eta_m$, $\eta^m \leq \eta^i < \eta^j$.

**Proof of Proposition 2:** (a). We first show that $w^m \geq w^i$ in step (a.i), followed by the comparison between $w^i$ and $w^j$ in step (a.ii). The rest of the proof is straightforward, so we omit it.

(a.i) $w^m \geq w^i$: In strategy $i$, after setting $w_1 = w_2 = w^m$, (6) implies

\[
y^i = 1 - \frac{k}{\eta} \sqrt{\frac{w_1 + w_2 - 2\lambda g}{(w_1 + w_2 - 2c)(w_1 + b + h + \lambda d - r)(w_2 + b + h + \lambda d - r)}},
\]

\[
y^m = 1 - \frac{k}{\eta(w^m + b + h + \lambda d - r)} \sqrt{\frac{w^m - \lambda g}{w^m - c}} = y^m,
\]
and $x_1^j = x_2^j = 1 - \frac{1}{\eta} \sqrt{\frac{w_1 + w_2 - 2c}{w_1 + w_2 - 2\lambda g}} \sqrt{\frac{w_2 + b + h + \lambda d - r}{w_1 + b + h + \lambda d - r}} = 1 - \frac{1}{\eta} \sqrt{\frac{w_m - c}{w_m - \lambda g}} = x_1^m = x_2^m$, so

$$
\frac{dU_1^j(w)}{dw_1}|_{w_1=w_m} = \frac{\partial U_1^j}{\partial w_1} + \frac{\partial U_1^j}{\partial y} \frac{dy^j}{dw_1} + \frac{\partial U_1^j}{\partial x_2} \frac{dx_2^j}{dw_1} + \frac{\partial U_1^j}{\partial x_1} \frac{dx_1^j}{dw_1}
$$

$$
= \frac{\partial U_1^j}{\partial w_1} + \frac{\partial U_1^j}{\partial y} \frac{dy^j}{dw_1} + \frac{\partial U_1^j}{\partial x_2} \frac{dx_2^j}{dw_1}
$$

$$
= (-y^j - (1 - y^j)(1 - \eta x_1^j)(1 - \eta x_2^j)) + (r - w_1 - b + (1 - \eta x_1^j)(1 - \eta x_2^j)(w_1 + b + h + \lambda d - r)) \frac{dy^j}{dw_1} + (k(w_1 - w_2 + h + \lambda d - r)) \frac{dx_2^j}{dw_1}
$$

$$
= (-y^m - (1 - y^m)(1 - \eta x_1^m)(1 - \eta x_2^m)) + (r - w_m - b + (1 - \eta x_1^m)(1 - \eta x_2^m)(w_m + b + h + \lambda d - r)) \frac{dy^j}{dw_1} + (k(w_1 + b + h + \lambda d - r)) \frac{dx_2^j}{dw_1}
$$

$$
= (-y^m - (1 - y^m)(1 - \eta x_1^m)(1 - \eta x_2^m)) + (r - w_m - b + (1 - \eta x_1^m)(1 - \eta x_2^m)(w_m + b + h + \lambda d - r)) \frac{dy^j}{dw_1} + k \frac{4\eta}{\sqrt{w_m - \lambda g} \frac{1}{(w_m - \lambda g)(w_m - c)} + (c - \lambda g) \frac{2(w_m - \lambda g)}{w_m + b + h + \lambda d - r}}
$$

$$
+ k \frac{\sqrt{w_m - \lambda g} \frac{1}{w_m + b + h + \lambda d - r}}{4\eta \sqrt{(w_m - \lambda g)(w_m - c)}(w_m + b + h + \lambda d - r)} \left( -\frac{(c - \lambda g)}{w_m - \lambda g} \frac{2(w_m - c)}{w_m + b + h + \lambda d - r} \right)
$$

$$
\leq (-y^m - (1 - y^m)(1 - \eta x_1^m)(1 - \eta x_2^m)) + (r - w_m - b + (1 - \eta x_1^m)(1 - \eta x_2^m)(w_m + b + h + \lambda d - r))\frac{dy^j}{dw_1} + \frac{k}{\eta} \left( \frac{w_m - \lambda g}{w_m - c} \frac{1}{(w_m + b + h + \lambda d - r)^2} + \frac{1}{2(w_m + b + h + \lambda d - r)} \sqrt{\frac{w_m - c}{w_m - \lambda g} \frac{c - \lambda g}{(w_m - c)^2}} \right)
$$

$$
- \frac{k}{2\eta} \sqrt{\frac{w_m - \lambda g}{w_m - c} \frac{c - \lambda g}{(w_m - \lambda g)^2}}
$$

$$
= \left( \frac{dU_1^j(w)}{dw} \right)_{w=w_m} = 0,
$$

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where as implied by \( r - w^m - b \geq 0 \) and \( w^m + b + h + \lambda d - r \geq 0 \) the inequality is from

\[
(r - w^m - b + (1 - \eta x^m_r)(1 - \eta x^m_l)(w^m + b + h + \lambda d - r))\frac{dy^j}{d\omega_1}
\]

\[
= (r - w^m - b + (1 - \eta x^m_l)(1 - \eta x^m_r)(w^m + b + h + \lambda d - r))
\]

\[
\frac{k}{\eta} \left( \sqrt{\frac{w^m - \lambda g}{w^m - c}} - \frac{1}{w^m + b + \lambda d - r} \right)^2 + \frac{1}{w^m + b + h + \lambda d - r} \sqrt{\frac{w^m - c}{w^m - \lambda g} (w^m - c)^2}
\]

\[
= \frac{k(r - w^m - b + \frac{w^m - c}{w^m - \lambda g} (w^m + b + h + \lambda d - r))}{4\eta (w^m + b + h + \lambda d - r)} \sqrt{\frac{1}{(w^m - c)(w^m - \lambda g)} \left( \frac{c - \lambda g}{w^m - c} + \frac{2(w^m - \lambda g)}{w^m + b + h + \lambda d - r} \right)}
\]

Then, together with \( \frac{dU^j_b}{d\omega_1} |_{\omega_1=\omega_l} = 0 \), the concavity of \( U^j_b(\omega) \) implies \( \omega^i \leq w^m \).

(a.ii) Next, we compare \( \omega^j \) with \( \omega^l \). Given the procurement price \( \omega^l \) in strategy \( j \), i.e., the optimal procurement price in this case, then, with \( y^l = 1 - \frac{k}{2\eta (w^l + b + h + \lambda d - r)} \sqrt{\frac{w^l - c}{w^l - \lambda g}} \) and \( x^l = \frac{1 - \frac{w^l - c}{w^l - \lambda g}}{\eta} \),

\[
\frac{dU^j_b}{d\omega} |_{\omega=\omega^l} = 0 \quad \text{implies}
\]

\[
0 = \frac{\partial U^j_b}{\partial \omega} |_{\omega=\omega^l} + \frac{\partial U^j_b}{\partial y} |_{y=y^l} \frac{dy^j}{d\omega} |_{\omega=\omega^l} + \frac{\partial U^j_b}{\partial x} |_{x=x^l} \frac{dx^j}{d\omega} |_{\omega=\omega^l}
\]

\[
= \frac{\partial U^j_b}{\partial \omega} |_{\omega=\omega^l} + \frac{\partial U^j_b}{\partial y} |_{y=y^l} \frac{dy^j}{d\omega} |_{\omega=\omega^l}
\]

\[
= 2 \left[ (-y^l - \frac{(1 - y^l)(w - c)}{w - \lambda g}) + (r - w^l - b + \frac{(w^l - c)(w^l + b + h + \lambda d - r)}{w^l - \lambda g}) \frac{dy^j}{d\omega} \right]
\]

\[
= 2 \left[ (-y^l - \frac{(1 - y^l)(w^l - c)}{w^l - \lambda g}) + (r - w^l - b + \frac{(w^l - c)(w^l + b + h + \lambda d - r)}{w^l - \lambda g}) \right]
\]

\[
\frac{k}{4\eta (w^l + b + h + \lambda d - r)} \sqrt{\frac{1}{(w^l - c)(w^l - \lambda g)} \left( \frac{c - \lambda g}{w^l - c} + \frac{2(w^l - \lambda g)}{w^l + b + h + \lambda d - r} \right)}
\]

where the second equality is from \( \frac{\partial U^j_b}{\partial x} |_{x=x^l} = \frac{2\eta (1 - y^l)(w^l + b + h + \lambda d - r) - k}{2\eta (w^l + b + h + \lambda d - r)} = 0 \) in equilibrium with mixed strategy.

In strategy \( i \), after setting \( \omega_1 = w_2 = \omega^i \), (6) implies \( y^l = 1 - \frac{k}{\eta} \sqrt{\frac{w^2 - \lambda g}{w^2 - c}} < y^l = 1 - \frac{k}{\eta} \sqrt{\frac{w^2 - \lambda g}{w^2 - c}} \) (because \( \sqrt{\frac{w^2 - \lambda g}{w^2 - c}} \geq 1 > \frac{1}{2} \)) and
\[ x_1^i = x_2^i = 1 - \frac{1}{\eta} \sqrt{\frac{w_1 + x_2 - 2c}{w_1 + x_2 - 2\lambda g}} \left( \frac{w_2 + b + h + \lambda d - r}{w_1 + b + h + \lambda d - r} \right) = 1 - \frac{1}{\eta} \sqrt{\frac{w_1 - c}{w_1 - \lambda g}}. \]

\[ \frac{dU_1^i(w)}{dw} \bigg|_{w = w^i} = \frac{\partial U_1^i}{\partial w_1} + \frac{\partial U_1^i}{\partial y} \frac{dy^i}{dw_1} + \frac{\partial U_1^i}{\partial x_2} \frac{dx_2^i}{dw_1} + \frac{\partial U_1^i}{\partial x_1} \frac{dx_1^i}{dw_1} = \frac{\partial U_1^i}{\partial w_1} + \frac{\partial U_1^i}{\partial y} \frac{dy^i}{dw_1} + \frac{\partial U_1^i}{\partial x_2} \frac{dx_2^i}{dw_1} + \frac{\partial U_1^i}{\partial x_1} \frac{dx_1^i}{dw_1} = \frac{-y^i - (1 - y^i)(1 - \eta x_1^i)(1 - \eta x_2^i) + (r - w_1 - b + (1 - \eta x_1^i)(1 - \eta x_2^i)(w_1 + b + h + \lambda d - r) + \frac{k(w_1 + b + h + \lambda d - r)}{w_1 + b + h + \lambda d - r} \frac{dy^i}{dw_1} + \frac{k}{4\eta \sqrt{w_1 - \lambda g}} \frac{1}{(w_1 - \lambda g)(w_1 - c)} \left( \frac{c - \lambda g}{w_1 - c} \left( \frac{2(w_1 - \lambda g)}{w_1 - \lambda g} \left( \frac{w_1 - \lambda g}{w_1 - c} \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) = \frac{-y^i - (1 - y^i)(w_1 - c)}{w_1 - \lambda g} \left( r - w_1 - b + \frac{(w_1 - c)(w_1 + b + h + \lambda d - r)}{w_1 - \lambda g} \right) + \frac{k}{4\eta \sqrt{w_1 - \lambda g}} \frac{1}{(w_1 - \lambda g)(w_1 - c)} \left( \frac{c - \lambda g}{w_1 - c} \left( \frac{2(w_1 - \lambda g)}{w_1 - \lambda g} \left( \frac{w_1 - \lambda g}{w_1 - c} \right) \right) \right) \right) \right) \right) \right) \right) = \frac{\frac{dU_1^i(w)}{dw}}{2} \bigg|_{w = w^i} = 0, \]

where the first inequality is because in addition to \( y^i \leq y^j, -y - \frac{(1 - y)(w_1 - c)}{w_1 - \lambda g} \) decreases in \( y \), while the last inequality holds if \( \left( \frac{c - \lambda g}{w_1 - \lambda g} + \frac{2(w_1 - c)}{w_1 + b + h + \lambda d - r} \right) \geq 0 \). Because \( \left( \frac{c - \lambda g}{w_1 - \lambda g} + \frac{2(w_1 - c)}{w_1 + b + h + \lambda d - r} \right) \geq 0 \) when \( c = \lambda g \). Given \( c \geq \lambda g \) and the continuity of \( \left( \frac{c - \lambda g}{w_1 - \lambda g} + \frac{2(w_1 - c)}{w_1 + b + h + \lambda d - r} \right) \) with respect to \( c \), there exists a threshold \( c_w \) such that \( \left( \frac{c - \lambda g}{w_1 - \lambda g} + \frac{2(w_1 - c)}{w_1 + b + h + \lambda d - r} \right) \geq 0 \) holds when \( c \leq c_w \). Then, together with \( \frac{dU_1^i(w)}{dw} \bigg|_{w = w^i} = 0 \), the concavity of \( U_1^i(w) \) implies \( w^i \geq w_1 \) when \( c \leq c_w \).

(b) Additionally, when \( c \leq c_w, w_m \geq w_1 \geq w_i, \) which implies that \( x_z^m \leq x_z^i \leq x_i, z = 1, 2, \) and \( y^m \geq y^i \).
Proof of Proposition 3: (a). The optimal solutions in strategy $i$ are feasible in strategy $m$ by letting $x^m_z = x^i_z$, $z = 1, 2$, and $w^m = w^i_1 = w^i_2$, so $U^m_b - 2b \geq 2(U^i_z - b)$, resulting in $\eta^m \leq \eta^i$.

(b). For the comparison between strategy $j$ and strategy $m$, we start considering a case $jf$, which is similar to case $j$ but without the fixed cost $F$. For exposition, we denote by $U^{jf}_b(\eta)$ the brand’s optimal utility in this case $jf$ given the inspection accuracy $\eta$, and by $U^m_b(\eta)$ the brand’s optimal utility in strategy $m$ given the inspection accuracy $\eta$. Because the optimal solutions for case $m$ are feasible for case $jf$ by letting $x^{jf} = x^m_1 = x^m_2$ and $w^{jf} = w^m$, $U^{jf}_b(\eta) \geq U^m_b(\eta)$.

On the other hand, when $\eta \geq \max\{\eta^j, \eta^m\} = \eta^m$, the risky supplier is chosen in either strategy $j$ or strategy $m$. Thus, the brand’s utility in strategy $j$ is

$$U^j_b(w^j) = 2(r - w^j) + \frac{k(1 - \sqrt{\alpha(w^j)})^2}{\eta \sqrt{\alpha(w^j)}} - \frac{\beta(w^j) \lambda d}{\sqrt{\alpha(w^j)}} - F,$$

and the brand’s utility in strategy $m$ is

$$U^m_b(w^m) = 2U^j_b(w^m) = 2[r - w^m + \frac{(1 - \sqrt{\alpha(w^m)})k}{\eta \sqrt{\alpha(w^m)}} - \frac{\beta(w^m) \lambda d}{\sqrt{\alpha(w^m)}}].$$

In particular, in the presence of the interior solutions of procurement prices for both strategy $j$ and strategy $m$, the envelope theorem implies $\frac{dU^j_b(w^j)}{dw} |_{w = w^j} = \frac{dU^m_b(w^m)}{dw} |_{w = w^m} = 0$. Accordingly,

$$\frac{\partial U^m_b(w^m)}{\partial \eta} - \frac{\partial U^j_b(w^j)}{\partial \eta} = -\frac{2}{\eta^2} \frac{(1 - \sqrt{\alpha(w^m)})k}{\eta \sqrt{\alpha(w^m)}} \left( \frac{k \lambda d}{\sqrt{\alpha(w^m)} (w^m + b + h + \lambda d - r) \sqrt{\alpha(w^m)}} \right) + \frac{1}{\eta^2} \frac{k(1 - \sqrt{\alpha(w^j)})^2}{\sqrt{\alpha(w^j)}} \left( \frac{k \lambda d}{\sqrt{\alpha(w^j)} (w^j + b + h + \lambda d - r) \sqrt{\alpha(w^j)}} \right) - \frac{1}{\eta} \frac{2(1 - \sqrt{\alpha(w^m)})k}{\eta \sqrt{\alpha(w^m)}} \left( \frac{\beta(w^m) \lambda d}{\sqrt{\alpha(w^m)}} - \frac{\beta(w^j) \lambda d}{\sqrt{\alpha(w^j)}} \right) \geq 0$$

because $\frac{k(1 - \sqrt{\alpha(w^j)})^2}{\eta \sqrt{\alpha(w^j)}} - \frac{\beta(w^j) \lambda d}{\sqrt{\alpha(w^j)}} \geq 2\left[ \frac{(1 - \sqrt{\alpha(w^m)})k}{\eta \sqrt{\alpha(w^m)}} - \frac{\beta(w^m) \lambda d}{\sqrt{\alpha(w^m)}} \right]$, so

$$2(r - w^j) + \frac{k(1 - \sqrt{\alpha(w^j)})^2}{\eta \sqrt{\alpha(w^j)}} - \frac{\beta(w^j) \lambda d}{\sqrt{\alpha(w^j)}} \geq 2(r - w^m) + \frac{k(1 - \sqrt{\alpha(w^m)})^2}{\eta \sqrt{\alpha(w^m)}} - \frac{\beta(w^m) \lambda d}{\sqrt{\alpha(w^m)}} \geq 2(r - w^m) + \frac{(1 - \sqrt{\alpha(w^m)})k}{\eta \sqrt{\alpha(w^m)}} - \frac{\beta(w^m) \lambda d}{\sqrt{\alpha(w^m)}}$$

where the first inequality holds since $U^j_b(w^j) + F \geq U^m_b(w^m)$, i.e., the optimal solutions for strategy $m$ are feasible for strategy $j$ by letting $x^j = x^m_1 = x^m_2$ and $w^j = w^m$, and the second inequality holds since $U^m_b(w^m) - 2b \geq 2(U^j_z - b)$, resulting in $\eta^m \leq \eta^j$.}

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holds because for a feasible \( w, U_b^i(w) + F \geq U_b^m(w) \) according to the definitions of \( U_b^i(\cdot) \) and \( U_b^m(\cdot) \). Thus, \( U_b^m(w^m) - U_b^i(w^i) \) increases in \( \eta \) when the risky supplier is chosen in both strategy \( j \) and strategy \( m \), and the procurement prices are characterized by the interior solutions for both strategy \( j \) and strategy \( m \).

(b.i). Denote by \( E := \min \{ \min_{\eta} [U_b^j(\eta) - U_b^i(\eta)], F^\eta \} \). If \( F \geq E \), \( U_b^j(\eta) - F \geq U_b^m(\eta) \) for all \( \eta \). Because the optimal solutions for case \( j \) are feasible for case \( ij \), if \( F \leq E \), the brand’s utility in strategy \( m \) is always lower than in strategy \( j \).

(b.ii). If \( F > E^i \), Proposition 1(b) implies \( \eta^i > \eta^m \). Thus, the proof of Proposition 3(b) implies that there exists a threshold \( \eta \geq \eta^j > \min \{ \eta^m, \eta^i \} \) such that when \( \eta < \eta^j \), the brand’s utility in strategy \( m \) is higher than in strategy \( j \). In particular, for \( \eta \leq \eta^m \) the backup supplier is used in both strategies \( j \) and \( m \), so the brand’s utility in strategy \( j \) is 2b, which is the same as in strategy \( m \). For \( \eta^m < \eta \leq \eta^j \), the backup supplier is used in strategy \( j \) resulting in a utility 2b for the brand, which is lower than that in strategy \( m \), since the risky supplier is used in strategy \( m \) (i.e., \( U_b^m(w^m) \geq 2b \)). Given the continuity of the brand’s utility in either strategy \( j \) or strategy \( m \) and the fact that the brand’s utility in strategy \( j \) is smaller than in strategy \( m \), because \( U_b^m(w^m) - U_b^i(w^i) \) increases in \( \eta > \eta^j \) when the risky supplier is chosen in both strategy \( j \) and strategy \( m \), \( U_b^i(w^i) \leq U_b^m(w^m) \) when \( \eta^j < \eta \).

(b.iii). If \( F < F \leq E^i \), Proposition 1(b) implies \( \eta^i \leq \eta^m \). When the inspection accuracy is very low (i.e., \( \eta^i = \min \{ \eta^m, \eta^i \} \)), provided that the backup supplier is used in both strategies \( j \) and \( m \), strategy \( j \) is equivalent to strategy \( m \). When the inspection accuracy is medium (i.e., \( \eta^j \leq \eta \leq \eta^m \)), the active risky supplier in strategy \( j \) implies that \( U_b^j(w^i) \geq 2b \) (otherwise, the backup supplier is used in strategy \( j \)), while the backup supplier is used in strategy \( m \) resulting in a utility 2b for the brand. When the inspection accuracy is high (i.e., \( \eta > \eta^m \)), because \( U_b^m(w^m) - U_b^i(w^i) \) increases in \( \eta > \eta^j \) when the risky supplier is chosen in both strategy \( j \) and strategy \( m \), there exists a critical number \( \eta^j > \eta^i \) such that \( U_b^i(w^i) \leq U_b^m(w^m) \) when \( \eta < \eta^j \).

The rest of the proof is straightforward, so we omit it.

**Proof of Proposition 4:** When \( \eta \geq \eta^j \), the risky supplier is chosen in both strategy \( i \) and strategy \( m \), the quantity of low-quality products sold to the market in strategy \( i \) is \( e^i(w^i) = 2\sqrt{\alpha(w^i)}\beta(w^i) = \frac{2}{\sqrt{(w^i + b + h + \lambda d - r)^2 \left( \frac{w^i - \lambda g}{w - c} \right)}} \), and in strategy \( m \) is \( e^m(w^m) = 2\sqrt{\alpha(w^m)}\beta(w^m) = \frac{2}{\sqrt{(w^m + b + h + \lambda d - r)^2 \left( \frac{w^m - \lambda g}{w^m - c} \right)}} \). Because \( (w + b + h + \lambda d - r)^2 \left( \frac{w - \lambda g}{w - c} \right) \) increases in \( w \), \( \frac{2}{\sqrt{(w^m + b + h + \lambda d - r)^2 \left( \frac{w^m - \lambda g}{w^m - c} \right)}} \) decreases in \( w \). This, together with \( w^i \leq w^m \), implies that \( e^i(w^i) \geq e^m(w^m) \).
Moreover, once the risky supplier is chosen both strategy $j$ and strategy $m$, when $c = \lambda g$, (7) implies $\frac{k\lambda d}{\eta(w^m + b + h + \lambda d - r)} = 2$, and (8) implies $\frac{k\lambda d}{\eta(w^m + b + h + \lambda d - r)^2} = 1$. As such, when $c = \lambda g$, $e^m(w^m) \geq e^j(w^j) = \frac{2k}{\eta(w^m + b + h + \lambda d - r)} - \frac{k}{\eta(w^m + b + h + \lambda d - r)} = 2[(w^m + b + h + \lambda d - r) - (w^j + b + h + \lambda d - r)] = 2(w^m - w^j) \geq 0$.

Given $c \geq \lambda g$ and the fact that both $e^m(w^m)$ and $e^j(w^j)$ are continuous in $c$, there exist a threshold $\bar{c} \geq \lambda g$ such that for $c \leq \bar{c}$, $e^j(w^j) \geq e^m(w^m) \geq e^j(w^j)$.

**Proof of Proposition 5**: When the risky supplier is chosen in both strategy $j$ and strategy $m$, and the procurement prices are characterized by the interior solutions for both strategy $j$ and strategy $m$:

$$
\frac{\partial U^m_b(w^m)}{\partial h} - \frac{\partial U^j_b(w^j)}{\partial h} = \frac{2k\lambda d}{\eta(w^m + b + h + \lambda d - r)^2\sqrt{a(w^m)}} - \frac{k\lambda d}{\eta(w^m + b + h + \lambda d - r)^2\sqrt{a(w^m)}} > \frac{2k\lambda d}{\eta(w^m + b + h + \lambda d - r)^2\sqrt{a(w^m)}} - 2 = 2\left[\frac{k\lambda d}{\eta(w^m + b + h + \lambda d - r)^2\sqrt{a(w^m)}} - 1\right],
$$

where the inequality holds because the proof of Lemma 2 shows that $\frac{\lambda d(w^j - \lambda g)}{(w^j + b + h + \lambda d - r)} - (c - \lambda g) > 0$ (or $\frac{2\lambda d(w^j - \lambda g)(w^j - c)^2}{\eta(w^j + b + h + \lambda d - r)^2\sqrt{a(w^j)}} - (c - \lambda g) > 0$), which, together with (7), implies that

$$\frac{k\lambda d}{\eta(w^j + b + h + \lambda d - r)^2\sqrt{a(w^j)}} > 2.$$

Note that when $c = \lambda g$, $a(w^m) = 1$ implies $\frac{k\lambda d}{\eta(w^m + b + h + \lambda d - r)^2\sqrt{a(w^m)}} = 1 = 0$, where the last equality holds since (8) under $c = \lambda g$ leads to $\frac{k\lambda d}{\eta(w^m + b + h + \lambda d - r)^2\sqrt{a(w^m)}} = 1$. That is, given $c = \lambda g$, $\frac{\partial U^m_b(w^m)}{\partial h} - \frac{\partial U^j_b(w^j)}{\partial h} > 0$.

Given $c \geq \lambda g$ and the fact that both $\frac{\partial U^m_b(w^m)}{\partial h}$ and $\frac{\partial U^j_b(w^j)}{\partial h}$ are continuous in $c$, there exist a threshold $\bar{c} \geq \lambda g$ such that for $c \leq \bar{c}$, $\frac{\partial U^m_b(w^m)}{\partial h} \geq \frac{\partial U^j_b(w^j)}{\partial h}$. Thus, there exists a threshold $\bar{h}$ such that when $h \geq \bar{h}$, the brand’s utility in strategy $m$ is larger than in strategy $j$, i.e., $U^m_b(w^m) < U^j_b(w^j)$.

The rest of the proof is straightforward, so we omit it. 

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