Strategic Recommendation Algorithms: Overselling and Demarketing Information Designs*

Ron Berman† Hangcheng Zhao‡ Yi Zhu§

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†The Wharton School of the University of Pennsylvania. ronber@wharton.upenn.edu
‡The Wharton School of the University of Pennsylvania. zhaohc@wharton.upenn.edu
§University of Minnesota, Carlson School of Management. yizhu@umn.edu

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Abstract

We analyze recommendation algorithms that firms can engineer to strategically provide information to consumers about products with uncertain matches. Monopolists who cannot alter prices can design recommendation algorithms to oversell the product instead of algorithmically recommending perfectly matching products. However, when prices are endogenous or when competition is rampant, firms opt to lower their persuasive claims and instead choose to fully reveal the product’s match (i.e., maximize recall and precision). As competition strengthens, the algorithms will shift to demarket their products in order to soften competition. When a platform designs a recommendation algorithm for products sold by third party sellers we find that overselling is not an equilibrium strategy of the platform, but demarketing might be. Overselling entails designing an algorithm that recommends badly fitting products to consumers, which would lower the consumers’ ex-ante willingness to pay, and thus increase competition among the sellers and lower the platform’s profit. Demarketing, in contrast, softens the competition among sellers from the information perspective, which can be lucrative for the platform.

Keywords: Product Recommendation, Algorithmic Bias, Bayesian Persuasion, Information Competition, Information Design, Platform Design, Demarketing.

1 Introduction

Recommendation algorithms are a popular tool to help personalize the experience of online users. In two famous examples, it was estimated that 35 percent of Amazon’s sales and 75 percent of what consumers stream on Netflix could be attributed to product recommendations.¹

The proliferation of recommendation algorithms and their perceived influence on consumer behavior has led to abundant research on their business impact. Recommendation algorithms have been shown to affect consumption levels (Pathak et al. 2010), the diversity of products consumed (Fleder and Hosanagar 2009, Hosanagar et al. 2014, Nguyen et al. 2014, Möller et al. 2018, Lee and Hosanagar 2019) as well as which products are the most popular (Brynjolfsson et al. 2011, Oestreicher-Singer and Sundararajan 2012).

Despite their importance for modern e-commerce and content websites, the majority of the research on recommendation algorithms has been focused on designing better algorithms, while assuming that firms always benefit from recommending the most fitting product for the consumer. However, for many customers product recommendations often appear as arbitrary and misleading, causing many to question the value of such AI automation.\textsuperscript{2} Such reactions seem at odds with the claims of many online firms that they only recommend the most relevant products to consumers. This contrast between consumer experience and firm claims raises the question of whether it is in the firm’s best interest to always algorithmically recommend the product that fits consumers the best.

From the firm’s perspective, using an algorithm to recommend the most fitting product for a consumer might not maximize the objectives of the firm, namely its demand and its profit. Consider for example a website that recommends clothing items for consumers in a competitive market where prices are exogenous. If the consumer trusts the recommendations of the algorithm and follows them, then the website naturally has an incentive to also recommend potentially less fitting products and oversell them. By showing potentially less fitting products to the consumer, the website might sell more items and increase the demand from consumers, and thus increase its profits. Figure 1 presents an example of product recommendations that have been customized for a consumer, stating “you may also like”. If a consumer believes the recommendation and assumes that these products will be a good fit for their needs or style, and picks one, then the seller will have an incentive to add products to those lists, even if those products might not fit well.

Such overselling by a retailer, however, can be a double edged sword. Consumers who anticipate the firms’ monetary incentives might rationally discount the information in the product recommendations and potentially ignore it, resulting in lower sales. In fact, some argue that websites recommend poor fitting products and that consumers shouldn’t blindly trust these product recommendations.\textsuperscript{3}

In another instance of recommendation algorithm design, Figure 2 shows how Netflix provides an individualized prediction of the match between the content and the consumer (e.g., 60%), while Amazon Prime Video only provides an average star rating which is not customized per consumer.


Given the different practices of firms, we ask how should a firm strategically design its product recommendation algorithms in a competitive environment. We use the intuition from the examples above to challenge the notion that recommendation algorithms always benefit firms if they recommend a product that best matches consumer tastes. To do so, we analyze the design of recommendation algorithms from an information design perspective and analyze the quality of recommendations that the algorithms produce as a strategic choice of the firm.

When making their algorithmic design choices, the firm needs to consider multiple opposing forces. First, firms have to decide how precise their algorithm will be and how well the recommended content items or products will fit the consumer’s tastes. Second, given the quality of past recommendations and the uncertainty about the recommended product’s fit, the consumers will decide whether to consume it or not, and how much they are willing to pay for it. Third, if a competitor also provides a recommendation, or if a platform controls the recommendation algorithm, then consumers might take these other recommendations into account as well. All of these forces affect the degrees of accuracy of recommendations that the firm would like to provide.
We develop a game theoretical model to analyze the design of recommendation algorithms of firms. Our benchmark model consists of a monopolist firm who sells products of uncertain match to consumers. A product can be a good or a bad match to the consumer’s taste, and the firm can design a recommendation algorithm that will provide a signal to the consumer about the match of the product to its tastes. To design a recommendation algorithm, the firm needs to pick the algorithm’s precision and recall, which are two common parameters used to describe and compare the performance of predictive machine learning and AI algorithms. Precision is the fraction of recommended products that will have a good fit. Recall is the fraction of well fitting products that are recommended. We assume that the firm is strategic in how it picks these parameters, and that its goal is to maximize its own profit.

The consumers, in turn, observe the firm’s recommended products and update their beliefs
about the match of the product while taking into account the precision and the recall selected by
the firm. If the expected utility from the product is higher than its price, the consumer purchases
the product.

Our framework allows us to study the microeconomics of recommendation algorithms which
are designed by firms seeking to maximize profits from consumers. Notable features of product
recommendation algorithms that our model captures include: (1) costlessness of making recom-
mendations: the marginal cost of sending signals to an additional consumer is almost zero, (2)
non-binding recommendations: the recommendations do not dictate consumer choices, and (3) one
algorithm for all: although recommendations may vary by consumer, the algorithm that generates
them is designed once. Importantly, the uncertainty of product match precludes the firm from
strategically signaling the fit of the product to the consumer.

These features of recommendation algorithms are naturally captured in a Bayesian Persuasion
framework (Kamenica and Gentzkow 2011, Bergemann and Morris 2019), which we believe is an
ideal model choice in this setting. The basic benchmark model allows us to show that the task
of designing a recommendation algorithm is equivalent to the information design task of Bayesian
Persuasion, and that commonly known results from the Bayesian Persuasion literature apply to
strategic recommendation algorithm design. Specifically, we find that when prices are exogenous,
a monopolist recommender has incentive to deviate from always algorithmically recommending
perfectly matching products and instead is better off overselling them, since this approach can
increase demand. This means that from an information perspective, the signals provided to the
consumer should not reveal the true match of the product, and that precision and recall should not
be maximized.4

The major contributions of our paper uncover when we extend the benchmark model to al-
low firms to endogenously set their prices, as well as once we add competition to the mix. Our
analysis shows that the traditional results from the Bayesian Persuasion literature no longer apply.
The results allow us to characterize the benefits and boundaries of Bayesian Persuasion and its
applicability to recommendation algorithm design. We find that when prices are endogenous, a
monopolist would like to design a fully revealing recommendation algorithm (i.e., maximize recall

4We note that when designing recommendation algorithms there is often a tradeoff between precision and recall
due to inherent noise in the data (Buckland and Gey 1994), but even if we assume that both can theoretically be
maximized concurrently, we show that this might not always be desirable.
and precision), where only fitting products are recommended for a consumer who ends up consuming only products with a good fit. The intuition for not trying to oversell is that using an overselling algorithm might indeed increase demand, but this increase in demand will be counter-veniled by decreased willingness to pay of the consumer because of the decreased ex-ante expected utility of consuming the recommended item.

We then extend the analysis to competing firms, where each firm designs its own recommendation algorithm as well as sets its prices for the products. We find that firms will often elect to use mixed-strategies for pricing, and in some cases might choose to either oversell to consumers by over-recommending when the product’s fit is not very high, but in some other cases might choose to demarket their products by under-recommending well fitting products. Among the set of subgame perfect equilibria we find, the demarketing one is quite interesting. In that case the firms choose to undersell in order to differentiate from one another and soften the price competition.

We conclude the analysis with the case of a platform that designs a recommendation algorithm for products being sold by other sellers on the platform, while the sellers select their own prices. A surprising result we find in this case is that overselling is not an equilibrium strategy of the platform, but demarketing might be. Overselling would entail the platform designing an algorithm that recommends badly fitting products to consumers, which would lower the consumers’ ex-ante willingness to pay, and thus increase competition among the sellers and lower the platform’s profit. Demarketing, in contrast, softens the competition among sellers from the information perspective, which can be lucrative for the platform.

Our paper makes a contribution to several streams of literature, but also has implications beyond academic research. For the research on recommendation algorithms and their design we contribute by taking a strategic perspective and analyze the recommendation algorithms implementation choices of firms under different competitive structures, and also analyze the impact on consumers. Other research by Hagiu and Jullien (2011; 2014) and De Corniere and Taylor (2014) have shown that search engines might have an incentive to provide biased search results, while Drugov and Jeon (2017) and Bourreau and Gaudin (2022) show that bias can arise in equilibrium when content platforms face different upstream costs by content creators. Given the tradeoff between precision and recall, one might be frustrated at not being able to design algorithms that can perfectly predict the fit of products, but our results provide some insight into why that might not be de-
sirable. Because we compare three common market structures (monopoly, duopoly and platform), we are also able to provide predictions about the prevalence of different types of recommendation algorithms, the expected equilibrium quality of recommendations displayed to consumers, and their impact on prices and the diversity and quality of content and products consumed.

Within the research on Information Design and Bayesian Persuasion, we provide a concrete application that shows how the results might translate to the field, but also provide insight into the boundaries and disadvantages of using algorithmic recommendations for persuasion. The marketing literature has started to adopt Bayesian Persuasion structures only recently (Iyer and Zhong 2020, Jerath and Ren 2021, Yao 2022, Pei and Mayzlin 2022), often in monopolistic settings. Our results on demarketing are especially notable, as they provide potential predictions for the existence of this phenomenon that has been previously documented in Miklós-Thal and Zhang (2013), Kim and Shin (2016) and Harbaugh and To (2020).

Our research also relates to the literature on strategic information provision. Prior research studied a firm’s incentives to provide information about a product’s valuation from various aspects including price discrimination (Lewis et al. 1994) product returns (Shulman et al. 2009), and quality information (Kuksov and Lin 2010). Like these papers, our framework shows the importance of studying strategic information provision. Our work, however, is distinct from previous research in three important aspects: (1) Substantively, our paper studies the design of algorithmic product recommendations, and how pricing decisions and competition interact with these designs. The prior literature has not investigated such strategic decisions to the best of our knowledge. (2) We focus on the recommendation decision before the firms know consumer private tastes, as recommendation algorithms are predetermined and cannot change the recommendation by observing consumers’ idiosyncratic preferences. This additional strategic consideration alters the firms’ product recommendation strategies and consumers’ updated beliefs, leading to a non-trivial impact on the competition between firms. (3) In terms of results, our framework shows that firms may not only have an incentive to design algorithms which are not fully revealing, they may even undersell the product in order to avoid head-to-head competition.

Finally, there has been a voluminous discussion recently about recommendation algorithms of platforms (mostly in the sphere of content recommendation) and their impact on societal outcomes such as increased online polarization and the emergence of filter bubbles and echo chambers.
(Hosanagar et al. 2014, Nguyen et al. 2014, Möller et al. 2018, Berman and Katona 2020). Our analysis provides insights and predictions about the impact of these algorithms on retail outcomes, with some good news for consumers. In retail settings, where platforms design recommendation algorithms but sellers are free to set their prices, the setup might benefit consumers because the incentives to oversell are diminished.

2 Model

The market consists of online sellers (indexed by \( j \)) who offer items for sale to a mass one of consumers. Each customer wants to consume a single item, e.g., buy a single clothing item or watch one movie. Each item provides ex-ante uncertain utility \( u_j \) which is unknown to both the sellers and the consumers. The item can be a good match (\( G \)) with probability \( \alpha \) or a bad match (\( B \)) with probability \( 1 - \alpha \). The item, if purchased at a price \( m_j \), provides the consumer with a realized utility \( u_j - m_j \) where \( u_j = u_G \) if the product matches and \( u_j = u_B \) otherwise, with \( u_G = 1 \) and \( u_B < u_G \).

Although neither the sellers nor the consumers know the match of the product before it is consumed, a recommendation algorithm can be employed to predict the match of the product for the consumer. The recommendation algorithm will provide a signal \( s_j = g \) (recommendation) if it predicts that the product will be a match (\( G \)), and will provide a signal \( s_j = b \) (no recommendation) otherwise. The seller can then display to the consumer the product with no recommendation, with a recommendation \( g \) or with a recommendation \( b \). A recommendation algorithm is characterized by its precision and recall, which are parameters that can be chosen by the seller (Buckland and Gey 1994). Precision is the share of items with utility \( u_G \) among items that received a recommendation \( g \). Recall is the share of items with utility \( u_G \) that received a positive recommendation \( g \).

We denote the probability that the recommendation algorithm provides a signal \( g \) when the product is a match by \( p_G = Pr(g|G) \),\(^6\) and the probability that the algorithm provides a signal \( g \) when the product does not match by \( p_B = Pr(g|B) \) (see Table 1).

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\(^5\)Normalizing \( u_G = 1 \) is without loss of generality and simplifies exposition.

\(^6\)To simplify notation, we write the state where \( u = u_G \) simply as \( G \), and the recommendation of \( j, s_j = g \) as \( g \).
Table 1: Website Recommendation Algorithm design.

<table>
<thead>
<tr>
<th>State</th>
<th>( G )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realized Signal</td>
<td>( g )</td>
<td>( p_G )</td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>( 1 - p_G )</td>
</tr>
</tbody>
</table>

Using this notation we can rewrite precision and recall as:

\[
\text{precision} = \frac{Pr(g, G)}{Pr(g)} = \frac{\alpha p_G}{\alpha p_G + (1 - \alpha)p_B} \quad \text{recall} = \frac{Pr(g, G)}{Pr(G)} = \frac{\alpha p_G}{\alpha} = p_G
\]  

(1)

Because there’s a one-to-one mapping between precision, recall, and \( p_G \) and \( p_B \), then \( p_G \) and \( p_B \) can fully characterize a recommendation algorithm’s properties. We will also interchangeably call \( p_G \) and \( p_B \) the algorithm’s information design.

After seeing the product and the information displayed along with it, the consumers can update their beliefs about the product’s expected utility and decide whether to buy it or not. We denote the expected utility of consuming the product after observing the recommendation \( s_j \) as \( u_j(s_j) \), i.e., \( u_j(s_j) = E[u_j|s_j] \). Given the recommendation and the price, the consumer will purchase the product that provides the highest surplus compared to other sellers and to an outside option normalized to 0. In the case of a single monopolist seller, the consumer will purchase if \( u(s) - m > 0 \). In the case of two sellers, the consumer will purchase product \( j \) if \( u_j(s_j) - m_j > 0 \) and \( u_j(s_j) - m_j > u_j(s_{-j}) - m_{-j} \), where \( -j \) indicates the other seller.

We call a recommendation algorithm \textit{perfect} or \textit{fully revealing} if the signal perfectly reveals the match of the product. That is, if \( Pr(G|s = g) = 1 \) and \( Pr(B|s = b) = 1 \), the recommendation is perfect. Using Bayes rule, this is equivalent to stating that \( Pr(s = g|G) = 1 \) and \( Pr(s = b|B) = 1 \).

Our focus is on the incentives of sellers (and of an intermediating platform in Section 4) to pick algorithms that are more or less revealing of the true state of items to consumers. We will analyze the case of a monopolist, of competing duopolists, and of competing duopolists who sell through a platform. In our analysis, we will call an algorithm \textit{overselling} if in equilibrium \( p_B > 0 \), that is, if sellers design an algorithm that predicts and recommends an item as matching even if it does not. We will call an algorithm \textit{demarketing} if \( p_G < 1 \), that is, if the algorithm will display a negative recommendation next to a matching product. Demarketing under-recommends well fitting.

\footnote{In the case of a single seller we drop the index \( j \).}
products, while overselling over-recommends badly fitting products.

2.1 Algorithmic recommendation under monopoly with exogenous prices

We begin the analysis with the case of a single seller (a monopolist) and an exogenous price $m$ for the items. The analysis allows us to introduce the main solution concept we utilize throughout the paper as well as establish the relationship between the research on Bayesian Persuasion and Information Design (Kamenica and Gentzkow 2011, Bergemann and Morris 2019), and our research on strategic design of recommendation algorithms.

The timing of the game is as follows:

1. The monopolist decides whether to provide recommendations next to items, and if so, they design a recommendation algorithm by picking values for $p_G$ and $p_B$.

2. If recommendations are displayed, consumers observe the signal realization $s$ and purchase the product if $u(s) - m > 0$. If recommendations are not displayed, they purchase if $E[u] - m \geq 0$.

Since designing recommendation algorithms is a complex long-term task, we assume that consumers observe the recommendation strategy of the platform and know the values of $p_G$ and $p_B$ when they make their purchase decisions. One way consumers can learn these values is through multiple interactions with the sellers, which will allow the consumers to infer the correlation between the signals and the realized utilities. We abstract away from how consumers learn about the algorithm’s quality of predictions for simplicity, but will require the seller to pick the optimal algorithm from a profit perspective.

Given a recommendation algorithm design, the seller’s expected profit when providing recommendations is:

$$mE_s[\mathbb{I}(u(s) - m > 0)]$$

where $\mathbb{I}(\cdot)$ is the indicator function.

We seek to characterize the following types of equilibria. In separating equilibria, the firm will provide a recommendation and consumers will buy an item if it comes with a signal $g$, but will not buy if it comes with a signal $b$. In pooling equilibria, the firm will not provide a recommendation and the consumer will choose to buy the product. In later sections, when we analyze competition, we will also consider hybrid equilibria (which we also call asymmetric equilibria) where one firm
provides a recommendation while the other does not. We note that not providing a recommendation is equivalent to setting \( p_G = p_B \), since when \( p_G = p_B \), the expected utility of the item after observing the signal equals the prior expected utility, as we show below. In all of these equilibria sellers pick the recommendation strategy that maximizes their profit with incentive compatibility, while consumers make incentive compatible choices and buy the product only if it provides positive expected surplus compared to the other options.

For a monopolist, the equilibrium recommendation strategy \((p^*_G, p^*_B)\) would solve:

\[
\max_{p_G, p_B} m \mathbb{E}[\mathbb{I}(u(s) - m > 0)]
\]

subject to the following individual incentive compatibility constraints: (ICS) if \( p_G \neq p_B \) then \( u(g) - m \geq 0 \) and \( u(b) - m < 0 \); (ICP) while if \( p_G = p_B \) then \( \mathbb{E}[u] - m \geq 0 \).

A separating equilibrium that satisfies (ICS) is also called an obedient equilibrium by Bergemann and Morris (2019). As a tie-breaking rule, we assume that if sellers are indifferent between not providing a recommendation or using a recommendation algorithm, they will choose to not provide a recommendation. This assumption is consistent with the fact that designing a recommendation algorithm often incurs fixed costs.

After observing a signal, the consumers update their beliefs about the utility of the product. Using Bayes’ rule, the updated expected utility from consuming when observing \( s = g \) is:

\[
u(g) = \frac{Pr(G|g)u_G + Pr(B|g)u_B}{Pr(g)} = \frac{\alpha p_G u_G + (1 - \alpha) p_B u_B}{\alpha p_G + (1 - \alpha) p_B}, \]

and similarly \( u(b) = \frac{\alpha (1 - p_G) u_G + (1 - \alpha) (1 - p_B) u_B}{\alpha (1 - p_G) + (1 - \alpha) (1 - p_B)} \). If we let \( p_G = p_B \), then \( u(g) = \alpha u_G + (1 - \alpha) u_B \) and similarly \( u(b) = \alpha u_G + (1 - \alpha) u_B \). Hence, setting \( p_G = p_B \) is equivalent to not providing any recommendation.

Maximizing the firm’s profit in (2) subject to the incentive compatibility constraints (3) yields the following:

**Proposition 1.** When \( \mathbb{E}[u] - m < 0 \), i.e., when \( \alpha < \frac{m - u_B}{u_G - u_B} \), there is a unique separating equilibrium of overselling in which the product recommendation algorithm’s precision is less than one with perfect recall, i.e., \( p^*_G = 1 \) and \( p^*_B = \frac{\alpha (u_G - m)}{(1 - \alpha) (m - u_B)} \in (0, 1) \).
When \( E[u] - m \geq 0 \), i.e., when \( \alpha \geq \frac{m - u_B}{u_G - u_B} \), the firm will not provide a recommendation and a pooling equilibrium will ensue.

Proof. All proofs appear in the Appendix.

Proposition 1 shows that when \( \alpha \) is large and the consumer has a high a priori expected value for the product, the firm does not need to invest in informative recommendations since the consumer will consume the product anyway. When \( \alpha \) is smaller and \( u_B \) is smaller than \( m \), the firm always has an incentive to reveal the true state of the product if it is good, and oversell to consumers in a bad state by setting \( p_B \) to be between zero and one. Setting \( p_G^* = 1 \) is optimal because it increases demand, and does not make the consumer trust the signal less. Setting \( p_B \) too high, however, will lower the expected utility from buying and will cause the consumer to not buy even when receiving a good signal because they would trust the signal less. The firm would like to increase \( p_B \) as high as possible while still making sure that the consumer would buy if they see a good signal. Hence, in equilibrium the monopolist will oversell by setting a positive level of \( p_B \) which is lower than 1.

This result is not surprising by itself and echoes previous results in Kamenica and Gentzkow (2011) and Bergemann and Morris (2019) about the conditions for Bayesian persuasion to be an equilibrium strategy. We use it as a benchmark to demonstrate how pricing and competition incentives dramatically alter the incentives to provide unbiased recommendations in the next sections.

### 2.2 Algorithmic recommendation under monopoly with endogenous pricing

Now, assume that the monopolist can choose the price of its products after it designs its recommendation algorithm. Unlike the case with exogenous prices, the firm faces a trade-off between increasing the consumer’s expected utility of a product by providing more revealing information, and persuading the consumer to buy the product by overselling. Increasing the expected utility allows the firm to charge higher prices and extract more rent, at the risk of lowered demand, while overselling increases demand, but lowers the willingness to pay of the consumer.

The timing of the game matches that of the previous section, but in step 1 the firm also chooses the price \( m \) for its products. Because information is symmetric and the firm does not have a priori private information about the product’s utility, it cannot use prices to signal the utility of its products.
The firm’s profit is the same as in Equation (2), the only difference being that $m$ is an endogenous choice variable. The consumer’s incentive compatibility constraints in separating and pooling equilibria are also the same as with exogenous prices.

Because the price $m$ is endogenous, in a separating equilibrium the firm can extract the maximum revenue by charging a price $m^* = u(g)$. If we plug in the equilibrium price $m^*$ into the expression for the firm’s profit, the expression for profit becomes:

$$\alpha p_G u_G + (1 - \alpha) p_B u_B$$

Compared to the case of exogenous prices, the ability to pick prices removes the conditioning of the profit on the purchase probability for the customer, as prices will be endogenously selected to induce buying. Consequently, in equilibrium all consumers will purchase the product. Hence, the algorithm design $p_G$ and $p_B$ now solely control the willingness to pay of the customer. Solving for the optimal algorithm design, we find:

**Proposition 2.** When $u_B > 0$ the unique equilibrium is pooling. The firm does not want to provide information and consumers will always buy the product. When $u_B < 0$ the unique equilibrium is separating and fully revealing with perfect precision and recall. i.e., $p^*_G = 1$ and $p^*_B = 0$.

Proposition 2 provides new insight about the effects of pricing on algorithm design. In this framework pricing serves as a strategic substitute to persuasion within a certain range of values for $u_B$. Moreover, there is no case where overselling or demarketing are viable, unlike the typical case analyzed in the literature on information design. This means that the pricing and persuasion trade-off may often be strong enough to dissuade the monopolist from persuading consumers to buy products of low quality. Potentially, this implies that consumers should not worry as much from being “falsely” persuaded when the seller is a monopolist, because the pricing power of the monopolist makes overselling a less valuable strategy.

### 3 Algorithmic recommendation under duopoly

We now proceed to analyze the recommendation algorithm design under duopolistic competition. The full game with endogenous algorithm design has two stages – an algorithm design stage followed by pricing stage. We assume that both firms first decide on the information design $(p_{G1}, p_{B1})$
and \((p_{G2}, p_{B2})\) for their recommendation algorithm in the first stage, and then they choose their prices \(m_1\) and \(m_2\) after observing the designs chosen. The consumers then observe the firm recommendations \(s_1\) and \(s_2\), and make their purchase decisions.

A consumer will consume product \(j\) only if \(u(s_j = g) - m_j > u(s_{-j}) - m_{-j}\) and \(u(s_j = g) \geq m\), while we assume that if the expected surpluses are tied, the consumer will choose either product with equal probability. As before, we also assume that if a seller does provide information, in equilibrium a consumer will not purchase the product if they observe a signal \(b\).

The signals \(s_1\) and \(s_2\) segment consumers into four segments with sizes detailed in Table 2.\(^8\)

<table>
<thead>
<tr>
<th>Signals</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g_1, g_2)</td>
<td>((\alpha p_{G1} + (1 - \alpha)p_{B1})(\alpha p_{G2} + (1 - \alpha)p_{B2}))</td>
</tr>
<tr>
<td>(g_1, b_2)</td>
<td>((\alpha p_{G1} + (1 - \alpha)p_{B1})(\alpha(1 - p_{G2}) + (1 - \alpha)(1 - p_{B2})))</td>
</tr>
<tr>
<td>(b_1, g_2)</td>
<td>((\alpha(1 - p_{G1}) + (1 - \alpha)(1 - p_{B1}))(\alpha p_{G2} + (1 - \alpha)p_{B2}))</td>
</tr>
<tr>
<td>(b_1, b_2)</td>
<td>((\alpha(1 - p_{G1}) + (1 - \alpha)(1 - p_{B1}))(\alpha(1 - p_{G2}) + (1 - \alpha)(1 - p_{B2})))</td>
</tr>
</tbody>
</table>

Firm \(j\)'s profit when the other firm uses information design \(p_{G-j}\) and \(p_{B-j}\) as its strategy and price \(m_{-j}\) is therefore:

\[
\pi_j(p_{Gj}, p_{Bj}, m_j) = m_j/2(\alpha p_{Gj} + (1 - \alpha)p_{Bj})(\alpha p_{G-j} + (1 - \alpha)p_{B-j})\mathbb{I}(u_j(g) - m_j = u_{-j}(g) - m_{-j}) \\
+ m_j(\alpha p_{Gj} + (1 - \alpha)p_{Bj})(\alpha p_{G-j} + (1 - \alpha)p_{B-j})\mathbb{I}(u_j(g) - m_j > u_{-j}(g) - m_{-j}) \\
+ m_j(\alpha p_{Gj} + (1 - \alpha)p_{Bj})(\alpha(1 - p_{G-j}) + (1 - \alpha)(1 - p_{B-j}))
\]

(6)

The first term is firm \(j\)'s profit when both firms generate a signal \(g\) and provide equal surplus, the second is the profit when both firms generate a signal \(g\) and firm \(j\) provides higher surplus, and the third is the profit when firm \(-j\) produces a signal \(b\). There is no profit when firm \(j\) generates a signal \(b\) as consumers will not buy after seeing \(b\).

We solve the game using backwards induction, first focusing on the second pricing stage, and then solving for the optimal recommendation algorithm information design.

To aid the analysis, we first study pricing in a subgame where duopoly firms have chosen a fully revealing information design in the design stage. We then generalize the result by endogenizing the information design in the first stage. The following lemma describes the pricing equilibrium when

\(^8\)We shorten the notation \(s_j = g\) to \(g_j\).
both firms have chosen a fully revealing information design:

**Lemma 1.** When firms fully reveal their product match, i.e., set \( p_{Gj} = 1 \) and \( p_{Bj} = 0 \), and \( u_B < \frac{\alpha(1-\alpha)}{1-\alpha+\alpha^2} \), there exists a mixed strategy pricing equilibrium such that each firm will randomize its prices within \([u_G - (1-\alpha)u_G, u_G]\), with CDF \( F(m) = \frac{m-(1-\alpha)u_G}{\alpha m} \).

The pricing equilibrium described in Lemma 1 is similar in spirit to that of Varian (1980) and Narasimhan (1988) where consumers are segmented into a loyal segment of each store and a switchers segment that chooses among the two stores. In our case the loyals are those who receive only one good signal, and the switchers are those who receive two good signals. A higher chance of receiving good signals from the firms makes consumers more price sensitive on average. Therefore, the game has no pure strategy equilibrium in prices and each firm in equilibrium randomizes its prices in the range \([u_G - (1-\alpha)u_G, u_G]\). The upper bound \( u_G \) is the reservation price of consumers, \( u_j(g) = u_G \), yielding profit \( \alpha(1-\alpha)u_G \).

When prices are endogenous, the restriction that consumers do not buy the product if they observe a signal \( b \) can only be rationalized if the sellers do not have an incentive to price at a price low enough such that \( u(b) - m > 0 \). We therefore require this restriction to hold in the pricing subgame.

The non-existence of pure strategy pricing equilibria will also hold when the information design decision is endogenized in the design stage, which we now turn to analyze. Similarly to the previous section, in the design stage, firm \( j \) decides whether to send out information (by setting \( p_{Gj} \neq p_{Bj} \)) or not to send any information and let consumers use their prior beliefs (by setting \( p_{Gj} = p_{Bj} \)).

The firm choices of information design in the first stage define three types of scenarios: (1) both firms do not provide a recommendation which we call (passive, passive), (2) both firms provide a recommendation (active, active), and (3) one firm provides a recommendation while the other does not (active, passive). These three scenarios, if they consist an equilibrium, define a pooling, separating, and hybrid\(^{10}\) equilibria, respectively.

\(^9\)Unlike in Varian (1980) and Narasimhan (1988) the sizes of these segments are stochastic and endogenous, but the behavior of the consumers in these segments is similar to loyals and switchers.

\(^{10}\)A hybrid equilibrium is one where one player plays a separating strategy, and the other plays a pooling strategy. This is in contrast to a semi-separating equilibrium where the same player might switch from one strategy to the other for different parameter values.
3.1 Scenario 1: Both firms are passive

Because consumers do not update their beliefs in this subgame, they will have symmetric willingness
to pay for the products from both firms. After observing the price, the consumers will buy from
the firm with the lowest price, leading to Bertrand duopoly competition. In equilibrium both firms
will set prices to zero and make zero profit.

3.2 Scenario 2: Both firms are active

In the pricing stage, the profit of firm $j$ will follow (6). In any pricing equilibrium firms will mix
their pricing following the same reasoning as in Lemma 1. The highest price they can charge will
be $u_j(g)$, and the expected equilibrium profit will be:

$$E[\pi_j] = (\alpha p_{Gj} + (1 - \alpha)p_{Bj}) (\alpha(1 - p_{Gj}) + (1 - \alpha)(1 - p_{Bj})) u_j(g)$$

(7)

Plugging in $u_j(g) = \frac{\alpha p_{Gj} u_G + (1-\alpha) p_{Bj} u_B}{\alpha p_{Gj} + (1-\alpha) p_{Bj}}$ and using $u_G = 1$ we prove the following:

**Lemma 2.** In a separating Nash equilibrium of this subgame, both firms will set the recommendation
algorithm to have perfect recall, i.e., set $p_{Gj}^* = 1$.

Lemma 2 shows that in equilibrium both firms will design their signal to fully reveal the match
of the product if it is good by setting $p_{Gj} = 1$. The reason is that the profit of firm $j$ increases in
$p_{Gj}$ for any value of $\alpha$ and $p_{Bj}$. Using the result that $p_{Gj}^* = 1$, we solve for the full Nash equilibrium
and prove the following:

**Lemma 3.** In the subgame where both firms are active:

- When $u_B \leq 0$, firms will set $p_{Bj}^* = 0$ such that the firms will be fully revealing and not engage
  in any persuasion.

- When $u_B > 0$, there is no equilibrium where both firms are active.

Lemma 3 shows that there isn’t a pure strategy active-active information design equilibrium
when $u_B$ is positive. The reason is that firms could always deviate to being passive, change their
pricing strategies and achieve higher profits when the other firm is fixed in its recommendation
algorithm design. When $u_B < 0$, in contrast, we find that full revelation, with perfect precision
and recall, is the only possible equilibrium, similarly to the results in the monopoly case with endogenous pricing. In this case, increasing \( p_B \) might indeed increase demand, but will also intensify competition, making overselling non-profitable.

Figure 3 illustrates the range of parameters where equilibrium exists in the subgame in which both firms provide information signals. When \( u_B < 0 \) (Region F), the equilibrium information design is \( p_{Gj}^* = 1 \) and \( p_{Bj}^* = 0 \).

Figure 3: Parameter ranges for equilibria in Subgame 2

Region F: Both firms choose full revelation (\( p_{Gj}^* = 1 \) and \( p_{Bj}^* = 0 \)).

3.3 Scenario 3: One firm is active and the other passive

In this scenario, only one firm provides a signal and sets \( p_B \) and \( p_G \) in the design stage, while the other remains quiet. For this result to be possible in equilibrium, it is required that \( E[u] \geq 0 \). Otherwise, the firm that does not provide a signal will have zero demand, and a subgame perfect equilibrium will not include this information design structure. Without loss of generality, we assume that firm 2 chooses to be passive, while firm 1 is active and sets \( p_{G1} \) and \( p_{B1} \) and provides information. The next lemma describes the resulting equilibrium price distributions when the firms compete in the pricing stage:

**Lemma 4.** There is a mixed strategy pricing equilibrium such that the passive firm will randomize prices between \([\Pr(b) \cdot E[u], E[u]]\) and the active firm will randomize prices between \([u(g) - \)
Lemma 4 shows that in the hybrid passive-active equilibrium both firms will mix their pricing within two separate price ranges. The active firm will charge higher prices on average and will sell to consumers who receive a good signal. The passive firm will charge lower prices on average and will sell to consumers who receive a bad signal from the active firm. Because the firms provide different levels of information to consumers and sell to consumers of different segments, the active firm can charge up to \( u(g) \), while the passive firm can charge up to \( E[u] \). In order to make sure that the passive firm does not poach too many customers from the active firm, the lowest price that the active firm charges is equal to the highest surplus the consumers can receive if they switch to buying from the passive firm. Overall, the mixing strategies are similar to the benchmark analyzed in Lemma 1, with asymmetry in prices due to the different levels of information provided to consumers. The proof of Lemma 4 in the Appendix fully characterizes the price distributions of firms 1 and 2.

Next, we use backward induction to find firm 1’s information design policy in the first stage. The profit maximization problem for firm 1 is:

\[
\max_{p_{G1}, p_{B1}} \pi_1(p_{G1}, p_{B1}) = \Pr(g) \cdot (u(g) - \Pr(g)E[u]) \\
= \left( (1 - \alpha)p_{B1} + \alpha p_{G1} \right) \left( \frac{(1 - \alpha)u_B p_{B1} + \alpha u_G p_{G1}}{(1 - \alpha)p_{B1} + \alpha p_{G1}} - ((1 - \alpha)p_{B1} + \alpha p_{G1}) (1 - \alpha)u_B + \alpha u_G \right)
\]

\[\text{s.t.} \quad 0 \leq p_{B1} \leq 1, 0 \leq p_{G1} \leq 1\]

The analysis of this constrained maximization problem yields the following result:

**Lemma 5.** When \( E[u] \geq 0 \) (which is required for a passive-active equilibrium), the active firm will select:

- **Overselling:** imperfect precision but perfect recall \((p_B > 0, p_G = 1)\) for low values of \( \alpha \) and high values of \( u_B \).
- **Full revelation:** perfect precision and recall \((p_B = 0, p_G = 1)\) for intermediate values of \( \alpha \).
- **Demarketing:** perfect precision but imperfect recall \((p_B = 0, p_G < 1)\) for high values of \( \alpha \).
The surprising result in Lemma 5 is that the active firm may choose to oversell \((p_B > 0)\) when \(\alpha\) is low, but also to demarket \((p_G < 1)\) when \(\alpha\) is high, such that it sometimes sends a bad signal when the product is actually a match and yields utility \(u_G\). The intuition can be gleaned from Lemma 4, and from figure 4(b) that illustrates the ranges of prices and average prices in equilibrium for different levels of \(\alpha\). When \(\alpha\) is very low, the active firm does not face strong competition from the passive firm, and both firms can increase their prices with \(\alpha\) without too much competition. As \(\alpha\) increases even more, the active firm now faces a passive firm that provides enough expected utility to the customers who receive a negative signal to pose stronger competition. This requires the active firm to both provide information to the customer and reveal the true match of the product, and also to lower prices in order to keep demand at a desired level. Finally, for high values of \(\alpha\), the competition is even stronger, and when the passive firm receives lower demand, it lowers prices and competes more aggressively. The active firm finds it optimal to respond by lowering \(p_G\) which gives more demand to the passive firm and softens price competition, which allows the active firm to raise its prices. Hence, when there is a high chance that the products in the market will match consumer preferences, an active firm might choose to demarket its products.

Figure 4 illustrates the range of parameters where an equilibrium exists in the subgame with one passive and one active firm. Regions P/F/D are where the active firm chooses \(p_B > 0\) (overselling), \(p_B = 0\) (full revelation) and \(p_G < 1\) (demarketing), respectively. Figure 4(b) shows the dynamics of price competition between the two firms when \(\alpha\) increases. As we can see, while firm 2’s average price always increases in \(\alpha\), firm 1’s average price is non-monotonic and will first increase but then decrease. The intuition is that an increase in \(\alpha\) causes the range of prices that the firms use to overlap, which results in stronger competition. The only firm that can respond through changing its recommendation algorithm’s information design is the active firm, and as a result, it would gradually transition from overselling to full revelation and eventually to demarketing, to avoid head-to-head competition.

3.4 Equilibrium selection

In scenario 1, the profits of both firms are zero. Hence we can focus on the equilibria in scenarios 2 and 3 only. A firms’ strategy in the first stage is to choose to be either passive (not provide information) or active (provide information) and set values for \(p_G\) and \(p_B\). We will call the outcome
Figure 4: Equilibria and Price Distributions in Subgame 3

(a) Parameter ranges for equilibria

(b) Price distributions when $u_B = 0.1$

(Left) Figure (a) shows the range where equilibria exist and the corresponding strategy of the active firm in each range. The active firm selects: (i) Overselling (Region P), (ii) Full revelation (Region F), or (iii) Demarketing (Region D). (Right) Figure (b) shows the maximum, minimum, and expected prices of both firms when $u_B = 0.1$. Solid black lines indicate firm 1 (active) and black dashed lines indicate firm 2 (passive).

in scenario 2 an active-active equilibrium (aa for short) while it will be called a passive-active (pa) in scenario 3. Using backward induction, we arrive at the following subgame perfect equilibrium (SPE):

**Proposition 3.** When $u_B < 0$, the active-active equilibrium is the subgame perfect equilibrium, and both firms choose full revelation with perfect precision and recall. When $u_B > 0$, for low values of $\alpha$ the hybrid passive-active equilibrium is the SPE. Specifically, the passive-active equilibrium shifts from overselling, through full revelation and ends with demarketing as $\alpha$ increases.

Figure 5 illustrates these results. The colored area contains the range of parameters where a subgame perfect equilibrium in pure information-design strategies exists. The boundary of the areas in color is determined by the region where profitable deviations do not exist in the second stage. When $u_B < 0$, region F^{aa} has an active-active equilibrium with full revelation. When $u_B > 0$, we observe the shift from overselling to full revelation to demarketing through regions P^{pa}, F^{pa} and D^{pa} as $\alpha$ increases.
To understand these results, we first note that when $E[u] < 0$, a passive firm will not make any profit, and hence a passive-active equilibrium is infeasible. When $E[u] > 0$ and $u_B < 0$, if a firm deviates from the active-active equilibrium, the profit is lower, hence an active-active equilibrium is the subgame perfect equilibrium. For $u_B > 0$, we see a more nuanced result. When $\alpha$ is small, having one firm as passive and one firm as active softens the competition between the firms, as they split the market such that the active firm sells to those who received a good signal, while the passive firm can sell to those who received a bad signal from the active firm. In that sense, a passive-active equilibrium enlarges the market. If both firms were active, they could only sell when they provide a good signal, which makes them compete more fiercely and lowers their profit. We also find that the active firm will transition from overselling towards full revelation, and even to demarketing as $\alpha$ increases. Essentially, for the active firm, information design creates a trade-off between demand enhancement and price competition. On the one hand, overselling increases sales. On the other hand, the passive firm who is facing a smaller demand is forced to price more aggressively in order to compete for market share. When $\alpha$ is small, the chance of a good product match is small so the demand enhancement effect dominates for the active firm, while as $\alpha$ gets larger, the price competition effect slowly becomes more substantial. In this case, the active firm will then turn to full revelation or even demarketing in order to avoid head to head price competition.
4 Algorithmic recommendation within a platform

We now analyze a shopping platform that intermediates the sales of products between the two competing sellers and buyers. Examples that match this scenario are Amazon and eBay that design what the product description or listing environment looks like for sellers, as well as Airbnb that designs what a host’s listing appears like. The platform cannot dictate the price each seller is going to charge, but can decide which recommendations will be displayed next to products. We explore whether the platform has an incentive to intervene in the sales process by deciding what information will be presented to customers and how.

We have two goals for the analysis. First, we investigate the optimal recommendation algorithm design for the platform. Second, we analyze the level of persuasion the platform chooses. Specifically, we ask: (1) Does the platform bias recommendations away from perfect precision and recall by not selecting full revelation, i.e., by selecting \((p_{Gj}, p_{Bj}) \neq (1, 0)\), and (2) does the platform persuade more or less compared to the Duopoly case in Section 3.

Similarly to the case of duopoly competition, when prices are endogenous the game has two stages. In the first stage, the platform selects a recommendation algorithm characterized by \(p_{Gj}\) and \(p_{Bj}\) for each seller.\(^{11}\) After observing the information design environment, firms then choose optimal prices \(m_j\). We assume that the platform obtains profit based on a fixed share of the total sales from both firms. We solve for the platform’s optimal recommendations strategy by using backward induction. We consider two types of scenarios for the platform: A design where the platform provides information for both sellers (which we call symmetric), and an asymmetric one, where the platform only provides information for one of the sellers. If the platform does not provide information for both of the sellers, we are in a similar scenario to that in Section 3.1, where both sellers (and the platform) will make zero profit.

The symmetric design applies to a case where the platform designs the recommendation environment, but does not strategically intervene in how information is presented to customers once it is created. The asymmetric design allows the platform to also favor one seller over the other (for example, by adding more information to their listings, or by only featuring the top seller), and this will result in one seller providing more information to consumers than the other.

\(^{11}\)The platform can also decide not to provide information, e.g., by setting \(p_{Gj} = p_{Bj}\).
4.1 Symmetric design: The platform provides information for both sellers

When we focus on separating equilibria where consumers only buy products after they receive a good signal, similarly to the previous results, the equilibrium prices in the second stage will be in mixed strategies given an information design. Therefore, plugging in \( u_j(g) = \frac{\alpha p_G u_G + (1-\alpha) p_B u_B}{\alpha p_G + (1-\alpha) p_B} \) the platform’s optimization problem will be:

\[
\max_{p_G, p_B} \pi_P = \pi_j + \pi_{-j} = \\
\sum_{j=1}^{2} \{ (\alpha (1-p_G) + (1-\alpha) (1-p_B)) (\alpha p_G u_G + (1-\alpha) p_B u_B) \} \\
\text{s.t. } 0 \leq p_B \leq 1, 0 \leq p_G \leq 1
\] (9)

The analysis of the solution to the platform’s optimization problem (9) yields the following results:

**Lemma 6.** When \( \alpha \in [0,0.5] \), the platform will choose perfect precision and recall and will fully reveal the product’s match. When \( \alpha \in (0.5,1] \), the platform will choose to demarket and set \( p_G < 1 \), \( p_B = 0 \), i.e., perfect precision but imperfect recall.

Figure 6 illustrates the range of parameters where equilibrium exists. Region F indicates full revelation while Region D indicates demarketing. The upper curve is the boundary that support an equilibrium in the second pricing stage.

The platform’s choice to demarket under a symmetric design is interesting. The intuition is that the platform wants to avoid head to head competition between the two sellers that will lower equilibrium prices resulting in lower platform revenue. By constraining both firms’ ability to reveal the product’s match when the product matches, it is essentially softening the price war, which has little effect on demand, but increases total revenue.

4.2 Asymmetric design: The platform provides recommendations only for one seller

We perform the analysis in two parts: (i) when \( \mathbb{E}[u] \geq 0 \) and (ii) when \( \mathbb{E}[u] < 0 \). When \( \mathbb{E}[u] \geq 0 \), we know from Lemma 4 that the profits of the two firms are \( \Pr(g) \ast (u(g) - \mathbb{E}[u]) \) and \( \Pr(b)\mathbb{E}[u] \),
The left figure presents the range of parameters $\alpha$ and $u_B$ where equilibria exist and their type (Full revelation or Demarketing), while the right figure shows the maximum, minimum and expected prices of both firms when $u_B = 0.1$.

respectively. We assume w.l.o.g that the platform does not provide recommendations for seller 2, while it sets $p_{G1}$ and $p_{B1}$ for seller 1.

In this case, the platform’s optimization problem becomes:

$$
\max_{p_{G1}, p_{B1}} \pi_P = \pi_1 + \pi_2 \\
= \Pr(b)\mathbb{E}[u] + \Pr(g) * (u(g) - \Pr(g)\mathbb{E}[u]) \\
= \left( (1 - \alpha)p_{B1} + \alpha p_{G1} \right) \left( \frac{(1 - \alpha)u_B p_{B1} + \alpha p_{G1}}{(1 - \alpha)p_{B1} + \alpha p_{G1}} - (\alpha + (1 - \alpha)u_B)((1 - \alpha)p_{B1} + \alpha p_{G1}) \right) \\
+ (\alpha + (1 - \alpha)u_B)((1 - \alpha)(1 - p_{B1}) + \alpha (1 - p_{G1})) \\
s.t. \quad 0 \leq p_{B1} \leq 1, 0 \leq p_{G1} \leq 1
$$

Next, we consider the case where $\mathbb{E}[u] < 0$. In the second stage, firm 2 will price at zero, earn zero profit, and no consumer will buy from firm 2 since $\mathbb{E}[u] < 0$. In a separating equilibrium, the consumer would buy from firm 1 if $u(g) - m_1 \geq 0$. As a result, firm 1 can extract the maximum
revenue by charging a price $m_1$ equal to $u(g)$. The platform’s optimization problem now becomes:

$$\max_{p_{G1},p_{B1}} \pi_P = \pi_1 + \pi_2$$

$$= 0 + \Pr(g) * u(g)$$

$$= ((1 - \alpha)p_{B1} + \alpha p_{G1}) \left( \frac{(1 - \alpha)u_B p_{B1} + \alpha p_{G1}}{(1 - \alpha)p_{B1} + \alpha p_{G1}} \right)$$

$$\text{s.t. } 0 \leq p_{B1} \leq 1, 0 \leq p_{G1} \leq 1$$

The platform will choose full revelation and set $p_{B1}^* = 0$ and $p_{G1}^* = 1$ since when $\mathbb{E}[u] < 0$, the platform’s profit is monotonically increasing in $p_{G1}$ and decreasing in $p_{B1}$. Combining the results of the analysis from these two cases, we find the following:

**Lemma 7.** When the platform sets an asymmetric information design, if $\mathbb{E}[u] > 0$, it will select full revelation with perfect precision and recall when $\alpha$ is small, and demarket when $\alpha$ is large. If $\mathbb{E}[u] < 0$, it will select full revelation.

Figure 7 illustrates these results. Region F and F’ are where the platform will pick full revelation, while Region D indicates where demarketing will be used. The upper curve is the condition required to support an equilibrium in the second stage. The intuition behind the results is similar to the case with a symmetric information design—when $\alpha$ is high, the platform would like the firms to avoid price competition. By using demarketing, the platform restricts the ability of the stronger firm to compete over customers by lowering its prices and increasing consumer surplus.

### 4.3 The optimal algorithm design of the platform

Having analyzed the two possible strategies of the platform, we now shift to finding the equilibrium design. As mentioned, we can rule out the case in which the platform provides no recommendations for both sellers, since then the revenue of both sellers and then the platform is zero due to convergence to Bertrand competition. The next proposition fully characterizes the equilibrium choice of the platform.

**Proposition 4.** The optimal recommendation algorithm design for the platform is:

- For any $u_B < 0$, there are three cutoff points $\alpha_1(u_B) < \alpha_2(u_B) < \alpha_3(u_B)$ and the platform selects:
The left figure shows the range of parameters and corresponding strategy that the platform chooses for the seller that provides information. The platform selects: (i) Full revelation (Regions F and F'), or (ii) Demarketing (Region D). In Region F, $\mathbb{E}[u] > 0$. In Region F', $\mathbb{E}[u] < 0$ and firm 2 does not sell, in contrast to region F. The right figure shows the maximum, minimum, and expected prices of both firms when $u_B = 0.1$. The solid lines characterize prices for firm 1 which provides information and the dashed lines characterize prices for firm 2.

- **Full revelation with symmetric design when** $\alpha < \alpha_1$, or asymmetric design when $\alpha_1 \leq \alpha < \alpha_2$. In both cases precision and recall are perfect.

- **Demarketing with asymmetric design when** $\alpha_2 \leq \alpha < \alpha_3$, in which case precision is perfect and recall is imperfect.

- For any $0 < u_B < \bar{u}_B$,\(^{12}\) there are two cutoff points $\alpha_2(u_B) < \alpha_3(u_B)$ and the platform selects:

  - **Full revelation with asymmetric design when** $\alpha < \alpha_2$.
  
  - **Demarketing with asymmetric design when** $\alpha_2 \leq \alpha < \alpha_3$.

Figure 8 illustrates these results ("F" represents Full Revelation, "D" represents Demarketing; "s" represents a symmetric design and "a" stands for asymmetric.). In Region F^s (F^a), the platform chooses full revelation for both (one of the) sellers. In Region D^a, the platform chooses demarketing for one of the sellers.

\(^{12}\)The expressions for $\bar{u}_B$ appear in the proof.
The results not only echo our previous findings, but also add new insights about the impact of the platform that chooses the recommendation algorithm. Similarly to before, when $\alpha$ increases, the platform moves away from full revelation to demarketing, in order to deescalate price competition between the two firms. The additional insight is that centralizing the information design at the platform level allows it to select the equilibrium that the sellers will play in the pricing game. This allows the platform to make sure the sellers do not compete too strongly, even in parameter ranges where they would have deviated under duopoly competition. We see that overselling is never an equilibrium strategy when a platform intermediates sales, while it might be possible in duopoly competition. The reason is that a platform can serve as a commitment mechanism for sellers not to oversell and increase demand, which allows them to compete less and increase their prices. This effect is beneficial to both the platform and the sellers.

4.4 The impact of platforms on consumer experience

Much of the online interaction of consumers with sellers has shifted towards platforms in the past few years. One interesting question is how this shift affects the user experience given the different recommendation incentives between platforms and duopolies.

The results of the previous sections show that overselling will be more prevalent in duopolistic
markets not operating through platforms, and in that sense consumers benefit from the consolidation in the market. However, demarketing might also become more prevalent when shifting to platforms, meaning that consumers might experience less exposure to higher quality items.

We can further analyze the consumer experience by comparing the realized variance and utility of consumed items under a platform structure and under a duopolistic structure. We summarize the results as follows:

**Proposition 5.** When \( \alpha < \bar{\alpha} \approx 0.273 \) and \( \frac{2\alpha^2}{2\alpha^2 - 2\alpha + 1} < u_B < \frac{(1-\alpha)\alpha}{\alpha - \alpha^2 + 1} \), the realized expected utility of consumed items is higher, and the realized variance is lower under platform competition, compared with duopoly competition.

Proposition 5 shows that when \( \alpha \) is small and when \( u_B \) is in an intermediate range, then the platform provides consumers with higher expected utilities and less realized match variance than competing duopolists. The intuition is that in this range the platform provides full revelation and perfect match to consumers, while duopolists are in a hybrid equilibrium, which allows them to commit to higher prices, but not better recommendations.

A second interesting implication is evident in Figure 9. When \( \alpha < 0.273 \) and \( \frac{2\alpha^2}{2\alpha^2 - 2\alpha + 1} < u_B < \frac{(1-\alpha)\alpha}{\alpha - \alpha^2 + 1} \) (orange horizontal lines), the variance of realized utility is lower in platform competition. In the regions with blue vertical lines, the variances are the same. For the rest of the parameter ranges, the variance is larger with platform competition.

Higher realized variance means that consumers experience more diverse content and products in terms of their fit. Potentially, this might be bad for consumers if fit implies quality. However, this might also be a good outcome if, for example, higher diversity of content implies exposure to new ideas and a lower chance of creating echo-chambers. In that sense, consolidating content to a platform and allowing the platform to design a recommendation algorithm might lower polarization and increase the diversity of content consumed, echoing results in Berman and Katona (2020).

5 Conclusion

It is often true that providing precise recommendations to customers about a product can help them make better decisions and make a marketer more successful. This fact, however, raises a dilemma for marketers—should they provide fully revealing recommendations about a product’s
match, or should they be more strategic about the design of recommendation algorithms?

In other settings, such as sales and advertising, marketers are often tempted to try to oversell the product and persuade consumers to purchase, assuming that they are benefiting the firm or the platform in the process. The recent literature on Bayesian persuasion has provided a formal argument underpinning this decision—the results show that firms can increase their demand by credibly persuading consumers to purchase products even if the products is not a good fit to consumers’ preferences.

Our analysis revealed that when designing recommendation algorithms, biasing the recommendations towards overselling is not necessarily the best approach, and full revelation of the match (or even lowering recall through demarketing) are often preferred. The results show that when pricing and competition incentives are added to the mix, firms who choose to bias their algorithms towards overselling often lose profits because persuaded consumers are either willing to pay less (monopoly), or switch more easily to competitors (duopoly). These results also apply to a platform that designs a central recommendation algorithm for sellers on the platform, since overselling increases competition and lowers the platform’s revenue. The ability to change the algorithm’s design allows the platform to make the firms appear to be more differentiated, allowing them to charge higher prices and making overselling unnecessary. Often this will result in an asymmetric equilibrium where one
firm provides information and the other does not. An interesting implication of this result is that platforms may want to restrict how many sellers they recommend to consumers.

In extreme cases, when competing products are expected to match consumer tastes with high a priori probability, firms can further differentiate by demarketing, or “hiding” information about good products, because this approach increases perceived differentiation by consumers. The result is decreased price competition even further beyond full revelation. This finding provides a possible explanation to a recent trend where a few well-known brands have made it harder to recognize that their products are part of the brand’s family, which may cause a consumer to confuse the product with an inferior brand. However, confirming that observed recommendations stem from demarketing might not be easy. In the example in Figure 2, “The Town” had a 60% match to one of the authors tastes, but the author very enjoyed watching the movie. It is hard to say whether this mismatch between recommendation and outcome is because of a noisy prediction of the algorithm, or demarketing. Identifying conditions that will separate demarketing from noise is an interesting direction for future research.

The analysis we performed is limited to the fundamental cases of monopoly, duopoly and a platform to maintain parsimony, but there are additional avenues to explore the limits of algorithmic bias in recommendations. For example, we did not consider the implications of heterogeneity in information design, e.g., consumers with different tastes. The analysis of heterogeneity provides a fruitful avenue for future research, but remains an open question due to its technical complexity, even in simple models. Another promising avenue for future research is targeted information design where firms can personalize the algorithm that provides recommendations to customers based on their tastes.

Our analytical results have implications to empirical research and can help explain some of the observed choices of firms online in utilization of recommendation algorithms. The results predict that overselling will be less common with competition or when firms have pricing power. The fact that pricing power reduces the incentive to bias information provision is a general result that would apply to other contexts beyond recommendation algorithms. We also help to explain the choice of platforms and online sellers in how they disclose product information, and how they design search experiences (Dukes and Liu 2016). Because recommendation algorithms can be biased strategically to alter the level of competition in a market or on a platform, firms should realize that they
might not want to use persuasive techniques like their competitors, and sometimes even refrain from providing any information. In addition, platforms might create implicit differentiation by not featuring all sellers, or by demarketing the products of some of them.

An interesting aspect of our findings is their implications for how platforms and regulators should respond to persuasive information which might have negative societal consequences (e.g., fake reviews, misleading native advertising, influencer marketing). For example, when considering the level of enforcement against fake reviews (He et al. 2020) platforms might realize that allowing persuasive fake reviews might lower their revenues. Similarly, regulators have mandated that social media influencers disclose financial relationships with the sponsoring brands to discourage misleading consumers. Our model predicts that the level of disclosure should affect the persuasiveness of the messages used by influencers, but will also affect prices in this market. These effects should be taken into account carefully when considering legislation or other disclosure rules.

References


He, S., B. Hollenbeck, and D. Proserpio (2020). The market for fake reviews. Available at SSRN 3664992.


**Appendix**

*Proof of Proposition 1.*

When $\alpha \geq \frac{m-u_B}{u_G-u_B}$, $E[u] \geq 0$, and since the price is exogenous, the firm does not need to provide information and profits will be maximized. When $\alpha < \frac{m-u_B}{u_G-u_B}$, there will not be consumption without additional information since $E[u] < 0$. Hence, a sufficient condition for a separating equilibrium to exist is an information design where $u(g) \geq 0$. Notice the condition on $\alpha$ only holds if $u_B < m$. The revenue of the firm increases in $p_G$, hence it is optimal to set $p_G^* = 1$. As the expected utility of the consumer $u(g)$ increases in $p_G$ and $u(b)$ decreases in $p_G$, setting $p_G = 1$ will also not violate the IC constraints.
The incentive compatibility constraints can be written as:

\[ p_B \leq \frac{\alpha(u_G - m)}{(1 - \alpha)(m - u_B)} \quad (12) \]

\[ (1 - \alpha)(1 - p_B)(u_B - m) \leq 0 \quad (13) \]

The second condition holds for any \( u_B < m \), and hence in equilibrium, only the first constraint might be binding. The unconstrained problem yields a solution \( p_B = 1 \), hence in equilibrium the IC constraint will bind and the firm will set \( p_B^* = \frac{\alpha(u_G - m)}{(1 - \alpha)(m - u_B)} \). Finally, we note that for every \( u_B < m < u_G \) and \( \alpha < \frac{m - u_B}{u_G - u_B} \), \( 0 < p_B^* < 1 \).

Proof of Proposition 2.

The revenue increases in \( p_G \), leading to \( p_G^* = 1 \). When \( u_B > 0 \), the profit from setting \( 1 > p_B > 0 \) is \( (1 - \alpha)p_Bu_B \), but without providing information the profit \( (1 - \alpha)u_B \) is higher or equal. Hence, the firm will not provide information when \( u_B > 0 \).

When \( u_B < 0 \), the revenue decreases in \( p_B \). Consequently the firm will set \( p_B \) as low as possible at \( p_B^* = 0 \). The profit in this case is higher than the expected profit without a signal.

Proof of Lemma 1.

Since only the consumer who obtains a good signal will buy, all firms will set prices above \( u_j(b) \). Then the profit of a firm setting price \( m_j \) when the competitor sets price \( m_{-j} \) is:

\[ \pi_j(m_j, m_{-j}) = (\alpha(1 - \alpha) + \alpha^2 \mathbb{I}[m_j < m_{-j}]) m_j \quad (14) \]

where \( \mathbb{I}[:] \) is the indicator function.

Because in a mixed strategy the firm should be indifferent between setting different prices, \( m_{\min} \) will satisfy:

\[ \pi_j(m_{\min}, m_{-j}) = (\alpha^2 + \alpha(1 - \alpha)) m_{\min} = \alpha(1 - \alpha)u_j(g) \quad (15) \]

resulting in \( m_{\min} = (1 - \alpha)u_G \). The lowest price should be higher than \( u_B \) (otherwise some consumers may buy with a bad signal), which means we need to restrict \( u_j(b) < (1 - \alpha)u_j(g) \). This implies \( u_B < (1 - \alpha)u_G \).

Finally, if we denote by \( F(\cdot) \) the cdf of the equilibrium prices in \([m_{\min}, m_{\max}]\), then we can solve for \( F(\cdot) \) in:

\[ \pi_j(m) = (\alpha^2(1 - F(m)) + \alpha(1 - \alpha)) m = \alpha(1 - \alpha)u_j(g) \quad (16) \]

resulting in the equilibrium mixed strategy price distribution \( F(m) = \frac{m-(1-\alpha)u_j(g)}{\alpha m} = \frac{m-(1-\alpha)u_G}{\alpha m} \) given that \( p_G = 1 \) and \( p_B = 0 \).

A final step in the analysis is to verify that no firm has a profitable deviation. In our case, a firm could charge a price lower than \( u_j(b) = u_B \), sell products to consumers who received a bad signal, and potentially make higher profit.
If a firm sets a price lower than \( u_B \), when the competitor mixes her pricing according to \( F(m) \), the deviating firm’s expected profit will be:

\[
\pi_j(m) = (\alpha^2 + \alpha(1 - \alpha) + (1 - \alpha)^2 + (1 - \alpha)\alpha(1 - F(u_j(g) - u_j(b) + m))) \ m
\]  

(17)

The second derivative of the profit with respect to price is \( \frac{2(1-\alpha)^2u_j(g)(u_j(b)-u_j(g))}{(p-u_j(b)+u_j(g))^3} \), which is negative for any \( m \leq u_j(b) \). Hence the function is concave. The first derivative at \( m = u_j(b) \) is \( 1 - \alpha(1-\alpha) - \frac{(1-\alpha)^2u_j(b)}{u_j(g)} > 1 - \alpha(1-\alpha) - (1-\alpha)^2 > 0 \). As a result, a deviating firm can maximize its profit by setting the price to \( u_j(b) \), yielding a profit

\[
(\alpha^2 + \alpha(1 - \alpha) + (1 - \alpha)^2)u_j(b)
\]

If this deviation profit is lower than the equilibrium profit of \( \alpha(1-\alpha)u_j(g) \), no firm will want to deviate. Hence, we require that \( u_j(b) = u_B < \min\left((1-\alpha)u_j(g), \frac{\alpha(1-\alpha)}{1-\alpha+\alpha^2}u_j(g)\right) = \frac{\alpha(1-\alpha)}{1-\alpha+\alpha^2}u_j(g) \)

when \( u_j(g) = u_G = 1 \) and \( 0 < \alpha < 1 \) to maintain an equilibrium where consumers do not buy with a bad signal.

Proof of Lemma 2.

We note that \( \frac{\partial \pi_j}{\partial p_{Gj}} = \alpha ((1-\alpha)(1-p_{B-j}) + \alpha(1-p_{G-j})) \) is larger than zero for all \( p_{Gj}, p_{G-j}, p_{Bj}, p_{B-j} \in [0, 1] \). Hence, it is maximized at the corner where \( p_{Gj} = 1 \).

Proof of Lemma 3.

Lemma 2 shows that both firms will set \( p_G^* = 1 \) in any separating equilibrium. When \( u_B \leq 0 \), Firm’s \( j \)'s payoff has the FOC

\[
\frac{\partial \pi_j}{\partial p_{Bj}} \bigg|_{p_{Gj}=1,p_{G-j}=1} = u_B(1-\alpha)^2(1-p_{B-j}) \leq 0, \forall p_{Bj}, p_{B-j} \in [0, 1]
\]

implying that the firms will set \( p_{Bj}^* = 0 \) and \( p_{Gj}^* = 1 \). Since \( u_B < 0 \), the condition \( u_B < \frac{\alpha(1-\alpha)}{\alpha^2-\alpha+1}u_G \)

always holds and ensures that no firm will deviate in the second stage to a lower price given \( p_{Gj} \) and \( p_{Bj} \). When \( u_B > 0 \) we have

\[
\frac{\partial \pi_j}{\partial p_{Bj}} \bigg|_{p_{Gj}=1,p_{G-j}=1} = u_B(1-\alpha)^2(1-p_{B-j}) \geq 0.
\]

Both firms would like to set \( p_{Bj} = 1 \) which is equivalent to the case of not providing information, leading to Bertrand duopoly competition.

Proof of Lemma 4.

We assume a separating equilibrium where firm 1 does not price below \( u(b) \) to make consumers buy with a low signal, and will verify that this condition holds in the equilibrium candidate we find through backward induction. First, suppose both firms use mixed strategies without a mass point. Firm 2 mixes between \([m_{2\min}, \mathbb{E}[u]]\). When firm 2 charges \( \mathbb{E}[u] \), it can only sell to consumers who received a bad signal from firm 1. Its profit is \( \Pr(b) \mathbb{E}[u] \). When firm 2 charges \( m_{2\min} \), its demand

Electronic copy available at: https://ssrn.com/abstract=4301489
will be the entire market, implying that \( m_{2\min} = \Pr(b) \cdot \mathbb{E}[u] \). For prices \( m_2 \in [\Pr(b) \cdot \mathbb{E}[u], \mathbb{E}[u]] \), the indifference condition for firm 2 implies:

\[
\pi_2(m_2) = m_2 \cdot [\Pr(b) + \Pr(g)(1 - F_1(u(g) - \mathbb{E}[u] + m_2))] = \Pr(b) \cdot \mathbb{E}[u] \tag{18}
\]

where \( F_1(\cdot) \) is the cdf of firm 1’s prices. In order for this equilibrium to hold, it is required that \( \mathbb{E}[u] \geq 0 \). Otherwise, the profit of firm 2 is negative.

Moving to firm 1, it will not charge prices lower than \( u(g) - \Pr(g)\mathbb{E}[u] \), because we assume firm 1 will sell only to consumers who received a good signal. Since firm 2 is mixing between \([\Pr(b) \cdot \mathbb{E}[u], \mathbb{E}[u]]\), the maximum surplus firm 2 can give a consumer is \( \mathbb{E}[u] - \Pr(b) \cdot \mathbb{E}[u] = \Pr(g) \cdot \mathbb{E}[u] \). Hence, when firm 1 charges \( u(g) - (\mathbb{E}[u] - \Pr(g)\mathbb{E}[u]) = u(g) - \Pr(g)\mathbb{E}[u] \), it provides a surplus of \( \Pr(g) \cdot \mathbb{E}[u] \) to consumers who received a good signal, and it will obtain demand from all these consumers since firm 2 has no mass point at the lowest price \( \Pr(b)\mathbb{E}[u] \). The profit for firm 1 is \( \Pr(g) \cdot (u(g) - \Pr(g)\mathbb{E}[u]) \) at this point. For prices \( u(g) - \Pr(g)\mathbb{E}[u] \leq m_1 \leq u(g) \), firm 1 is facing a trade-off between setting a higher price and losing a proportion of the \( \Pr(g) \) potential consumers. The indifference condition for firm 1 implies that:

\[
\pi_1(m_1) = m_1 \cdot \Pr(g) \cdot (1 - F_2(m_1 - (u(g) - \mathbb{E}[u]))) = \Pr(g) \cdot (u(g) - \Pr(g)\mathbb{E}[u]) \tag{19}
\]

where \( F_2(\cdot) \) is the CDF of firm 2’s prices.

Solving Equation (18) yields:

\[
F_1(m_1) = \begin{cases} 
0 & m_1 \leq m_{2\min} + (u(g) - \mathbb{E}[u]) = u(g) - \Pr(g)\mathbb{E}[u] \\
1 + \frac{\Pr(b) \cdot \mathbb{E}[u]}{\Pr(g)(m_1 - (u(g) - \mathbb{E}[u]))} & u(g) - \Pr(g)\mathbb{E}[u] \leq m_1 \leq u(g) \\
1 & m_1 \geq u(g)
\end{cases}
\]

(20)

Plugging in \( m_1 = u(g) - \Pr(g)\mathbb{E}[u] \) results in

\[
F_1(u(g) - \Pr(g)\mathbb{E}[u]) = 1 + \frac{\Pr(b)}{\Pr(g)} - \frac{\Pr(b)\mathbb{E}[u]}{\Pr(g)(u(g) - \Pr(g)\mathbb{E}[u] - (u(g) - \mathbb{E}[u]))} = 1 + \frac{\Pr(b)}{\Pr(g)} - \frac{\Pr(b)\mathbb{E}[u]}{\Pr(g) \cdot \Pr(b) \cdot \mathbb{E}[u]} = 0
\]

Plugging in \( m_1 = u(g) \) results in

\[
F_1(u(g)) = 1 + \frac{\Pr(b)}{\Pr(g)} - \frac{\Pr(b)\mathbb{E}[u]}{\Pr(g)(u(g) - (u(g) - \mathbb{E}[u]))} = 1
\]

This suggests that firm 1 has no mass point in \( F_1(\cdot) \) as assumed. The expectation of firm 1’s price
\[ \mathbb{E}(m_1) = \int_{u(g) - \Pr(g)\mathbb{E}[u]}^{u(g)} f_1(m_1) * m_1 \, dm_1 = \int_{u(g) - \Pr(g)\mathbb{E}[u]}^{u(g)} \frac{\Pr(b)\mathbb{E}[u]}{\Pr(g)(m_1 - (u(g) - \mathbb{E}[u]))^2} * m_1 \, dm_1 \] (21)

Moving to firm 2, the solution to Equation (19) yields:

\[
F_2(m_2) = \begin{cases} 
0 & m_2 \leq \Pr(b) * \mathbb{E}[u] \\
1 - \frac{u(g) - \Pr(g)\mathbb{E}[u]}{(m_2 + (u(g) - \mathbb{E}[u]))} & \Pr(b) * \mathbb{E}[u] \leq m_2 \leq \mathbb{E}[u] \\
1 & m_2 \geq \mathbb{E}[u]
\end{cases}
\] (22)

When plugging \( m_2 = \mathbb{E}[u] \) we note that \( F_2(\mathbb{E}[u]) = 1 - \frac{u(g) - \Pr(g)\mathbb{E}[u]}{(\mathbb{E}[u] + (u(g) - \mathbb{E}[u]))} = \frac{\Pr(g)\mathbb{E}[u]}{u(g)} < 1 \). This implies that firm 2 has a mass point at the highest price \( \mathbb{E}[u] \) with a probability \( 1 - \frac{\Pr(g)\mathbb{E}[u]}{u(g)} \). Since firm 2 has a mass point at the highest price \( \mathbb{E}[u] \) and firm 1 has no mass point, equation (19) remains unchanged. The expectation of firm 2’s price \( \mathbb{E}(m_2) \) is

\[
\mathbb{E}(m_2) = \int_{\Pr(b)\mathbb{E}[u]}^{\mathbb{E}[u]} f_2(m_2) * m_2 \, dm_2 + \mathbb{E}[u] * \left(1 - \frac{\Pr(g)\mathbb{E}[u]}{u(g)}\right)
\]

\[
= \int_{\Pr(b)\mathbb{E}[u]}^{\mathbb{E}[u]} \frac{u(g) - \Pr(g)\mathbb{E}[u]}{(m_2 + (u(g) - \mathbb{E}[u]))^2} * m_2 \, dm_2 + \mathbb{E}[u] * \left(1 - \frac{\Pr(g)\mathbb{E}[u]}{u(g)}\right)
\] (23)

**Proof of Lemma 5.**

Plugging in \( u_G = 1 \), the Lagrangian of firm 1’s maximization problem becomes:

\[
\mathcal{L}(p_{G1}, p_{B1}) = ((1 - \alpha)p_{B1} + \alpha p_{G1}) \left(\frac{(1 - \alpha)p_{B1}u_B + \alpha p_{G1}}{(1 - \alpha)p_{B1} + \alpha p_{G1}} - (1 - \alpha)p_{B1} + \alpha p_{G1}\right) ((1 - \alpha)u_B + \alpha)
\]

\[
- \lambda_1(p_{G1} - 1) - \lambda_2(p_{B1} - 1) - \lambda_3(-p_{G1}) - \lambda_4(-p_{B1})
\]

s.t. \( 0 \leq p_{Bj} \leq 1, 0 \leq p_{Gj} \leq 1 \)

The Kuhn-Tucker (KKT) conditions for firm 1’s maximization problem are:

\[
\frac{\partial \mathcal{L}}{\partial p_{G1}} = \alpha - 2(\alpha - 1)\alpha p_{B1} ((\alpha - 1)u_B - \alpha) + p_{G1} (-2\alpha^3 + 2(\alpha - 1)\alpha^2 u_B) - \lambda_1 + \lambda_3 = 0
\] (25)

\[
\frac{\partial \mathcal{L}}{\partial p_{B1}} = p_{B1} (-2\alpha(\alpha - 1)^2 + 2(\alpha - 1)^3 u_B) - (\alpha - 1) (2\alpha p_{G1} ((\alpha - 1)u_B - \alpha) + u_B) - \lambda_2 + \lambda_4 = 0
\]

\[
\lambda_i \geq 0, 0 \leq p_{G1} \leq 1, 0 \leq p_{B1} \leq 1, \text{ and,}
\]

\[
\lambda_1(p_{G1} - 1) = 0, \lambda_2(p_{B1} - 1) = 0, \lambda_3(-p_{G1}) = 0, \lambda_4(-p_{B1}) = 0
\]

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Solving the KKT conditions yields the following solutions. The constraints on \( u_B \) and the ranges for \( \alpha \) are detailed in Web Appendix WA.1:

1. \( p_{B1}^* = \frac{(2\alpha^2 - 2\alpha + 1)u_B - 2\alpha^2}{2(\alpha - 1)(1 - \alpha)u_B} \) > 0 and \( p_{G1}^* = 1 \) (Case 7 of the proof).

2. \( p_{B1}^* = 0 \) and \( p_{G1}^* = 1 \) (Case 8 of the proof).

3. \( p_{B1}^* = 0 \) and \( p_{G1}^* = \frac{1}{2\alpha - (\alpha - 1)u_B} \) \( \in (0, 1) \) (Case 9 of the proof).

\[ \square \]

**Proof of Proposition 3.**

We first consider the case where \( E[u] \leq 0 \), and define the following notation for payoffs:

<table>
<thead>
<tr>
<th>passive</th>
<th>active</th>
</tr>
</thead>
<tbody>
<tr>
<td>passive</td>
<td>( 0,0 )</td>
</tr>
<tr>
<td>active</td>
<td>( \pi_1^M((p_{G1}^M, p_{B1}^M), u_1(g)), (\emptyset, 0) )</td>
</tr>
</tbody>
</table>

First, (passive, active) cannot be a SPE because firm 1 can profitably deviate by setting \( p_{G1} = 1 \), \( p_{B1} = 0 \) and \( m > 0 \), which yields a non-negative profit. Second, suppose (active, active) is a SPE; we verify that firm 1 (similarly for firm 2 because of symmetry) does not have an incentive to deviate using the following conditions:

1. Firm 1 does will not deviate to other values of \( p_{G1}^* \) and \( p_{B1}^* \) because the current values are best replies in a Nash equilibrium.

2. Firm 1 will not have the incentive to deviate to not providing any signal with a deviation profit of \( \pi_1((\emptyset, F_1^1(m)), ((p_{G2}^D, p_{B2}^D), F_2^1(m))) = 0 \) since \( E[u] \leq 0 \) while \( \pi_1^D(((p_{G2}^D, p_{B2}^D), F_1^1(m)), ((p_{G1}^D, p_{B1}^D), F_2^1(m))) > 0 \). Hence this condition also holds.

To conclude, when \( E[u] \leq 0 \), (active, active) is a SPE.

Next, we consider the case where \( E[u] > 0 \), and the payoffs for the firms are:

<table>
<thead>
<tr>
<th>passive</th>
<th>active</th>
</tr>
</thead>
<tbody>
<tr>
<td>passive</td>
<td>( 0,0 )</td>
</tr>
<tr>
<td>active</td>
<td>( \pi_1^D(((p_{G1}^D, p_{B1}^D), F_1^1(m)), (\emptyset, F_2^1(m))), \pi_2^D((p_{G1}^D, p_{B1}^D), F_1^1(m)), (\emptyset, F_2^1(m))) )</td>
</tr>
</tbody>
</table>
1. Suppose (active, active) is a SPE, we will verify the following no-deviation conditions for firm 1 (which apply for firm 2 because of symmetry):

   (a) Firm 1 will not have an incentive to deviate to setting other values of \( p'_{G_1} \) and \( p'_{B_1} \), with a deviation profit of \( \pi_1^D (((p'_{G_1}, p'_{B_1}), F'_1(m)), ((p^D_{G}, p^D_{B}), F'_2(m))) \)

   \[ = (\alpha p'_{G_1} + (1 - \alpha)p'_{B_1}) (\alpha (1 - p^D_{G}) + (1 - \alpha) (1 - p^D_{B})) u_1(g)' \]

   (b) Firm 1 will not deviate to not providing any signal with a deviation profit of:

   \[ \pi_1^D ((\emptyset, F'_1(m)), ((p^D_{G}, p^D_{B}), F'_2(m))) = (\alpha (1 - p^D_{G}) + (1 - \alpha) (1 - p^D_{B})) E[u] \]

2. If (passive, active) is a SPE, we need to verify the no-deviation conditions for both firms:

   (a) Firm 1 might deviate to providing a signal with deviation profit:

   \[ \pi_1^D (((p'_{G_1}, p'_{B_1}), F'_1(m)), ((p^D_{G_2}, p^D_{B_2}), F'_2(m))) \]

   \[ = (\alpha p'_{G_1} + (1 - \alpha)p'_{B_1}) (\alpha (1 - p^D_{G_2}) + (1 - \alpha) (1 - p^D_{B_2})) u_1(g)' \]

   (b) Firm 2 might deviate to setting other values of \( p'_{G_2} \) and \( p'_{B_2} \) with profit

   \[ \pi_2^D ((\emptyset, F'_1(m)), ((p^D_{G_2}, p^D_{B_2}), F'_2(m))) > \pi_2^D ((\emptyset, F'_1(m)), ((p'_{G_2}, p'_{B_2}), F'_2(m))) \]

We first consider the case where \( E[u] > 0 \) and \( u_B < 0 \):

1. (active, active) is a SPE, since condition (1a) holds by the properties of the Nash equilibrium. For condition (1b), the equilibrium profit is \( \alpha (1 - \alpha) \), while the deviation profit when not providing a signal is \((1 - \alpha)(\alpha + (1 - \alpha)u_B)\). When \( u_B < 0 \), \((\alpha + (1 - \alpha)u_B) < \alpha \), and deviating is not profitable.

2. (passive, active) is not a SPE. We show that condition (2b) does not hold. Firm 1’s profit in a (passive, active) equilibrium is \((\alpha (1 - p^D_{G_2}) + (1 - \alpha) (1 - p^D_{B_2})) (\alpha + (1 - \alpha)u_B)\). Deviating to provide signals \( p'_{G_1} \) and \( p'_{B_1} \) yields profit

   \[ (\alpha p'_{G_1} + (1 - \alpha)p'_{B_1}) (\alpha (1 - p^D_{G_2}) + (1 - \alpha) (1 - p^D_{B_2})) u_1(g)' \]

   The deviation profit is monotonically increasing in \( p'_{G_1} \), yielding \( p'_{G_1} = 1 \), and is monotonically decreasing in \( p'_{B_1} \) when \( u_B < 0 \), yielding \( p'_{B_1} = 0 \). The corresponding profit is \( \alpha (\alpha (1 - p^D_{G_2}) + (1 - \alpha) (1 - p^D_{B_2})) \) which is higher than the (passive, active) equilibrium profit.

Combining these results, we conclude that when \( E[u] > 0 \) and \( u_B < 0 \), (active, active) is the unique SPE.

Next, we consider the case where \( u_B > 0 \):

1. (passive, active) is a SPE: Condition (2b) holds because of the properties of the Nash equilibrium. For condition (2a), the (passive, active) equilibrium profit is
The solutions to the KKT conditions and the constraints on $u_B$ yield:

$$(\alpha (1-p'_{G2}) + (1-\alpha)(1-p'_{B2})) (\alpha + (1-\alpha)u_B)$$

Similarly to before, the deviation profit is monotonically increasing in $p'_{G1}$, and the firm will set $p'_{G1} = 1$ if it deviates. When $u_B > 0$, firm 1 will maximize its deviation profit by setting $p'_{B1} = 1$. For these values of $p'_{G1}$ and $p'_{B1}$, the deviation profit is the same as the (passive, active) equilibrium profit, so firm 1 does not have the incentive to deviate. Hence, (passive, active) is a SPE.

2. (active, active) is not SPE.

We conclude that when $u_B > 0$, (passive, active) is a SPE. \[ \square \]

**Proof of Lemma 6.**

The Lagrangian of the platform’s payoff is:

$$L(p_{G1}, p_{B1}) = ((1-\alpha)(1-p_{B2}) + \alpha (1-p_{G2}))((1-\alpha)u_B p_{B1} + \alpha u_G p_{G1})$$
$$+ ((1-\alpha)(1-p_{B1}) + \alpha (1-p_{G1}))((1-\alpha)u_B p_{B2} + \alpha u_G p_{G2})$$
$$- \lambda_1(p_{G1} - 1) - \lambda_2(p_{G2} - 1) - \lambda_3(p_{B1} - 1) - \lambda_4(p_{B2} - 1)$$
$$- \lambda_5(-p_{G1}) - \lambda_6(-p_{G2}) - \lambda_7(-p_{B1}) - \lambda_8(-p_{B2})$$

where $\lambda_i \geq 0, p_{G1} \leq 1, p_{G2} \leq 1, p_{B1} \leq 1, p_{B2} \leq 1, -p_{G1} \leq 0, -p_{G2} \leq 0, -p_{B1} \leq 0, -p_{B2} \leq 0$ and,  

$$\lambda_1(p_{G1} - 1) = 0, \lambda_2(p_{G2} - 1) = 0, \lambda_3(p_{B1} - 1) = 0, \lambda_4(p_{B2} - 1) = 0, \lambda_5(-p_{G1}) = 0, \lambda_6(-p_{G2}) = 0, \lambda_7(-p_{B1}) = 0, \lambda_8(-p_{B2}) = 0$$

The solutions to the KKT conditions and the constraints on $u_B$ appear in Web Appendix WA.2, and yield:

1. $p'_{G1} = 0$ and $p^*_G = 1$ when $\alpha \in [0, \frac{1}{2}]$

2. $p'_{Bj} = 0$ and $p^*_G = \frac{\alpha + 1}{2\alpha + 1}$ when $\alpha \in [\frac{1}{2}, 1]$
Proof of Lemma 7.

The case of $\mathbb{E}[u] < 0$ was discussed in the main text. For $\mathbb{E}[u] \geq 0$ to hold, $\alpha + (1 - \alpha)u_B \geq 0 \Rightarrow u_B \geq -\frac{\alpha}{1 - \alpha}$. The Lagrangian of the platform’s payoff is:

$$L(p_{G1}, p_{B1}) = ((1 - \alpha)p_{B1} + \alpha p_{G1}) \left(\frac{(1 - \alpha)p_{B1}u_B + \alpha p_{G1}}{(1 - \alpha)p_{B1} + \alpha p_{G1}} - ((1 - \alpha)p_{B1} + \alpha p_{G1})((1 - \alpha)u_B + \alpha)\right)$$

$$+ (\alpha + (1 - \alpha)u_B)((1 - \alpha)(1 - p_{B1}) + \alpha(1 - p_{G1}))$$

$$- \lambda_1(p_{G1} - 1) - \lambda_2(p_{B1} - 1) - \lambda_3(-p_{G1}) - \lambda_4(-p_{B1})$$

Yielding the KKT conditions:

$$\frac{\partial L}{\partial p_{G1}} = \alpha - 2\alpha((\alpha - 1)u_B - \alpha)((\alpha - 1)p_{B1} - \alpha p_{G1}) + \alpha((\alpha - 1)u_B - \alpha) - \lambda_1 + \lambda_3 = 0$$

(33)

$$\frac{\partial L}{\partial p_{B1}} = p_{B1}(-2\alpha^3 + 4\alpha^2 - 2\alpha + 2(\alpha - 1)^2u_B) - (\alpha - 1)\alpha(2p_{G1}((\alpha - 1)u_B - \alpha) + u_B - 1) - \lambda_2 + \lambda_4 = 0$$

(34)

$$\lambda_i \geq 0, p_{G1} \leq 1, p_{B1} \leq 1, -p_{G1} \leq 0, -p_{B1} \leq 0, \text{ and,}$$

$$\lambda_1(p_{G1} - 1) = 0, \lambda_2(p_{B1} - 1) = 0, \lambda_3(-p_{G1}) = 0, \lambda_4(-p_{B1}) = 0$$

Solutions to the KKT conditions, and constraints on $u_B$ for the different ranges of $\alpha$ appear in Web Appendix WA.3 and yield as follows:

1. $p_{B1}^* = 0$ and $p_{G1}^* = 1$ (case 8 of the proof).
2. $p_{B1} = 0$ and $p_{G1} = -\frac{(\alpha - 1)(u_B - 1)}{(\alpha - 1)(\alpha - 1)u_B - \alpha}) \in (0, 1)$ (case 9 of the proof).

Proof of Proposition 4.

If the platform chooses a symmetric design:

1. When $\alpha \in [0, 0.5]$ and $u_B < \frac{(1 - \alpha)}{\alpha^2 - \alpha + 1}$ (conditions supporting an equilibrium in the second stage), the optimal information design for the platform is $p_{Gj} = 1, p_{Bj} = 0$ with profit $\pi_p^*|_{p_{Gj}=1, p_{Bj}=0} = 2(1 - \alpha)\alpha$.

2. If $\alpha \in (0.5, 1]$ and $u_B < \frac{3\alpha - 2}{3\alpha - 3}$, then $p_{Gj} = \frac{1}{\alpha} < 1, p_{Bj} = 0$ with profit $\pi_p^*|_{p_{Gj}=\frac{1}{\alpha}, p_{Bj}=0} = \frac{1}{2}$.

If the platform chooses an asymmetric design with $p_{Gj}$ and $p_{Bj}$ for only one firm and no information for the other:  

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1. When $u_B > \frac{-2\alpha^2 - \alpha + 1}{-2\alpha^2 - \alpha + 1}$, the platform sets $p_{G1} = -\frac{(\alpha - 1)(u_B - 1)}{2\alpha((\alpha - 1)u_B - \alpha)}$ and $p_{B1} = 0$.

When $\alpha \in [0, 0.5]$, $\pi_{p \text{as}} = -\frac{5\alpha^2 + 2\alpha + 2(5\alpha^2 - 6\alpha + 1)u_B - 5(\alpha - 1)^2u_B^2 - 1}{4(\alpha - 1)u_B - 4\alpha} > 2(1 - \alpha)\alpha = \pi_{p \text{ss}}$; When $\alpha \in (0.5, 1)$, $\pi_{p \text{as}} = -\frac{5\alpha^2 + 2\alpha + 2(5\alpha^2 - 6\alpha + 1)u_B - 5(\alpha - 1)^2u_B^2 - 1}{4(\alpha - 1)u_B - 4\alpha} > \frac{1}{2} = \pi_{p \text{ss}}$.

The profit under an asymmetric design is always higher than under a symmetric design. The platform chooses an asymmetric design when $u_B > \frac{-2\alpha^2 - \alpha + 1}{-2\alpha^2 - \alpha + 1}$.

2. When $-\frac{\alpha}{1 - \alpha} \leq u_B \leq \frac{-2\alpha^2 - \alpha + 1}{-2\alpha^2 + \alpha + 1}$, the platform sets $p_{G1} = 1$ and $p_{B1} = 0$. When $\alpha \in [0, 0.5)$, $\pi_{p \text{ss}} = (1 - \alpha)(\alpha - (1 - \alpha)u_B) > 2(1 - \alpha)\alpha = \pi_{p \text{as}}$ if $u_B > \frac{\alpha^2}{\alpha^2 + \alpha - 1}$ and $\pi_{p \text{as}} > \pi_{p \text{ss}}$ if $u_B < \frac{\alpha^2}{\alpha^2 + \alpha - 1}$. When $\alpha \in (0.5, 1]$, $\pi_{p \text{as}} = (1 - \alpha)(\alpha - (1 - \alpha)u_B) + \alpha(1 - \alpha(\alpha + (1 - \alpha)u_B)) > \frac{1}{2} = \pi_{p \text{ss}}$.

Hence, when $-\frac{\alpha}{1 - \alpha} \leq u_B \leq \frac{-2\alpha^2 - \alpha + 1}{-2\alpha^2 + \alpha + 1}$, for $\alpha \in (0, 0.5)$ and $u_B > \frac{\alpha^2}{\alpha^2 + \alpha - 1}$, the platform chooses an asymmetric design; for $\alpha \in [0, 0.5)$ and $u_B < \frac{\alpha^2}{\alpha^2 + \alpha - 1}$, the platform chooses a symmetric design; for $\alpha \in (0.5, 1]$, the platform chooses an asymmetric design.

3. When $u_B < -\frac{\alpha}{1 - \alpha}$, the platform sets $p_{G1} = 1$ and $p_{B1} = 0$. The other firm shuts down because $E[u] < 0$. When $\alpha \in [0, 0.5)$, $\pi_{p \text{ss}} = \alpha < 2\alpha(1 - \alpha) = \pi_{p \text{as}}$; When $\alpha \in (0.5, 1]$, $\pi_{p \text{as}} = \alpha > \frac{1}{2} = \pi_{p \text{ss}}$.

Hence, when $u_B < -\frac{\alpha}{1 - \alpha}$, for $\alpha \in [0, 0.5)$, the platform chooses a symmetric design; for $\alpha \in (0.5, 1]$, the platform chooses an asymmetric design.

\[ \square \]

\textbf{Proof of Proposition 5.}

We first compute the expected utility consumers receive and the variance of items they consume under endogenous duopoly pricing:

1. When both firms are active, a consumer who receives a good signal will buy, with ex-post utility $u(g) = \frac{\alpha p_G + (1 - \alpha) p_B u_B}{\alpha p_G + (1 - \alpha) p_B} = u_G$ and variance

   \[ \left( \frac{\alpha p_G}{\alpha p_G + (1 - \alpha) p_B} \right) u_G^2 + \left( \frac{(1 - \alpha) p_B}{\alpha p_G + (1 - \alpha) p_B} \right) u_B^2 - \left( \frac{\alpha p_G + (1 - \alpha) p_B u_B}{\alpha p_G + (1 - \alpha) p_B} \right)^2 = \frac{(1 - \alpha) \alpha p_B u_B - u_B u_G}{(\alpha p_G + (1 - \alpha) p_B)^2} = 0 \]

2. When only one firm is active, consumers buy from the active firm when receiving a good signal, and otherwise buy from the passive firm. The expected utility is $Pr(g) \cdot u(g) + Pr(b) \cdot E[u] = (\alpha p_G + (1 - \alpha) p_B) \cdot \frac{\alpha p_G u_G + (1 - \alpha) p_B u_B}{\alpha p_G + (1 - \alpha) p_B} + (\alpha(1 - p_G) + (1 - \alpha)(1 - p_B)) \cdot (\alpha u_G + (1 - \alpha) u_B)$, with variance

   \[ (\alpha p_G + (1 - \alpha) p_B) \cdot \left( \frac{(1 - \alpha) \alpha p_B u_B - u_B u_G}{(\alpha p_G + (1 - \alpha) p_B)^2} \right) + (\alpha(1 - p_G) + (1 - \alpha)(1 - p_B)) \cdot (\alpha u_G + (1 - \alpha) u_B)^2 = \frac{(1 - \alpha) \alpha (u_B - u_G)^2 (p_B (\alpha + (1 - 2\alpha^2 + 2\alpha - 1) p_G - 1) + (\alpha - 1)^2 p_B^2 + \alpha p_G (\alpha - 1))}{(\alpha - 1)p_B - \alpha p_G} \]

Plugging in the different equilibrium information designs we find:
When we compare the variance under duopoly and platform competitions. 

(a) When \( p^*_G = 0 \) and \( p^*_G = 1 \), the expected utility (EU) is \( \alpha + (1 - \alpha) \ast (\alpha + (1 - \alpha)u_B) \). The variance is \( (1 - \alpha)^2 \ast (u_B - 1)^2 \).

(b) When \( p^*_G = 0 \) and \( p^*_G = \frac{\alpha^2 - (2a^2 + a + 1)u_B - 2a^2}{2(\alpha - 1)(\alpha - 1)u_B - 2a} > 0 \) and \( p^*_G = 1 \), the EU is \( \frac{-4\alpha^2 + 4(\alpha^2 + 5\alpha - 5)}{\alpha - 1}u_B + (\alpha - 1)(u_B - 2) \), with variance \( \frac{\alpha (u_B - 1)^2 \ast (4a^2 - 2(4a^2 - 5a + 2)\alpha u_B + (4\alpha^3 - 10\alpha^2 + 9\alpha - 3)u_B^2)}{2u_B(\alpha - 1)u_B - 2a} \).

(c) When \( p^*_B = 0 \) and \( p^*_G = \frac{1}{2(\alpha - 1)u_B} \in (0, 1) \), the EU is \( \alpha - (\alpha - 1)u_B + \frac{1}{2(\alpha - 1)u_B} - 1 \), with variance \( (\alpha - 1)(u_B - 1)^2 \ast \left( \frac{1}{2(\alpha - 1)u_B} - 1 \right) \).

Similarly, with endogenous pricing under a platform:

1. When both firms are active, a consumer who receives a good signal will buy with ex-post utility \( u(g) = \frac{\alpha p_G u_G + (1 - \alpha) p_B u_B}{\alpha p_G + (1 - \alpha) p_B} \), and variance \( \left( \frac{\alpha p_G}{\alpha p_G + (1 - \alpha) p_B} \ast u_G^2 \right) + \left( \frac{(1 - \alpha) p_B}{\alpha p_G + (1 - \alpha) p_B} \ast u_B^2 \right) = \left( \frac{(1 - \alpha) p_B u_B - u_G}{(\alpha p_G + (1 - \alpha) p_B)^2} \right)^2 \) with variance \( \left( \frac{\alpha (u_B - u_G)^2}{(\alpha p_G + (1 - \alpha) p_B)^2} \right) \).

2. When only one firm is active, consumers buy from the active firm when receiving a good signal, and from the passive firm when receiving a bad signal. The EU is \( Pr(g) \ast u(g) + Pr(b) \ast E[u] = (\alpha p_G + (1 - \alpha) p_B) \ast \frac{\alpha p_G u_G + (1 - \alpha) p_B u_B}{\alpha p_G + (1 - \alpha) p_B} + (\alpha p_G + (1 - \alpha) p_B) \ast (\alpha u_G + (1 - \alpha) u_B) \), with variance \( (\alpha p_G + (1 - \alpha) p_B) \ast \left( \frac{(1 - \alpha) p_B u_B - u_G}{(\alpha p_G + (1 - \alpha) p_B)^2} \right)^2 + (\alpha p_G + (1 - \alpha) p_B) \ast (\alpha u_G + (1 - \alpha) u_B) \).

Plugging in the equilibrium information designs, we find:

(a) For \( p^*_G = 0 \) and \( p^*_G = 1 \), the EU is \( \alpha + (1 - \alpha) \ast (\alpha + (1 - \alpha)u_B) \) with variance \( \alpha + (1 - \alpha) \ast (\alpha + (1 - \alpha)u_B) \).

(b) When \( p^*_G = 0 \) and \( p^*_G = \frac{\alpha (u_B - 1)}{2a((\alpha - 1)u_B - a)} \in (0, 1) \), the EU is \( \frac{-3a^2 + 2a + 6a^2 - 8a + 2u_B - 3(\alpha - 1)^2u_B^2 - 1}{2(\alpha - 1)u_B - 2a} \), with variance \( \frac{(\alpha - 1)u_B - 1)^2 \ast \left( \frac{-3a + 3(\alpha - 1)u_B + 1}{2(\alpha - 1)u_B - 2a} \right) \).

When we compare the variance under duopoly and platform competitions.

1. When \( \frac{\alpha^2}{\alpha^2 + a - 1} < u_B < 0 \) and \( \alpha < 0.5 \), or \( u_B < 0 \) and \( \alpha > 0.5 \): Under duopoly, both firms choose full revelation with variance 0. Under a platform, one firm is active and the other is passive and the variance under platform is larger.

2. When \( u_B < \frac{\alpha^2}{\alpha^2 + a - 1} \) and \( \alpha < 0.5 \): Under duopoly and platform, both firms will have full revelation. Hence, the variances are the same.

3. When \( u_B < \frac{2a^2 - 2a + 1}{2a^2 + a + 1} \) and \( u_B < \frac{-2a^2 + a + 1}{2a^2 + a + 1} \) and \( u_B > 0 \) and \( u_B < \frac{(1 - \alpha)}{\alpha - a + 1} \): Under duopoly and platform, one firm will have full revelation while the other firm will not provide information. Hence, the variances are the same.

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4. When \( u_B < \frac{2\alpha^2}{2\alpha^2-2\alpha+1} \) and \( u_B < \frac{-2\alpha^2-\alpha+1}{-2\alpha^2+\alpha+1} \) and \( u_B > 0 \) and \( u_B < \frac{(1-\alpha)\alpha}{\alpha^2-\alpha+1} \): Under duopoly and platform, one firm will have full revelation while the other firm will not provide information. Hence, the variances are the same.

5. When \( u_B < \frac{\sqrt{3}-3\alpha}{3-3\alpha} \) and \( u_B > \frac{-2\alpha^2-\alpha+1}{-2\alpha^2+\alpha+1} \) and \( u_B > 0 \) and \( u_B < \frac{(1-\alpha)\alpha}{\alpha^2-\alpha+1} \): Under duopoly, one firm chooses full revelation while the other firm does not provide information. Under a platform, one firm will have full revelation while the other firm will be demarketed. Hence the variance under the platform is larger.

6. When \( u_B < \frac{\sqrt{3}-3\alpha}{3-3\alpha} \) and \( u_B > \frac{-2\alpha^2-\alpha+1}{-2\alpha^2+\alpha+1} \) and \( u_B > 0 \) and \( u_B < \frac{(1-\alpha)\alpha}{\alpha^2-\alpha+1} \): Under duopoly, one firm chooses full revelation while the other firm does not provide information. Under platform, one firm will be demarketed while the other firm will not provide information. Hence, the variance under a platform is larger.

7. When \( u_B > \frac{2\alpha^2}{2\alpha^2-2\alpha+1} \) and \( u_B < \frac{(1-\alpha)\alpha}{\alpha^2-\alpha+1} \) and \( u_B > 0 \): Under duopoly, one firm chooses persuasion while the other firm does not provide information. Under a platform, one firm will have full revelation while the other does not provide information. Hence, the variance under a platform is smaller.
Web appendix

WA.1 Solution of Kuhn-Tucker conditions for proof of Lemma 5

We solve the KKT conditions by cases based on possible values of $p_{G1}^*$ and $p_{B1}^*$:

1. When $p_{G1}^* = 0$ and $p_{B1}^* = 0$, the complementary slackness condition (CSC) $\lambda_1(p_{G1} - 1) = 0$ implies $\lambda_1 = 0$. The conditions in (25) become:

$$\frac{\partial \mathcal{L}}{\partial p_{G1}} = \alpha + \lambda_3 = 0$$

$$\lambda_3 \geq 0 \tag{35}$$

However, $\lambda_3 = -\alpha < 0$ when $\alpha \in [0, 1]$, contradicting $\lambda_3 \geq 0$, implying $p_{G1}^* = 0$ and $p_{B1}^* = 0$ is not a solution.

2. When $p_{G1}^* = 1$ and $p_{B1}^* = 1$, the CSC $\lambda_3(-p_{G1}) = 0$ and $\lambda_4(-p_{B1}) = 0$ implies $\lambda_3 = 0$ and $\lambda_4 = 0$. The conditions in (25) and (26) become:

$$\frac{\partial \mathcal{L}}{\partial p_{G1}} = \alpha (-2\alpha + 2(\alpha - 1)u_B + 1) - \lambda_1 = 0 \tag{37}$$

$$\frac{\partial \mathcal{L}}{\partial p_{B1}} = (1 - \alpha) ((\alpha - 1)u_B - 2\alpha) - \lambda_2 = 0 \tag{38}$$

However, $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ cannot hold at the same time, implying $p_{G1}^* = 1$ and $p_{B1}^* = 1$ is not a solution.

3. When $p_{G1}^* = 0$ and $p_{B1}^* = 1$, the CSC $\lambda_1(p_{G1} - 1) = 0$ and $\lambda_4(-p_{B1}) = 0$ implies $\lambda_1 = 0$ and $\lambda_4 = 0$. The conditions in (25) and (26) become:

$$\frac{\partial \mathcal{L}}{\partial p_{G1}} = \alpha - 2(\alpha - 1)\alpha ((\alpha - 1)u_B - \alpha) + \lambda_3 = 0 \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial p_{B1}} = (\alpha - 1) ((2\alpha^2 - 4\alpha + 1)u_B - 2(\alpha - 1)\alpha) - \lambda_2 = 0 \tag{40}$$

However, $\lambda_3 \geq 0$ and $\lambda_2 \geq 0$ cannot hold at the same time, implying $p_{G1}^* = 0$ and $p_{B1}^* = 1$ is not a solution.

4. When $p_{G1}^* = 0$ and $p_{B1}^* \in (0, 1)$, the CSC implies $\lambda_1 = 0$, $\lambda_2 = 0$ and $\lambda_4 = 0$. The condition in (26) becomes:

$$\frac{\partial \mathcal{L}}{\partial p_{B1}} = (\alpha - 1) (2(\alpha - 1)p_{B1} ((\alpha - 1)u_B - \alpha) - u_B) = 0 \tag{41}$$
yielding \( p_{B1} = \frac{u_B}{2(\alpha - 1)(-\alpha + \alpha u_B - u_B)} \). However, in this case, \( \frac{\partial L}{\partial p_{G1}} = \alpha - \alpha u_B + \lambda_3 > 0 \), implying \( p_{G1}^* = 0 \) and \( p_{B1}^* \in (0, 1) \) is not a solution.

5. When \( p_{G1}^* \in (0, 1) \) and \( p_{B1}^* = 1 \), the CSC implies \( \lambda_1 = 0 \), \( \lambda_3 = 0 \), \( \lambda_4 = 0 \). The condition in (26) becomes:

\[
\frac{\partial L}{\partial p_{G1}} = \alpha \left( (\alpha - 1)u_B - \alpha \right) - 2(\alpha - 1)\left( (\alpha - 1)u_B - \alpha \right) + 1 = 0 \tag{42}
\]

yielding \( p_{G1}^* = -\frac{2\alpha^2 + 2\alpha^2(\alpha - 1)u_B - 1}{2(\alpha - 1)} \). However, in this case, \( \frac{\partial L}{\partial p_{B1}} = (1 - \alpha)(u_B - 1) - \lambda_2 < 0 \), implying \( p_{G1}^* \in (0, 1) \) and \( p_{B1}^* = 1 \) is not a solution.

6. When \( p_{G1}^* \in (0, 1) \) and \( p_{B1}^* \in (0, 1) \), then \( \lambda_i = 0 \) for \( i = 1, 2, 3, 4 \). Conditions (25) and (26) become:

\[
\frac{\partial L}{\partial p_{G1}} = \alpha - 2(\alpha - 1)\alpha p_{B1} \left( (\alpha - 1)u_B - \alpha \right) + p_{G1} \left( -2\alpha^3 + 2(\alpha - 1)\alpha^2 u_B \right) = 0 \tag{43}
\]

\[
\frac{\partial L}{\partial p_{B1}} = p_{B1} \left( -2\alpha^3 + 4\alpha^2 - 2\alpha + 2(\alpha - 1)^3 u_B \right) - (\alpha - 1)\left( 2\alpha p_{G1} \left( (\alpha - 1)u_B - \alpha \right) + u_B \right) = 0 \tag{44}
\]

\[
p_{G1}^* \in (0, 1), p_{B1}^* \in (0, 1) \tag{45}
\]

Solving Equations (43), yields the solution candidates:

\[
p_{B1} = \frac{2\alpha^2 u_B p_{G1} - 2\alpha u_B p_{G1} - 2\alpha^2 p_{G1} + 1}{2(\alpha - 1)(-\alpha + \alpha u_B - u_B)}
\]

However, when \( p_{B1} = \frac{2\alpha^2 u_B p_{G1} - 2\alpha u_B p_{G1} - 2\alpha^2 p_{G1} + 1}{2(\alpha - 1)(-\alpha + \alpha u_B - u_B)} \), \( \frac{\partial L}{\partial p_{B1}} = (1 - \alpha)(u_B - 1) < 0 \). Hence \( p_{G1}^* \in (0, 1) \) and \( p_{B1}^* \in (0, 1) \) is not a solution.

7. When \( p_{G1}^* = 1, p_{B1}^* \in (0, 1) \), the CSC conditions imply \( \lambda_2 = 0 \), \( \lambda_3 = 0 \), \( \lambda_4 = 0 \). Equations (25) and (26) become:

\[
\frac{\partial L}{\partial p_{G1}} = \alpha \left( -2\alpha^2 - 2(\alpha - 1)p_{B1} \left( (\alpha - 1)u_B - \alpha \right) + 2(\alpha - 1)\alpha u_B + 1 \right) - \lambda_1 = 0 \tag{46}
\]

\[
\frac{\partial L}{\partial p_{B1}} = p_{B1} \left( -2\alpha^3 + 4\alpha^2 - 2\alpha + 2(\alpha - 1)^3 u_B - 1 \right) - (\alpha - 1)\left( 2\alpha \left( (\alpha - 1)u_B - \alpha \right) + u_B \right) = 0 \tag{47}
\]

Solving (47), we find the solution candidate \( p_{B1} = \frac{(2\alpha^2 - 2\alpha + 1)u_B - 2\alpha^2}{2(\alpha - 1)((\alpha - 1)u_B - \alpha)} \) and \( p_{G1} = 1 \). To find conditions for \( u_B \) under which the solution candidate is valid, we analyze the following cases:
8. When \( p_{B1} \in (0,1) \), \( E[u] \geq 0 \) and \( \lambda_1 \big|_{p_{G1}=1,p_{B1}=(2\alpha^2-2\alpha+1)u_B-2\alpha^2} \geq 0 \) require that \( u_B > \frac{2\alpha^2}{2\alpha^2-2\alpha+1} \) and \( \alpha \in [0,\frac{1}{2}] \).

(b) If firm 1 sets a price lower than \( u(b) \), when firm 2 uses mixed pricing strategies according to \( F_2(m) = 1 - \frac{u(g) - \Pr(g)E[u]}{(m + u(g) - E[q])} \) for \( \Pr(b) * E[u] \leq m_2 \leq E[u] \), firm 1’s expected profit will be:

\[
\pi_1(m_1) = m_1 \times \Pr(g) + \Pr(b)(1 - F_2(m_1 + (E[u] - u(b)))) \text{, for } m_1 \leq u(b) \quad (48)
\]

The second derivative of the profit with respect to price is negative for any \( m \leq u(b) \). Hence the function is concave. The first derivative at \( m = u(b) \) is positive when \( \alpha \in [0,1] \), \( u_B < 1 \). As a result, a deviating firm can maximize its profit by setting the price to \( u(b) \). The deviating profit should be lower than the equilibrium profit, yielding a constraint that \( u_B < \bar{u}_B \), where \( \bar{u}_B \) is the root of \( u_B^3(4\alpha^3 - \alpha + 1) + u_B^2(\alpha - 12\alpha^3) + 12u_B\alpha^3 - 4\alpha^3 = 0 \)

8. When \( p_{G1}^* = 1 \) and \( p_{B1}^* = 0 \) then \( \lambda_2 = 0 \) and \( \lambda_3 = 0 \). Conditions (25) and (26) become:

\[
\frac{\partial L}{\partial p_{G1}} = -2\alpha^3 + \alpha + 2(\alpha - 1)\alpha^2 u_B - \lambda_1 = 0 \quad (49)
\]
\[
\frac{\partial L}{\partial p_{B1}} = -(\alpha - 1)(2\alpha((\alpha - 1)u_B - \alpha) + u_B) + \lambda_4 = 0 \quad (50)
\]
\[
\lambda_1 \geq 0, \lambda_4 \geq 0 \quad (51)
\]

For this solution to be possible, the following conditions need to hold:

(a) \( E[u] \geq 0 \) implies \( \alpha + (1 - \alpha)u_B \geq 0 \) if \( u_B \geq -\frac{\alpha}{1-\alpha} \).

(b) For \( \alpha \in [\frac{1}{2}, \frac{1}{2} + \frac{\sqrt{3}}{2}] \), the condition \( \lambda_1 = -2\alpha^3 + \alpha + 2(\alpha - 1)\alpha^2 u_B \geq 0 \) implies \( u_B < \frac{2\alpha^2 - 1}{2(\alpha - 1)\alpha} \).

(c) For \( \alpha \in [0, \frac{1}{2}] \), the condition \( \lambda_4 = (\alpha - 1)(2\alpha((\alpha - 1)u_B - \alpha) + u_B) \geq 0 \) implies \( u_B < \frac{2\alpha^2}{2\alpha^2 - 2\alpha + 1} \).

(d) Consumers will not buy from firm 1 after seeing a bad signal, i.e., firm 1 will not price lower than \( u(b) \):

\[
m_{1\min} - u(b) = u(g) - \Pr(g) E[u] - u(b) = -\alpha (\alpha + (1 - \alpha)u_B) - u_B + 1 > 0 \Rightarrow u_B < \frac{\alpha^2 - 1}{\alpha^2 - \alpha - 1} \]

(e) If firm 1 deviates to setting a price lower than \( u(b) \), when firm 2 uses mixed pricing strategies according to \( F_2(m) = 1 - \frac{u(g) - \Pr(g)E[u]}{(m + u(g) - E[q])} \) for \( \Pr(b) * E[u] \leq m_2 \leq E[u] \), firm 1’s expected profit will be:

\[
\pi_1(m_1) = m_1 \times \Pr(g) + \Pr(b)(1 - F_2(m_1 + (E[u] - u(b)))) \text{, for } m_1 \leq u(b) \quad (52)
\]
The expected payoff is maximized at \( m_1 = u_1(b) \). The deviating profit is lower than the equilibrium profit when \( u_B < -\frac{\sqrt{4a^3 - 8a^2 + 8a^{3/2} - 4a^2 + 1 - 1}}{2(\alpha - 1) \alpha} \).

The intersection of above constraints yields \(-\frac{\alpha}{(1-\alpha)} \leq u_B < \frac{2a^2}{2(\alpha^3 - 2a^2 + 1)} \) for \( \alpha \in [0, 0.28398] \); \(-\frac{\alpha}{(1-\alpha)} \leq u_B < \frac{2a^2}{2(\alpha^3 - 2a^2 + 1)} \) for \( \alpha \in [0.28398, 0.640388] \); \(-\frac{\alpha}{(1-\alpha)} \leq u_B < \frac{2a^2 - 1}{2(\alpha - 1) \alpha} \) for \( \alpha \in [0.640388, \frac{1}{\sqrt{2}}] \).

9. When \( p_{B1}^* = 0 \) and \( p_{G1}^* \in (0, 1) \), \( \lambda_1 = \lambda_2 = \lambda_3 = 0 \) and conditions (25) and (26) become:

\[
\frac{\partial L}{\partial p_{G1}} = \alpha + p_{G1}(-2a^3 + 2(\alpha - 1)\alpha^2 u_B) = 0 \tag{53}
\]
\[
\frac{\partial L}{\partial p_{B1}} = -(\alpha - 1)(2\alpha p_{G1}((\alpha - 1)u_B - \alpha) + u_B) + \lambda_4 = 0 \tag{54}
\]
\[\lambda_4 \geq 0 \tag{55}\]

Solving (53) we find \( p_{G1} = \frac{1}{2\alpha(\alpha + (1-\alpha)u_B)} \) and \( p_{B1} = 0 \).

The solution needs to fulfill the following restrictions to be an equilibrium:

(a) For \( \alpha \in [\frac{1}{2}, \frac{1}{\sqrt{2}}] \), \( p_{G1} \in (0, 1) \) and \( \mathbb{E}[u] \geq 0 \) imply \( u_B > \frac{2a^2 - 1}{2(\alpha - 1) \alpha} \).

(b) For \( \alpha \in [0, \frac{1}{2}] \), \( \lambda_4 = (\alpha - 1)(2\alpha p_{G1}((\alpha - 1)u_B - \alpha) + u_B) = (1 - \alpha)(1 - u_B) \geq 0 \).

(c) If firm 1 deviates to pricing below \( u(b) \), its expected profit is:

\[
\pi_1(m_1) = m_1 \cdot [\text{Pr}(g) + \text{Pr}(b)(1 - F_2(m_1 + (\mathbb{E}[u] - u(b))))], \text{ for } m_1 \leq u(b) \tag{56}
\]

This expected profit is maximized at \( m_1 = u(b) \), and needs to be lower than the equilibrium profit, yielding a constraint that \( u_B < \frac{4a - \sqrt{17}}{4(\alpha - 1)} \).

WA.2 Solution of Kuhn-Tucker conditions for proof of Lemma 6

We solve the KKT conditions by cases based on possible values of \( p_{Gj}^* \) and \( p_{Bj}^* \):

1. When \( p_{Gj}^* = 0 \) and \( p_{Bj}^* = 0 \), the complementary slackness condition (CSC) \( \lambda_1(p_{G1} - 1) = 0 \) implies \( \lambda_1 = 0 \). The conditions in (28) become:

\[
\frac{\partial L}{\partial p_{G1}} = \alpha + \lambda_5 = 0 \tag{57}
\]
\[
\lambda_3 \geq 0 \tag{58}
\]

However, \( \lambda_5 = -\alpha < 0 \) when \( \alpha \in [0, 1] \), contradicting \( \lambda_3 \geq 0 \), implying \( p_{Gj}^* = 0 \) and \( p_{Bj}^* = 0 \) is not a solution.
2. When $p_{Gj}^* = 1$ and $p_{Bj}^* = 1$, the CSC implies $\lambda_i = 0$, $i=5$, 6, 7, 8. The conditions in (28) and (30) become:

$$\frac{\partial L}{\partial p_{G1}} = \alpha ((\alpha - 1)u_B - \alpha) - \lambda_1 = 0$$

However, $\lambda_1 = \alpha ((\alpha - 1)u_B - \alpha) < 0$, implying $p_{Gj}^* = 1$ and $p_{Bj}^* = 1$ is not a solution.

3. When $p_{Gj}^* = 0$ and $p_{Bj}^* = 1$, the CSC implies $\lambda_i = 0$, $i=1$, 2, 7, 8. The conditions in (28) and (30) become:

$$\frac{\partial L}{\partial p_{G1}} = \alpha (\alpha + (\alpha - 1)u_B) + \lambda_5 = 0$$

$$\frac{\partial L}{\partial p_{B1}} = -((\alpha - 1)(2\alpha - 1)u_B - \lambda_3 = 0$$

However, $\lambda_3 \geq 0$ and $\lambda_5 \geq 0$ cannot hold at the same time, implying $p_{Gj}^* = 0$ and $p_{Bj}^* = 1$ is not a solution.

4. When $p_{Gj}^* = 0$ and $p_{Bj}^* \in (0, 1)$, the CSC implies $\lambda_i = 0$, $i=1$, 2, 5, 6, 7, 8. The conditions in (30) become:

$$\frac{\partial L}{\partial p_{B1}} = (\alpha - 1)u_B (-2(\alpha - 1)p_{B2} - 1) = 0$$

yielding $p_{Bj} = -\frac{1}{2(\alpha-1)}$. However, in this case, $\frac{\partial L}{\partial p_{G1}} = \frac{1}{2}\alpha (1 - u_B) + \lambda_5 > 0$, implying $p_{Gj}^* = 0$ and $p_{Bj}^* \in (0, 1)$ is not a solution.

5. When $p_{Gj}^* \in (0, 1)$ and $p_{Bj}^* = 1$, the CSC implies $\lambda_i = 0$, $i=1$, 2, 5, 6, 7, 8. The conditions in (28) become:

$$\frac{\partial L}{\partial p_{G1}} = \alpha^2 (1 - p_{G2}) - \alpha ((1 - \alpha)u_B + \alpha p_{G2}) = 0$$

yielding $p_{Gj}^* = \frac{\alpha + u_B - u_B^*}{2\alpha}$. However, in this case, $\frac{\partial L}{\partial p_{B1}} = (\frac{1}{2})(\alpha - 1) (u_B - 1) ((\alpha - 1)u_B - \alpha) - \lambda_3 < 0$, implying $p_{Gj}^* \in (0, 1)$ and $p_{Bj}^* = 1$ is not a solution.

6. If $p_{Gj}^* \in (0, 1), p_{Bj}^* \in (0, 1)$. Then, $\lambda_i = 0$, $i = 1, 2, \ldots, 8$. Conditions (28) and (30) become:

$$\frac{\partial L}{\partial p_{G1}} = \alpha + (\alpha - 1)\alpha (u_B + 1) p_{B1} - 2\alpha^2 p_{G1} = 0$$

$$\frac{\partial L}{\partial p_{B1}} = (\alpha - 1) (u_B (-2(\alpha - 1)p_{B1} - 1) + \alpha (u_B + 1) p_{G1}) = 0$$

$p_{Gj}^* \in (0, 1), p_{Bj}^* \in (0, 1)$
Solving equations (64) and (65) yield the solution candidate:

\[ p_{Gj} = \frac{u_B}{\alpha (u_B - 1)} \]
\[ p_{Bj} = \frac{1}{(\alpha - 1)(u_B - 1)} \]

However, \( p_{Gj}^* \in (0, 1) \) and \( p_{Bj}^* \in (0, 1) \) cannot hold at the same time, hence \( p_{Gj}^* \in (0, 1) \), \( p_{Bj}^* \in (0, 1) \) is not a solution.

7. If \( p_{Gj}^* = 1, p_{Bj}^* \in (0, 1) \), then \( \lambda_3 = 0, \lambda_4 = 0, \lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0 \)

Conditions (28) and (30) become:

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial p_{G1}} &= \alpha (-2\alpha + (\alpha - 1)(u_B + 1)p_{B2} + 1) - \lambda_1 = 0 \quad (67) \\
\frac{\partial \mathcal{L}}{\partial p_{B1}} &= 0 \quad \Rightarrow \quad p_{Bj} = \frac{\alpha + \alpha u_B - u_B}{2(\alpha - 1)u_B} \quad (68) \\
\lambda_1 &\geq 0, p_{Bj} \in (0, 1) \quad (69)
\end{align*}
\]

First \( p_{Bj} = \frac{\alpha + \alpha u_B - u_B}{2(\alpha - 1)u_B} \in (0, 1) \), and the platform sets \( p_{Gj} \) and \( p_{Bj} \) such that consumers buy with a good signal, i.e., \( \alpha u_G p_{Gj} + (1 - \alpha)u_B p_{Bj} \geq 0 \). The intersection of the two conditions above is \( 0 < \alpha < \frac{1}{2} \wedge u_B > \frac{\alpha}{1 - \alpha} \).

To confirm the firm will not deviate to low prices that allow it to sell when consumers receive a bad signal, if a firm sets a price lower than \( u_j(b) \), when the competition mixes their pricing according to \( F(m) \), its expected profit will be:

\[
\pi_1(m) = m \left( ((1 - \alpha)(1 - p_{B1}) + \alpha (1 - p_{G1})) ((1 - \alpha)p_{B2} + \alpha p_{G2}) (1 - F(u_2(g) - u_1(b) + m)) + m ((1 - \alpha)(1 - p_{B1}) + \alpha (1 - p_{G1})) ((1 - \alpha)(1 - p_{B2}) + \alpha (1 - p_{G2})) + m ((1 - \alpha)p_{B1} + \alpha p_{G1}) ((1 - \alpha)(1 - p_{B2}) + \alpha (1 - p_{G2})) + m ((1 - \alpha)p_{B1} + \alpha p_{G1}) ((1 - \alpha)p_{B2} + \alpha p_{G2}) \right)
\]

A firm can deviate from the equilibrium by setting the price to \( u_j(b) \). The deviating profit should be smaller than the expected profit in the equilibrium. However, when \( u_B > \frac{\alpha}{1 - \alpha} \), this does not hold. A firm could profitably deviate from the equilibrium and set price \( u_j(b) \). Therefore, there is no value of \( u_B \) that can support such an equilibrium, and \( p_{Gj}^* = 1, p_{Bj}^* \in (0, 1) \) is not the solution.
8. If \( p_{Gj}^* = 1, p_{Bj}^* = 0 \), then \( \lambda_3 = 0, \lambda_4 = 0, \lambda_5 = 0, \lambda_6 = 0 \). Conditions (28) and (30) become:

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial p_{G1}} &= (\alpha - 2\alpha^2) - \lambda_1 = 0 \quad (70) \\
\frac{\partial \mathcal{L}}{\partial p_{B1}} &= (\alpha - 1) (\alpha (u_B + 1) - u_B) + \lambda_7 = 0 \quad (71)
\end{align*}
\]

\( \lambda_1 \geq 0, \lambda_7 \geq 0 \) \quad (72)

(a) \( \lambda_1 = 2(\alpha - 2\alpha^2) \geq 0 \) implies \( \alpha \leq \frac{1}{2} \).

(b) \( \lambda_7 = 2(1 - \alpha) (\alpha (u_B + 1) - u_B) \geq 0 \) implies \( u_B \leq \frac{\alpha}{1-\alpha} \).

(c) To support an equilibrium in the second stage, we need the condition \( u_j(b) < \frac{\alpha(1-\alpha)}{\alpha^2-\alpha+1} u_j(g) \) to hold, implying \( u_B < \frac{\alpha(1-\alpha)}{\alpha^2-\alpha+1} \) is the more restrictive constraint.

In summary, when \( \alpha \in [0, \frac{1}{2}] \) and \( u_B < \frac{\alpha(1-\alpha)}{\alpha^2-\alpha+1} \), \( p_{Gj}^* = 1 \) and \( p_{Bj}^* = 0 \) is a KKT solution.

9. If \( p_{Bj}^* = 0, p_{Gj}^* \in (0, 1) \), then \( \lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0, \lambda_5 = 0, \lambda_6 = 0 \). Conditions (28) and (30) become:

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial p_{G1}} &= 0 \Rightarrow p_{Gj} = \frac{1}{2\alpha} \quad (73) \\
\frac{\partial \mathcal{L}}{\partial p_{B1}} &= (\alpha - 1) (\alpha (u_B + 1) p_{G2} - u_B) + \lambda_7 = 0 \quad (74)
\end{align*}
\]

\( \lambda_7 \geq 0, p_{Gj}^* \in (0, 1) \) \quad (75)

(a) \( p_{Gj}^* \in (0, 1) \) implies \( \alpha \in (0.5, 1] \).

(b) \( \lambda_7 = (1 - \alpha) (\alpha (u_B + 1) p_{G2} - u_B) = \frac{1}{2}(1 - \alpha) (1 - u_B) \geq 0 \). So as long as \( u_B < 1 \), \( \lambda_7 \geq 0 \).

(c) In a mixed strategy equilibrium the firm should be indifferent between setting different prices, \( m_{\text{min}} \) will satisfy:

\[
\begin{align*}
[(\alpha p_{Gj} + (1 - \alpha)p_{Bj}) (\alpha p_{G-j} + (1 - \alpha)p_{B-j})] m_{\text{min}} \\
+ [(\alpha p_{Gj} + (1 - \alpha)p_{Bj}) (\alpha(1 - p_{G-j}) + (1 - \alpha)(1 - p_{B-j}))] m_{\text{min}} \\
= (\alpha p_{Gj} + (1 - \alpha)p_{Bj}) (\alpha(1 - p_{G-j}) + (1 - \alpha)(1 - p_{B-j})) u_j(g)
\end{align*}
\]

resulting in \( m_{\text{min}} = \frac{1}{2} \). The lowest price should be higher than \( u_j(b) \) (otherwise some consumers may buy with a bad signal), which means we need to restrict \( u_j(b) < \frac{1}{2} \), which implies \( u_B < \frac{4\alpha-3}{4\alpha-4} \).
We solve the KKT conditions by cases based on possible values of $\lambda$. Solution of Kuhn-Tucker conditions for proof of Lemma 7

1. When $p \lambda = 0$, however, $\sum_{j} u_j(d)$. The intersection of the above constraints is $G = \max_{b} B$. Hence the function is concave. The first derivative at $m = 0$. The conditions in (33) become:

$$
\pi_j(m) = (\alpha p_{Gj} + (1 - \alpha)p_{Bj}) (\alpha p_{G-j} + (1 - \alpha)p_{B-j}) (1 - F(m)) m
+ (\alpha p_{Gj} + (1 - \alpha)p_{Bj}) (\alpha(1 - p_{G-j}) + (1 - \alpha)(1 - p_{B-j})) m
= (\alpha p_{Gj} + (1 - \alpha)p_{Bj}) (\alpha(1 - p_{G-j}) + (1 - \alpha)(1 - p_{B-j})) u_j(g)
$$

resulting in the equilibrium mixed strategy price distribution $F(m) = \frac{2m - 1}{m}$.

Thus, each firm will randomize its prices between $[\frac{1}{2}, 1]$, with CDF $F(m) = \frac{2m - 1}{m}$.

If a firm sets a price lower than $u_j(b)$, when the competition mixes their pricing according to $F(m)$, its expected profit will be:

$$
\pi_1(m) = m ((1 - \alpha) (1 - p_{B1}) + \alpha (1 - p_{G1})) ((1 - \alpha)p_{B2} + \alpha p_{G2}) (1 - F(u_2(g) - u_1(b) + m))
+ m ((1 - \alpha) (1 - p_{B1}) + \alpha (1 - p_{G1})) ((1 - \alpha) (1 - p_{B2}) + \alpha (1 - p_{G2}))
+ m ((1 - \alpha)p_{B1} + \alpha p_{G1}) ((1 - \alpha) (1 - p_{B2}) + \alpha (1 - p_{G2}))
+ m ((1 - \alpha)p_{B1} + \alpha p_{G1}) ((1 - \alpha)p_{B2} + \alpha p_{G2})
$$

The second derivative of the profit with respect to price is negative for any $m \leq u_j(b)$. Hence the function is concave. The first derivative at $m = u_j(b)$ is positive when $\alpha \in (0.5, 1], u_B < 1$. As a result, a deviating firm can maximize its profit by setting the price to $u_j(b)$. The deviating profit should be lower than the equilibrium profit, which yields the constraint $u_B < \frac{3\alpha - 2}{3\alpha - 3}$.

(d) The intersection of the above constraints is $u_B < \frac{3\alpha - 2}{3\alpha - 3}$.

Summarizing, when $\alpha \in \left(\frac{1}{2}, 1\right]$ and $u_B < \frac{3\alpha - 2}{3\alpha - 3}$ then $p_{Gj}^* = \frac{1}{2\alpha}$ and $p_{Bj}^* = 0$ is a KKT solution.

**WA.3 Solution of Kuhn-Tucker conditions for proof of Lemma 7**

We solve the KKT conditions by cases based on possible values of $p_{G1}^*$ and $p_{B1}^*$:

1. When $p_{G1}^* = 0$ and $p_{B1}^* = 0$, the complementary slackness condition (CSC) $\lambda_1(p_{G1} - 1) = 0$ implies $\lambda_1 = 0$. The conditions in (33) become:

$$
\frac{\partial L}{\partial p_{G1}} = (1 - \alpha)\alpha (1 - u_B) + \lambda_3 = 0
$$

$$
\lambda_3 \geq 0
$$

However, $\lambda_3 = -(1 - \alpha)\alpha (1 - u_B) < 0$ when $\alpha \in [0, 1]$, contradicting $\lambda_3 \geq 0$, implying $p_{G1}^* = 0$ and $p_{B1}^* = 0$ is not a solution.
2. When $p_{G1}^* = 1$ and $p_{B1}^* = 1$, the CSC $\lambda_3(-p_{G1}) = 0$ and $\lambda_4(-p_{B1}) = 0$ implies $\lambda_3 = 0$ and $\lambda_4 = 0$. The conditions in (33) and (34) become:

\[
\frac{\partial L}{\partial p_{G1}} = \alpha (-3\alpha + 3(\alpha - 1)u_B + 1) - \lambda_1 = 0 
\] (78)

\[
\frac{\partial L}{\partial p_{B1}} = -(\alpha - 1) ((3\alpha - 2)u_B - 3\alpha) - \lambda_2 = 0 
\] (79)

However, $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ cannot hold at the same time, implying $p_{G1}^* = 1$ and $p_{B1}^* = 1$ is not a solution.

3. When $p_{G1}^* = 0$ and $p_{B1}^* = 1$, the CSC $\lambda_1(p_{G1} - 1) = 0$ and $\lambda_4(-p_{B1}) = 0$ implies $\lambda_1 = 0$ and $\lambda_4 = 0$. The conditions in (33) and (34) become:

\[
\frac{\partial L}{\partial p_{G1}} = -(\alpha - 1) \alpha (-2\alpha + (2\alpha - 3)u_B + 1) + \lambda_3 = 0 
\] (80)

\[
\frac{\partial L}{\partial p_{B1}} = (\alpha - 1) \alpha(3 - 2\alpha) + (2\alpha^2 - 5\alpha + 2) u_B) - \lambda_2 = 0 
\] (81)

However, $\lambda_3 \geq 0$ and $\lambda_2 \geq 0$ cannot hold at the same time, implying $p_{G1}^* = 0$ and $p_{B1}^* = 1$ is not a solution.

4. When $p_{G1}^* = 0$ and $p_{B1}^* \in (0, 1)$, the CSC implies $\lambda_1 = 0$, $\lambda_2 = 0$ and $\lambda_4 = 0$. The conditions in (34) become:

\[
\frac{\partial L}{\partial p_{B1}} = (\alpha - 1) (2(\alpha - 1)p_{B1} ((\alpha - 1)u_B - \alpha) + \alpha (1 - u_B)) = 0
\] (82)

yielding $p_{B1} = \frac{\alpha(u_B - 1)}{2(\alpha - 1)\alpha(i - 1)u_B - \alpha^2}$. However, in this case, $\frac{\partial L}{\partial p_{G1}} = \alpha - \alpha u_B + \lambda_3 > 0$, implying $p_{G1}^* = 0$ and $p_{B1}^* \in (0, 1)$ is not a solution.

5. When $p_{G1}^* \in (0, 1)$ and $p_{B1}^* = 1$, the CSC implies $\lambda_1 = 0$, $\lambda_3 = 0$, $\lambda_4 = 0$. The conditions in (34) become:

\[
\frac{\partial L}{\partial p_{G1}} = -\alpha (2\alpha p_{G1} (\alpha - (\alpha - 1)u_B) + (\alpha - 1) (-2\alpha + (2\alpha - 3)u_B + 1)) = 0
\] (83)

yielding $p_{G1}^* = \frac{(\alpha - 1)(2\alpha + 2\alpha u_B - 3u_B + 1)}{2\alpha(-\alpha + \alpha u_B - u_B)}$. However, in this case, $\frac{\partial L}{\partial p_{B1}} = (1 - \alpha) (u_B - 1) - \lambda_2 < 0$, implying $p_{G1}^* \in (0, 1)$ and $p_{B1}^* = 1$ is not a solution.

6. If $p_{G1}^* \in (0, 1)$, $p_{B1}^* \in (0, 1)$, then the CSCs imply that $\lambda_i = 0, i = 1, 2, 3, 4$. Conditions (33)
and (34) become:

\[
\frac{\partial L}{\partial p_{G1}} = \alpha - 2\alpha ((\alpha - 1)u_B - \alpha) ((\alpha - 1)p_{B1} - \alpha p_{G1}) + \alpha ((\alpha - 1)u_B - \alpha) = 0 \\
\frac{\partial L}{\partial p_{B1}} = p_{B1} (-2\alpha^2 + 4\alpha^2 - 2\alpha + 2(\alpha - 1)^3 u_B - 1) - (\alpha - 1)\alpha (2p_{G1} ((\alpha - 1)u_B - \alpha) + u_B) = 0
\]

(84)  

(85)  

\[p^*_{G1} \in (0,1), p^*_{B1} \in (0,1)\]

Solving the first equation, we have

\[p_{B1} = \frac{-\alpha + 2\alpha^2 u_B p_{G1} - 2\alpha u_B p_{G1} + \alpha u_B - u_B - 2\alpha^2 p_{G1} + 1}{2(\alpha - 1)(-\alpha + \alpha u_B - u_B)}\]

However, in this case \(\frac{\partial L}{\partial p_{B1}} = (1 - \alpha)(u_B - 1) < 0\). Hence, \(p^*_{G1} \in (0,1), p^*_{B1} \in (0,1)\) is not a solution.

7. If \(p^*_{G1} = 1, p^*_{B1} \in (0,1)\), then the CSCs imply that \(\lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0\). Conditions (33) and (34) become:

\[
\frac{\partial L}{\partial p_{G1}} = \alpha (-\alpha - 2((\alpha - 1)u_B - \alpha)((\alpha - 1)p_{B1} - \alpha) + (\alpha - 1)u_B + 1) - \lambda_1 = 0 \\
\frac{\partial L}{\partial p_{B1}} = p_{B1} (-2\alpha^3 + 4\alpha^2 - 2\alpha + 2(\alpha - 1)^3 u_B - 1) - (\alpha - 1)\alpha (-2\alpha + (2\alpha - 1)u_B) = 0
\]

(87)  

(88)  

The solutions are \(p_{B1} = \frac{\alpha(-2\alpha^2 + 2\alpha u_B - u_B - 1)}{2(\alpha - 1)(\alpha + \alpha u_B - u_B)}\) and \(p_{G1} = 1\). However, \(p_{B1} \in (0,1)\) is not possible when \(u_B \geq -\frac{\alpha}{1-\alpha}\). Hence, \(p^*_{G1} = 1, p^*_{B1} \in (0,1)\) is not a solution.

8. If \(p^*_{G1} = 1, p^*_{B1} = 0\), then \(\lambda_2 = 0, \lambda_3 = 0\). Conditions (33) and (34) become:

\[
\frac{\partial L}{\partial p_{G1}} = \alpha (-2\alpha^2 - \alpha + (2\alpha^2 - \alpha - 1) u_B + 1) - \lambda_1 = 0 \\
\frac{\partial L}{\partial p_{B1}} = -(\alpha - 1)\alpha (-2\alpha + (2\alpha - 1)u_B - 1) + \lambda_4 = 0
\]

\[\lambda_1 \geq 0, \lambda_4 \geq 0\]

(89)  

(90)  

(91)

(a) \(\lambda_1 = \alpha (-2\alpha^2 - \alpha + (2\alpha^2 - \alpha - 1) u_B + 1) \geq 0 \Rightarrow u_B \leq -\frac{\alpha^2 - \alpha + 1}{2\alpha^2 + \alpha + 1}\).

(b) \(\lambda_4 = (\alpha - 1)\alpha (-2\alpha + (2\alpha - 1)u_B - 1) \geq 0\) always holds when \(u_B \geq -\frac{\alpha}{1-\alpha}\).

(c) If firm 1 sets a price lower than \(u(b)\), when firm 2 uses mixed pricing strategies according
to $F_2(m) = 1 - \frac{u(g) - Pr(g)E[u]}{(m + (u(g) - E[u]))}$ for $Pr(b) * E[u] \leq m_2 \leq E[u]$, its expected profit will be:

$$\pi_1(m_1) = m_1 * [Pr(g) + Pr(b)(1 - F_2(m_1 + (E[u] - u(b))))], \text{ for } m_1 \leq u(b)$$ (92)

The deviation profit is maximized by setting the price to $u_j(b)$. The deviation profit should be lower than the equilibrium profit, yielding a constraint $u_B < -\frac{\sqrt{4(\alpha-8\alpha^2+8\alpha^3-4\alpha^4+1)} - 1}{2(\alpha-1)^2}$.

(d) The intersection of above constraints is $u_B < -\frac{-2\alpha^2 - \alpha + 1}{2\alpha^2 + \alpha + 1}$ for $\alpha \in [0, 0.366025]$; $u_B < -\frac{\alpha}{2(\alpha-1)^2}$ for $\alpha \in [0.366025, 1]$.

To conclude, $p_{G1}^* = 1, p_{B1}^* = 0$ is a possible solution.

9. If $p_{B1}^* = 0, p_{G1}^* \in (0, 1)$, then, $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0$. Conditions (33) and (34) become:

$$\frac{\partial L}{\partial p_{G1}} = -\alpha^2 + \alpha + p_{G1} (-2\alpha^3 + 2(\alpha - 1)\alpha^2 u_B) + (\alpha - 1)\alpha u_B = 0$$ (93)
$$\frac{\partial L}{\partial p_{B1}} = -(\alpha - 1)\alpha (2p_{G1} ((\alpha - 1)u_B - \alpha) + u_B - 1) + \lambda_4 = 0$$ (94)
$$\lambda_4 \geq 0$$ (95)

Solving (53) yields $p_{G1} = -\frac{(\alpha - 1)(u_B - 1)}{2\alpha((\alpha - 1)u_B - \alpha)}$ and $p_{B1} = 0$.

(a) $p_{G1} \in (0, 1)$ implies $u_B > -\frac{2\alpha^2 - \alpha + 1}{2\alpha^2 + \alpha + 1}$.

(b) $\lambda_4 = (1 - \alpha)(1 - u_B) \geq 0$ always holds when $u_B \geq -\frac{\alpha}{1 - \alpha}$.

(c) If firm 1 sets a price lower than $u(b)$, when firm 2 uses mixed pricing strategies according to $F_2(m) = 1 - \frac{u(g) - Pr(g)E[u]}{(m + (u(g) - E[u]))}$ for $Pr(b) * E[u] \leq m_2 \leq E[u]$, its expected profit will be:

$$\pi_1(m_1) = m_1 * [Pr(g) + Pr(b)(1 - F_2(m_1 + (E[u] - u(b))))], \text{ for } m_1 \leq u(b)$$ (96)

The deviation profit is maximized by setting the price to $u(b)$. The constraint that the deviating profit is lower than the equilibrium profit yields the condition $u_B < \frac{\sqrt{3} - 3\alpha}{3 - 3\alpha}$.