Stability-Equivalence of Bailouts and Bailins with Welfare consequences

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Abstract

In a global game, I show that creditor bailins, when well-designed, can attain the exact same level of bank stability as costly creditor bailouts. This result holds for both risk-averse and risk-neutral creditors. Because bailouts are costly but do not necessarily provide a stability advantage, a “stability-equivalent” bailin can yield higher welfare than a bailout either if the bank is small in the economy or if the bank is large and the ex ante stability level of the bank is high. This holds even though impatient creditor types exist that have to consume early and suffer from a bailin.

Key words: financial regulation, bank runs, global games, stability-equivalence, policy effectiveness, bank resolution, haircuts, bailout, withdrawal fees, money market mutual fund gates, suspension of convertibility

JEL Classification: G28,G21,G33, G38, D82, D81, E61

*Olin School of Business at Washington University in St Louis, lindas@wustl.edu. 1 Snow Way Dr, St. Louis, MO 63130. The existence and uniqueness result in Proposition 3.1, and the stability-equivalence result for the risk-neutral special case in section 4.2 had originally been part of the paper “Smooth regulatory intervention,” but are no longer a part of that paper.
1 Introduction

In times of crises, deposit insurance, guarantees, preferred equity injections, or generically “bailouts” of bank creditors are a successful and celebrated tool to stop or deter runs on financial institutions (Bryant, 1980; Diamond and Dybvig, 1983). Nevertheless, such bailouts are known to create a large range of moral hazard problems (Calomiris and Gorton, 1991; Farhi and Tirole, 2012; Dewatripont, 2014; Keister, 2016; Calomiris and Jaremski, 2016; Philippon and Wang, 2023; Dávila and Goldstein, 2023) and are expensive to the taxpayer. Most recently in 2023, a run on and subsequent failure of Silicon Valley Bank and Signature Bank prompted the FDIC to insure SVB and SB depositors in full beyond the $250,000 limit, stating a “systemic risk exception”. The FIDC estimates the cost of this controversial decision at $16.3bn to the deposit insurance fund. In 2008, the Troubled Asset Relief Program (TARP) was endowed with $700bn to stabilize the financial system by purchase or insurance of troubled assets, see Lucas (2019).


In parallel to such costly bailouts, also non-costly regulatory mechanisms exist to deter runs on financial institutions: redemption fees, haircuts, suspension of convertibility or generically creditor bail-ins are examples; see Diamond and Dybvig (1983); Green and Lin (2003); Greene, Hodges, and Rakowski (2007); Gorton and Metrick (2009); Schmidt, Timmermann, and Wermers (2016); Schilling (2023); Agarwal, Ren, Shen, and Zhao (2023).

The application of these distinct mechanisms for attaining the same goal raises the question of whether costly tools (“bailouts”) are more effective than non-costly tools (“bailins”) when it comes to reducing the run risk and whether they provide higher levels of welfare in the economy.

As the main contribution of the paper, I show that creditor bail-ins, when well-designed, can attain the exact same level of ex ante bank stability as costly creditor

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1 Technically, deposit insurance is not a bailout since the deposit insurance fund is financed via insurance premiums paid by banks. Banks can however forward these expenses to their customers who are predominantly taxpayers since deposit-taking is a local business.

2 Among the recipients were JPMorgan Chase and Wells Fargo, with $25 bn each, Bank of America with $15 billion; Morgan Stanley and Goldman Sachs with $10 billion each, and American International Group (AIG). Lucas (2019) also states the fair value costs of the bailout to creditors of Fannie Mae and Freddie Mac at $291bn.
bailouts. This result holds for both risk-averse and risk-neutral agents and allows for patient and impatient creditor types that differ in their preferred timing of consumption. While in this paper I refer to banks, the result likewise applies to money market mutual funds since the creditor payoffs in this model are left general.

“Stability-equivalence” of costly and non-costly policies has welfare implications. By the sameness of bank stability between a bailout and the accordingly designed bailin policy, bailouts do not necessarily provide a stability advantage. As a consequence, the costly taxation of all citizens and creditors to finance the bailout becomes the focal point of the welfare analysis. Naturally, I can show, that if the bank is small relative to the remaining economy then a stability-equivalent bailin provides a higher level of welfare than a bailout because regular citizens who are not affiliated with the bank suffer from taxation more than creditors benefit from the bailout. However, even if the bank is large in the economy, I show that a stability-equivalent bailin is preferred to the bailout if the ex ante stability level of the bank following either policy is high. This result holds even though there exist impatient creditor types that have to consume early and thus suffer extraordinarily from a bailin. The result obtains because conditional on no run, the bailin policy yields higher expected payoffs to all creditors than the bailout since the bailout requires high taxation. Only conditional on a run, the creditors prefer a bailout over the bailin. The welfare difference when imposing the bailout relative to the stability-equivalent bailin, thus, depends on how often a painful bailin occurs relative to good times during which a credible bailout requires the imposition of high taxes. These results continue to hold if bailouts are financed via increased levels of government debt which is just higher future taxation.

In conclusion, the results of the paper challenge the necessity of expensive bailouts given that bailins can be equally stable and provide higher levels of welfare.

This is to the best of my knowledge the first paper that tackles the possibility of “stability-equivalence” across different types of regulatory policy intervention. Here the term “stability-equivalence” of policies refers to the feature that the application of different regulatory policies can imply sameness of ex ante bank stability. To attain a well-defined endogenous run probability on the bank, I employ a global games framework (Carlsson and Van Damme, 1993; Morris and Shin, 2001; Frankel et al., 2003), generalizing the celebrated Goldstein and Pauzner (2005) model to allow for general payoffs.

The bailouts I consider can be thought of as a deposit guarantee (partial deposit insurance) or equity injection whereas the bailin should be interpreted as a withdrawal-
contingent haircut or gradual suspension of convertibility, as also analyzed in Wallace et al. (1988); Chari (1989); Peck and Shell (2003); Green and Lin (2003). In contrast to these papers, the generality of my model allows me to design stability-equivalent “bailins” for various types of bailouts and contracts between the bank and its creditors. I allow the original bailout amount to change with the aggregate withdrawals (size of the run). This means, the bailout amount may increase, be constant or decline as the run becomes larger, thus, impacting the value of debt in different ways. Because the value of the creditor contract with the bank, and the bailout are both dependent on the size of the run, the stability-equivalent bailin takes the form of withdrawal-contingent payoffs that are specific to the creditors’ utility function, the “shape” of the bailout, the creditors’ contract with the bank, and the bank asset’s payoff. While my main model keeps creditor payoffs general, I also consider a specific application, where I build a bailout in the form of a partial deposit guarantee into the Goldstein and Pauzner (2005) model with demand-deposit contracts, and derive the according stability-equivalent bailin for different levels of the guarantee.

For intuition on how I construct the least costly, feasible and stability-equivalent alternative policy to the bailout, I gradually reduce the original bailout guarantee $b$ to agents that roll over while keeping the ex ante run probability constant. To keep the run probability constant, I need to compensate for the reduction in the guarantee $b$ and the tax by simultaneously reducing the payoffs to agents that withdraw by the “right amount.” Here, the right amount of the payoff reduction (bailin) is such that the rollover incentives, that is, the payoff difference between roll-over and withdrawal remains exactly constant. Because the roll-over incentives differ depending on the size of the run, the stability-equivalent bailin needs to be designed for every possible size of the run. Ultimately, I show that the bailout to agents that roll over can feasibly be reduced to zero while keeping stability constant. Feasibility requires that the consumption levels of all agents, in particular those that incur the bailin, need to remain positive as I search for the least costly, stability-equivalent policy alternative, taking into account how the bailout reduction impacts the endogenous tax to all agents in the economy.

The main analysis disregards moral hazard, and assumes that the bank does not react as policy transitions from the bailout to the less costly policy. In section 7, I show how moral hazard can be incorporated into the derivation of stability-equivalent bailins. There, the bank is allowed to shift risk in the form of choosing a different asset depending
on the policy.

1.1 Literature

This paper contributes to three literature strands. With regard to the model, the closest related papers are Goldstein and Pauzner (2005), Morris and Shin (2009), and Rochet and Vives (2004).

The paper contributes to the large literature that analyzes the occurrence and prevention of runs on financial firms (Bryant (1980); Diamond and Dybvig (1983); Postlewaite and Vives (1987); Allen and Gale (2000); Ennis and Keister (2009); Green and Lin (2003); Allen and Gale (2004); Peck and Shell (2003); Rochet and Vives (2004); Goldstein and Pauzner (2005); Gorton and Metrick (2012); Schmidt et al. (2016); Andolfatto et al. (2017); Fernández-Villaverde et al. (2021); Liu (2023)).

This paper contributes to the literature on the effectiveness and drawbacks of financial regulation and policy intervention and their impact on bank stability. Keister (2016) shows that if financial intermediaries expect bailouts in times of crises, they choose illiquid and fragile asset positions which undermines the bailout provision. Fink and Scholl (2016) show that the prevention of sovereign default via bailouts in the short run may come at the cost of a higher default probability in the long run. Farhi and Tirole (2012) show that private leverage choices of banks become strategic complements if the policy response during crises is imperfectly targeted. Schilling (2023) studies the impact of suspension of convertibility policies on bank stability, showing that too conservative intervention can backfire by making runs more rather than less likely ex ante. In that same spirit, Zhong and Zhou (2021) study the impact of bankruptcy code design on run incentives in a dynamic setting, demonstrating possible front-running incentives. Schilling (2022) characterizes general features of policy intervention that backfire in terms of reducing rather than increasing bank stability. Bernard, Capponi, and Stiglitz (2022) show that banks are willing to participate in bail-ins if they fear contagion and if the regulator credibly commits to no bailouts. Likewise, Keister and Mitkov (2023) study the interplay between a government’s bailout provision and a bank’s willingness to impose losses, showing that the optimal policy requires the regulator to bound the size of the bailin, and delegate the decision on the size of the bailin to the bank.

The paper applies a global games framework (Carlsson and Van Damme, 1993; Morris and Shin, 2001; Frankel et al., 2003) to select a unique equilibrium, leading to an endogenous ex ante run-probability on the bank. For this purpose, I generalize the Goldstein and

2 Model

The model generalizes the Goldstein and Pauzner (2005) model, allowing for general payoffs. There are three time periods, $t = 0, 1, 2$ and four kinds of agents.

There exists a bank, a regulator, a continuum of bank creditors $i \in [0, 1] \times M_2$ and a continuum of citizens that are not affiliated with the bank, given by the measure $M_1$. All creditors and citizens are taxable to finance policy. Denote by $M = M_1 + M_2$ the measure of the taxable population. The bank creditors and citizens are risk-averse, with positive, strictly increasing and weakly concave utility $u(\cdot)$ over consumption. While I describe the firm as a bank it can likewise be a money market mutual fund, or any financial company whose investors roll-over funds on prespecified time periods.

At time zero, the bank creditors are symmetric, and are each endowed with one unit to invest in a bank contract. At $t = 1$, some investors learn that they are impatient and thus have to consume in $t = 1$. Let $\lambda \in [0, 1]$ denote the ex ante chance of being the impatient type. The remaining share $1 - \lambda$ of creditors is called patient and they can consume in both $t = 1$ or $t = 2$. Types are private information. The firm requires funding for investment, and for that purpose collects endowments from the investors in $t = 0$. I assume that investing is individually rational to investors. Returns to scale are constant. The initial firm investment and thus funding via investors is normalized to one unit.

**State** Let $\theta \sim U[0, 1]$ denote the unobservable, random state of the economy. The state realization is payoff relevant to investors. One may think of $\theta$ as parametrizing the quality of a risky bank asset, see below.
**Investor contract and payoffs**  In $t = 0$, the bank needs to raise funds for investment, and offers a contract to the investors. All investors invest their endowment in the contract, thus becoming a bank creditor. The bank invests the funds in a risky asset. For a one unit investment the asset pays a risky return $R(\theta)$ in $t = 2$ and the asset can be (partially) liquidated in $t = 1$ at a value $L(\theta)$. For the bank’s creditors, after investing in the bank contract in $t = 0$, at $t = 1$ they need to decide on their action. A creditor either “withdraws” her investment and thus opts for the short-term payoff $u(c_1(n, \theta))$ payable in $t = 1$, or she “rolls over” her investment until $t = 2$, opting for the payoff $u(c_2(n, \theta))$ payable in $t = 2$ where $n \in [0, 1]$ denotes the endogenous share of investors who withdraw in $t = 1$ (aggregate withdrawals). Creditors that roll over keep their funds in the bank, thus contributing to the bank’s stability whereas agents that withdraw contribute to a run of size $n$. The bank’s asset value $R(\theta)$ is relevant to the bank’s creditors in the following way. I assume that in $t = 1$, the bank finances withdrawals $n$ by liquidating assets. I assume that the bank asset’s value at state $\theta$ and aggregate withdrawals $n$ are such that the contracted payoffs $u(c_1(n, \theta))$ and $u(c_2(n, \theta))$ are feasible. This means, at every state $\theta$ and every withdrawal level $n \in [0, 1]$ the asset’s liquidation value $L(\theta)$ and the continuation value $R(\theta)$ satisfy both feasibility constraints

$$n \frac{u(c_1(n, \theta))}{L(\theta)} < 1 \quad (1)$$

and

$$R(\theta) \left( 1 - n \frac{u(c_1(n, \theta))}{L(\theta)} \right) > (1 - n) u(c_2(n, \theta)) \quad (2)$$

The first constraint says that the bank can satisfy all withdrawals by liquidating assets. The second constraint says that no matter how high the withdrawals, the remaining bank investment is sufficient to cover the payoffs to creditors that roll over. The feasibility of payoffs for every $(\theta, n)$ is common knowledge among all investors. Note that depending on the liquidity of assets, feasibility may require payoffs $u(c_2(n, \theta)) = 0$ or $u(c_1(n, \theta))$ small for $n$ high or $\theta$ low.

I model the creditors’ payoffs as dependent on the aggregate withdrawals because in classic models of bank runs (Diamond and Dybvig, 1983; Ennis and Keister, 2006; Goldstein and Pauzner, 2005), the liquidation of illiquid bank assets due to high volume withdrawals typically reduces the payoffs to agents that roll over (liquidation friction). In addition, this allows regulatory intervention to occur conditional on high volume withdrawals, thus, impacting the payoffs to all agents.

I do not model discounting explicitly but a discount factor can be accommodated
indirectly via the payoff \( u(c_2(n, \theta)) = \delta \hat{u}(c_2(n, \theta)) \).

I assume that the payoffs satisfy monotonicity conditions in the state \( \theta \), and the aggregate withdrawals \( n \), as summarized below in assumption 2.1. The functional forms of \( u(c_1(n, \theta)) \) and \( u(c_2(n, \theta)) \) are known to the depositors ex ante.

**Signals** Before the investors choose actions in \( t = 1 \), they observe noisy, private signals about the state \( \theta \),

\[
\theta_i = \theta + \epsilon_i, \; i \in [0, 1].
\]  

(3)

The idiosyncratic noise term \( \epsilon_i \) is independent of the state \( \theta \) and is distributed iid according to the uniform distribution \( \epsilon_i \sim U[-\epsilon, +\epsilon] \).

**Bailout Provision and Taxation** To improve the bank’s stability, I assume that in \( t = 0 \), the regulator sets and commits to providing a (partial) bailout \( b(n) \in \mathbb{R}_+ \), to creditors that roll over their deposit whenever a run on the bank starts to form, that is, if withdrawals exceed a threshold \( n \in [n_p, 1] \). Threshold \( n_p \) is set by the regulator, and is the smallest withdrawal level (size of the run) at which the regulator intervenes to provide the bailout. One should think of \( b(n) \) as a withdrawal-contingent payment function to investors in terms of consumption units, altering utility to \( u(c_2(n, \theta) + b(n)) \).

The regulator makes no payment to agents that withdraw. This bailout can take the specific form of partial deposit insurance which is common in Europe and the U.S., see the application section 6. I assume that the bailout function \( b(n) \) is bounded on \([n_p, 1]\) and piecewise continuous in the aggregate withdrawals \( n \), and thus integrable. Without loss of generality, I assume \( n_p > \lambda \), that is, intervention only occurs if the withdrawals are abnormally high.

Bailouts are costly. To finance the bailout, the regulator (government) symmetrically taxes the entire agent population in \( t = 0 \). Denote by \( \tau(b) \) the ex ante budget balancing lump-sum tax imposed on all agents, creditors and citizens, in \( t = 0 \). The tax itself is an equilibrium object since it depends on the endogenous ex ante run probability which in return depends on the payoffs and thus the tax. I pin down the tax below.

In the case where the bailout is paid in the form of deposit insurance, one might argue that it is not taxpayers but banks that finance the deposit insurance fund (DIF) in the form of insurance premia. In the U.S., the DIF is however backed by the United States government, and thus, ultimately by the U.S. taxpayer. Moreover, the costs of insurance premia can be forwarded to U.S. bank deposit customers, and thus, again, taxpayers. Alternatively one might think about financing bailouts via government debt rather than
immediate taxation. But increased levels of government debt correspond to higher future taxation, and thus also a higher net present value of future taxes. The analysis would be the same.

With the original bailout $b(n)$, payoffs are given by

$$ u_1(b,0) = \begin{cases} u(c_1(n) - \tau(b, \varepsilon)), & n \in [n_p, 1] \\ u(c_1(n) - \tau(b, \varepsilon)), & n \in [0, n_p] \end{cases} $$

$$ u_2(b,0) = \begin{cases} u(c_2(n) + b(n) - \tau(b, \varepsilon)), & n \in [n_p, 1] \\ u(c_2(n) - \tau(b, \varepsilon)), & n \in [0, n_p] \end{cases} $$

Recall that a credible bailout requires budget-balanced financing, that is, the bailout has to be readily available via ex ante taxation which necessarily means that in good times, $n \in [0, \lambda]$, the funds that are set aside seem inefficient. Note, however, that it is not possible to tax agents only in bad times when the bailout is required, because at that stage, in $t = 1$, the bank has already invested all of the agents’ funds, so taxation would require partial liquidation of deposits.

The payoff difference of rolling over versus withdrawing funds when granting the bailout is given as

$$ v_{BO}(n, \theta) = \begin{cases} u(c_2(n, \theta) + b(n) - \tau(b)) - u(c_1(n, \theta) - \tau(b)), & n \in [n_p, 1] \\ u(c_2(n, \theta) - \tau(b)) - u(c_1(n, \theta) - \tau(b)), & n \in [0, n_p] \end{cases} $$

where subscript $BO$ stands for bailout. Note that in equilibrium, impatient types always withdraw early which is why aggregate equilibrium withdrawals always realize in $[\lambda, 1]$.

I assume that the bailout $b$ is feasible. That is, the bailout is small enough such that the budget-balancing lump-sum tax satisfies

$$ \tau(b) \leq \begin{cases} c_2(n, \theta) + b(n), & n \in [n_p, 1] \text{ agents that roll over can pay the tax} \\ c_2(n, \theta), & n \in [\lambda, n_p] \text{ agents that roll over can pay the tax} \\ c_1(n, \theta), & \text{agents that withdraw can pay the tax} \\ \bar{c}, & \text{citizens can pay the tax} \end{cases} $$

where I denote by $\bar{c}$ the exogenous, optimal consumption level chosen by the symmetric citizens absent the bailout.

To guarantee equilibrium existence and uniqueness, I impose monotonicity conditions on the creditor’s payoff difference function following Morris and Shin (2001) section 2.2.2. and 2.2.3.
Assumption 2.1. Fix a bailout policy \( b(n) \in \mathbb{R}_+ \), \( n \in [n_p, 1] \). It holds

1. **(Strict state Monotonicity:)** \( v(n, \theta) \) is non-decreasing in \( \theta \), and strictly increasing in \( \theta \) for all \( \theta \in [\theta_p, \bar{\theta}_p] \).

2a. **(Action single crossing:)** For every state \( \theta \in [\theta_p, \theta_p] \), there exists \( n^*(p) \in (0, 1) \) such that \( v(n, \theta) > 0 \) for all \( n < n^*(p) \) and \( v(n, \theta) < 0 \) for all \( n > n^*(p) \).

2b. **(One-sided strategic complementarity:)** For every state \( \theta \in [\theta_p, \theta_p] \), whenever \( n \) is such that \( v(n, \theta) > 0 \), then \( v(n, \theta) \) is strictly decreasing in \( n \).

3. **(Uniform limit dominance:)** There exist upper and lower regions of action dominance: There exist \( \theta_p, \theta_p \in (0, 1) \) and \( \epsilon > 0 \) such that: if \( \theta \in [0, \theta_p] \), then withdraw is dominant, \( v(n, \theta) < -\epsilon \), for all \( n \in [0, 1] \) while for \( \theta \in [\theta_p, 1] \), roll-over is dominant \( v(n, \theta) > \epsilon \), for all \( n \in [0, 1] \).

Note, this assumption implies that the bailout is not large enough to entirely deter runs ex ante. There are two reasons for this assumption. First, it keeps the analysis interesting. Second, bailouts that deter runs entirely are very costly in terms of taxation, and may not be feasible.\(^3\) In the real world, for instance, deposit insurance is limited to $250,000, and a large share of U.S. deposits is uninsured.\(^4\)

Assumption 2.2. Given the bailout policy \( b(n), n \in [n_p, 1] \), the payoff difference function \( v(n, \theta) \) is continuous in \((n, \theta) \in [0, 1] \times [0, 1] \), and differentiable in \( \theta \in (\theta_p, \bar{\theta}_p) \).

Assumption 2.2 is important for equilibrium existence and uniqueness because it establishes continuity of the expected payoffs in the signal observed by investors. Because continuous functions on compact intervals are bounded, the assumption also implies that the payoff difference \( v(n, \theta) \) is Lebesgue integrable for all \((n, \theta) \in [0, 1] \times [0, 1] \).

Timing

- In \( t = 0 \), the regulator sets and fully commits to her bailout policy \( b(n), n \in [n_p, 1] \) without observing the state. The bailout policy \( b \) is common knowledge among all agents, and the policy choice conveys no information. The regulator taxes all citizens and creditors, requiring the payment \( \tau(b) \). Then, the state \( \theta \) realizes unobservably to all agents. All investors invest in the bank contract.

\(^3\)Designing bailouts respectively bailins to deter runs for sure is simple: For the bailin, announce to pay zero to every withdrawing agent. This makes “roll-over” a dominant action. For the bailout, announce to pay every agent that rolls over twice the amount you pay to a withdrawing agent. This, likewise, makes “roll over” a dominant action. The second policy is more expensive.

\(^4\)See for instance (Schilling, 2019) where the provision of high deposit insurance can lead to inefficient losses to the deposit insurance fund because the depositors roll over their deposits even for bad news on the bank’s assets.
In $t = 1$, all creditors observe their private signal $\theta_i$. Based on the signal and the bailout policy, they decide whether to withdraw. The bank observes the aggregate withdrawals $n \in [0, 1]$, and liquidates assets to pay $u(c_1(n, \theta))$. Moreover the regulator observes the withdrawals and gets ready to provide the bailout $b(n)$ to agents that roll over at $t = 2$ if withdrawals are above the cutoff $n_p$.

In $t = 2$, $\theta$ is revealed, and creditors that roll over receive the payoff $u(c_2(n, \theta))$ from the bank, and additionally receive the bailout $b(n)$ if withdrawals have realized in $n \in [n_p, 1]$.

The equilibrium concept is perfect Bayes Nash. Proofs that are not in the main text can be found in the appendix. Since the state $\theta$ is observed late in $t = 2$, the payoff to withdraw, $u(c_1(n, \theta))$, can depend on $\theta$ only if it is paid late in $t = 2$.

**3 Equilibrium Existence and Uniqueness**

The following result is the Goldstein and Pauzner (2005) existence and uniqueness result for equilibrium but for general payoffs subject to the monotonicity requirements in assumptions 2.1 and 2.2.

**Proposition 3.1** (Equilibrium Existence and Uniqueness at status quo)

*Fix a feasible bailout policy $b(n), n \in [n_p, 1]$. Assume, the preferences of investors satisfy assumptions (2.1) and (2.2). As noise vanishes, $\varepsilon \to 0$, the investor’s coordination game has a unique equilibrium, and the equilibrium is in trigger strategies. There exists a unique trigger signal $\theta^*(b)$ that makes an investor indifferent between rolling over the deposit or withdrawing. For signals below the trigger $\theta_i < \theta^*(b)$ an investor optimally withdraws. For signals above the trigger $\theta_i > \theta^*(b)$, roll-over is optimal.*

For tie-breaking reasons, I assume that an investor rolls over the investment whenever observing the equilibrium trigger, $\theta_i = \theta^*(b)$. To give more insight into the features of a trigger equilibrium, given an equilibrium trigger signal $\theta^*(b)$, the equilibrium withdrawals are a deterministic function of the state, given by

$$n(\theta, \theta^*(b)) = \begin{cases} 
\lambda + (1 - \lambda) \left( \frac{1}{2} + \frac{\theta^*(b) - \theta}{2\varepsilon} \right), & \theta \in [\theta^*(b) - \varepsilon, \theta^*(b) + \varepsilon] \\
1, & \theta < \theta^*(b) - \varepsilon \\
\lambda, & \theta > \theta^*(b) + \varepsilon 
\end{cases}$$

(7)
For a given bailout \( b(n) \), consider the expected payoff difference of the marginal investor who observes exactly the trigger signal \( \theta_i = \theta^* \),

\[
H(b, \theta^*(b)) = \frac{1}{2\varepsilon} \int_{\theta^*(b)-\varepsilon}^{\theta^*(b)+\varepsilon} v(n(\theta), \theta) d\theta
\]  

(8)

Given the observation \( \theta_i = \theta^* \), the true state can only be \( \varepsilon \) away in either direction. The following notation is sometimes more convenient,

\[
H(b, \theta^*(b)) = \frac{1}{1-\lambda} \int_{\lambda}^{1} v(n, \theta(n, \theta^*(b))) dn
\]  

(9)

and is obtained via substitution for \( \theta(n, \theta^*(b)) \) which is the inverse of (42), and equals the state consistent with measure \( n \) withdrawals if all depositors play a trigger strategy around \( \theta^*(b) \),

\[
\theta(n, \theta^*(b)) = \theta^*(b) + \varepsilon(1 - 2n), \theta^*(b) \in [\theta - \varepsilon, \theta + \varepsilon].
\]  

(10)

The expected payoff difference (9) reflects that from the perspective of the marginal investor, the aggregate withdrawals are uniformly distributed on \([\lambda, 1]\) (“Laplacian Belief”), because aggregate withdrawals equal, by a law of large numbers, the share of investors who observe signals below \( \theta^*(p) \).

As is standard in global games theory, the equilibrium trigger signal \( \theta^*(b) \) is implicitly characterized as the zero,

\[
H(b, \theta^*(b)) = 0.
\]  

(11)

Given the trigger signal \( \theta^*(b) \), a unique cut-off state \( \theta_p \in [\theta_p, \overline{\theta}_p] \), the critical state, exists at which the aggregate withdrawals realize such that the bailout intervention gets triggered:

\[
n(\theta_p, \theta^*(b)) = n_p.
\]  

(12)

For a given trigger signal, state realizations below \( \theta_p \) imply lower signal realizations and thus higher aggregate withdrawals. Therefore, if and only if \( \theta < \theta_p \), a bailout is provided. Because the state is uniformly distributed, the ex-ante probability of a regulatory bailout provision equals \( \theta_p \).

Because the unique equilibrium is a trigger equilibrium around signal \( \theta^* \), the equilibrium tax to finance bailout \( b(n) \) is given as

\[
\tau(b, \varepsilon) = \frac{M_2}{M} \int_{0}^{\theta_p} b(n(\theta, \theta^*(b))) d\theta
\]  

(13)
For states below $\theta_p$, the aggregate withdrawals realize above $n_p$, and trigger the bailout provision. Bank creditors of measure $M_2$ possibly have a claim on the bailout whenever it is provided. All agents of measure $M$ are taxed in $t = 0$ to finance the bailout.

4 Stability-equivalence of Bailins and Bailouts

In this section, I demonstrate that for the given costly bailout $b(n)$ there exist feasible, less costly alternative policies that attain ex ante identical run probabilities. These less costly policies reduce the bailout provision to investors that roll over, and thus require a smaller tax for financing. To keep the run-propensity of investors constant though, the commitment to a reduced bailout must be accompanied by a simultaneous commitment to partially bail in investors that withdraw.

Costly bailout policies are commonly implemented in the form of guarantees or deposit insurance, while the non-costly bailins are often named suspension of convertibility or haircuts. To reduce jargon,

**Definition 4.1** (Bailin and Bailout). I refer to a “bailout policy” to bank creditors that roll over, when money is raised via taxation from all agents in the economy, and is injected into the bank to increase the contract value to bank creditors that roll over, increasing $u_2(c)$. I refer to a “bailin” of agents that withdraw when no taxation is required, and the contract value to withdrawing investors is reduced, lowering $u_1(c)$.

I introduce the following notation: I write the original bailout policy as $(b(n), 0)$ since a bailout $b(n)$ is provided to agents that roll over and a zero bailin is imposed on withdrawing agents. For the less costly policy alternative I write $(\hat{b}(n), \kappa(\hat{b}, n))$ since a lower bailout $\hat{b}(n) \in [0, b(n)]$ is granted to agents that roll over whereas the bailin $\kappa(\hat{b}, n)$ is imposed on agents that withdraw.

I need to pin down what it means that a bailout provision is “less costly.”

**Definition 4.2** (Less costly policy). Consider the original bailout policy $b(n)$ for all $n \in [n_p, 1]$. A policy $\hat{b}(n) \in [0, b(n)]$ for all $n \in [n_p, 1]$ is less costly than $b(n)$ if

$$\hat{b}(n) \leq b(n), \; \text{for all } n \in [n_p, 1]$$

(14)

I further assume that the less costly bailout $\hat{b}(n)$ is piecewise continuous in the aggregate withdrawals on $[n_p, 1]$, and thus integrable.
Definition 4.3 (Stability-equivalence). Two distinct policies \((b(n), 0)\) and \((\hat{b}, \kappa(\hat{b}))\) are “stability-equivalent” if they imply bank creditor payoffs to roll-over versus withdraw such that the resulting trigger equilibria, and thus ex ante run probabilities (bank stability) are the same, \(\theta^*(b(n), 0) = \theta^*(\hat{b}, \kappa(\hat{b}))\) and \(\theta_p(b(n), 0) = \theta_p(\hat{b}, \kappa(\hat{b}))\).

Note, at this point I have not verified that the stability-equivalent equilibrium trigger signal \(\theta^*(\hat{b}, \kappa(\hat{b}))\) exists and is unique when imposing the alternative policy \((\hat{b}, \kappa(\hat{b}))\) on the bank’s creditors. The construction of that policy in the appendix however reveals that existence and uniqueness hold.

Given stability-equivalence of policy, if the alternative policy is less costly, \(\hat{b}(n) \in [0, b(n)]\) for all \(n \in [n_p, 1]\), also the ex ante tax is lower, given by

\[
\tau(\hat{b}, \varepsilon) = \frac{M_2}{M} \int_{0}^{\theta_p} \hat{b}(n(\theta, \theta^*(b)))d\theta \leq \tau(b, \varepsilon). \tag{15}
\]

When constructing a less costly, stability-equivalent alternative policy \((\hat{b}(n), \kappa(\hat{b}))\), I proceed the following way. I take as given the less costly bailout \(\hat{b}(n) \leq b(n)\), and then need to find the accompanying bailin function \(\kappa(\hat{b}, n)\) such that stability-equivalence holds. For the construction, it is important to notice that the reduced ex ante tax affects relative payoffs, and thus roll-over incentives in the entire withdrawal range \(n \in [0, 1]\). I therefore need to design the stability-equivalent bailin \(\kappa(\hat{b}, n)\) not only in the withdrawal range for which the original bailout is triggered, \([n_p, 1]\), but for the full range \(n \in [0, 1]\).

Once we have found the less costly, stability-equivalent policy \((\hat{b}(n), \kappa(\hat{b}))\), the creditors’ payoffs become

\[
u_1(\hat{b}, \kappa(\hat{b})) = \begin{cases} u(c_1(n) - \tau(\hat{b}, \varepsilon) - \kappa(\hat{b}, n)), & n \in [n_p, 1] \\ u(c_1(n) - \tau(\hat{b}, \varepsilon) - \kappa(\hat{b}, n)), & n \in [0, n_p) \end{cases} \tag{16}
\]

\[
u_2(\hat{b}, \kappa(\hat{b})) = \begin{cases} u(c_2(n) + \hat{b}(n) - \tau(\hat{b}, \varepsilon)), & n \in [n_p, 1] \\ u(c_2(n) - \tau(\hat{b}, \varepsilon)), & n \in [0, n_p) \end{cases} \tag{17}
\]

Here, I assume that the intervention threshold \(n_p\) is held constant when transitioning from the original bailout to the less costly policy. For a variation of this feature, see section 6 where the intervention threshold changes with the policy alternative.

The payoff difference to roll-over versus withdrawal following the less costly policy is
given as
\[
v_{BI}(n, \theta) = \begin{cases} 
  u(c_2(n, \theta) + \hat{b}(n) - \tau(\hat{b})) - u(c_1(n, \theta) - \tau(\hat{b}) - \kappa(\hat{b}, n)), & n \in [n_p, 1] \\
  u(c_2(n, \theta) - \tau(\hat{b}))) - u(c_1(n, \theta) - \tau(\hat{b}) - \kappa(\hat{b}, n)), & n \in [0, n_p]
\end{cases}
\] (18)

where subscript BI stands for bailin.

The following result is the main result in the paper, and I demonstrate an application of it in Proposition (6.1) below. It entails not only the existence of less costly feasible, and stability-equivalent policies but entails the exact design of the bailin policies \( \kappa(\hat{b}) \), including the least costly stability-equivalent policy.

**Proposition 4.1** (Existence of less costly, stability-equivalent policy)

Consider the original costly, feasible bailout that allocates the withdrawal-contingent payoff \( b(n) \geq 0 \) to all agents that roll over whenever withdrawals exceed the critical threshold \( n \in [n_p, 1] \). Then there exist many less costly, feasible and stability-equivalent alternative policies, that allocate the smaller bailout \( \hat{b}(n) \in [0,b(n)] \) to agents that roll over, thus requiring a lower ex ante tax \( \tau(\hat{b}) \), and a simultaneously bail in of agents that withdraw to keep the ex ante run probability constant. Every bailout reduction \( \hat{b}(n) \in [0,b(n)] \) jointly with the reduced tax \( \tau(\hat{b}, \varepsilon) \) is feasible and stability-equivalent when imposing the following withdrawal-contingent haircut \( \kappa(\hat{b}, n) > 0 \) on agents that withdraw:

\[
\kappa(\hat{b}, n) = \begin{cases} 
  c_1(n) - \tau(\hat{b}, \varepsilon) - u^{-1}\left(u(c_2(n) - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n)\right), & \text{for all } n \in [0, n_p] \\
  c_1(n) - \tau(\hat{b}, \varepsilon) - u^{-1}\left(u(c_2(n) + \hat{b}(n) - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n)\right), & \text{for all } n \in [n_p, 1]
\end{cases}
\] (19)

where \( v_{BO}(b, 0, n) \) is given in equation (5). The least-costly, feasible, stability-equivalent bailout is the zero bailout \( \hat{b} = 0 \), implying a zero tax \( \tau(0) = 0 \) and a stability-equivalent bailin

\[
\kappa(0, n) = c_1(n) - u^{-1}\left(u(c_2(n)) - v_{BO}(b, 0, n)\right)
\]

\[
= \begin{cases} 
  c_1(n) - u^{-1}\left(u(c_2(n)) - (u(c_2(n, \theta) + b(n) - \tau(b)) - u(c_1(n, \theta) - \tau(b)))\right), & n \in [n_p, 1] \\
  c_1(n) - u^{-1}\left(u(c_2(n)) - (u(c_2(n, \theta) - \tau(b)) - u(c_1(n, \theta) - \tau(b)))\right), & n \in [\lambda, n_p]
\end{cases}
\] (20)

where \( v_{BO}(b, 0, n) \) is given in (5). This existence result is important because credible, costly bailout policies require financing via the taxpayer also in good times when the bailout is not needed which has welfare consequences as analyzed below in section 5.

From a policy designer’s perspective, the result gives insight into how to reduce policy...
costs while maintaining the effectiveness of the policy with regard to stability. A bail-in can be accomplished with a zero government budget, whereas bailouts require a budget and thus taxpayer money. Sometimes it might not be possible to reduce the bailout down to zero and impose the bailin (20), possibly because of participation constraints of creditors. In that case, a less costly bailout $0 < \hat{b}(n) < b(n)$ in between the original costly bailout and the zero bailout with joint bailin given via (19) may be applicable.

The main proof of Proposition 4.1 is given in the appendix. For intuition on how to design stability-equivalent policy it is important to remember section 3: the equilibrium trigger following the original bailout $\theta^*(b)$ is determined as the zero of the expected payoff difference in equation (46),

$$0 = (1 - \lambda) H(b, \theta^*(b)) = \int_\lambda^1 v_{BO}(n, \theta(n, \theta^*(b))) \, dn.$$  

(21)

For a given bailout reduction $\hat{b} \in [0, b(n)]$ the bailin $\kappa(\hat{b}, n)$ grants stability-equivalence, if the bailin is such that for all $(n, \theta) \in [0, 1]^2$ the payoff difference functions coincide,

$$v_{BO}(n, \theta) = v_{BI}(n, \theta)$$  

(22)

where $v_{BO}(n, \theta)$ is given in (5) and $v_{BI}(n, \theta)$ is given in (18). To find a stability-equivalent bailin one needs to proceed as follows: For every given state $\theta \in [0, 1]$ and its implied aggregate withdrawal level $n \in [0, 1]$ via (42), one needs to find the according value of the payoff difference function following the original bailout policy, $v_{BO}(n, \theta)$, then set this value equal to the payoff difference function following the less costly policy, $v_{BO}(n, \theta) \equiv v_{BI}(n, \theta)$, and solve this equality for the bailin $\kappa$ by inverting the creditors’ utility function while also verifying feasibility. Feasibility requires that consumption levels to all agents in the economy are positive when accounting for the endogenous change in the tax and the bailin.

The construction in the appendix reveals that every less costly policy $(\hat{b}, \kappa(\hat{b}))$ given by (19) is feasible. In addition, the construction reveals that when implementing the stability-equivalent bailin the resulting equilibrium trigger is indeed the unique equilibrium in the setting following the alternate policy because, by design, the stability-equivalent bailin is such that all monotonicity properties of the original bailout are inherited.
4.1 Properties of the stability-equivalent bailin

It is worthwhile to analyze the properties of the stability-equivalent bailout provision \( \kappa \) given in (19). First, see that for a given withdrawal-contingent bailout provision \( \hat{b}(n) \) also the stability-equivalent haircut \( \kappa \) must change with the aggregate withdrawals. This is not surprising because both the less costly bailout \( \hat{b}(n) \) as well as the payoff difference function following the original bailout \( v_{BO}(n, \theta) \) are withdrawal contingent. This payoff difference function exhibits the monotonicity properties given in assumptions 2.1. That is, in bank run settings we typically believe that the payoffs to creditors that roll-over decline in relation to the payoffs of withdrawing creditors as the run gets larger, and the payoff difference eventually becomes negative for large runs.\(^5\) The stability-equivalent bailin \( \kappa \) needs to mimic these monotonicity properties, guaranteeing that also under the less costly policy \((\hat{b}, \kappa(\hat{b}))\) the same monotonicity properties hold for the payoffs while taking into account the withdrawal-contingency of the less costly bailout \( \hat{b}(n) \). Generically, the bailin \( \kappa(n) \) can be non-monotonic in the withdrawals because the less costly bailout \( \hat{b}(n) \in [0, b(n)) \) can be non-monotonic.

Second, clearly the stability-equivalent bailin function \( \kappa(n) \) must change as the bailout provision \( \hat{b}(n) \) to agents that roll over declines in the sense of Definition 4.2. Consider a sequence of bailout functions \( \{\hat{b}_j(n)\}_{j \geq 1} \). I say “the bailout declines in \( j \)” if the sequence decreases in \( j \) pointwise via \( \hat{b}_{j+1}(n) \leq \hat{b}_j(n) \), for all \( n \in [n_p, 1] \). I additionally require that the decline is such that \( \hat{b}_j(n) \) remains piecewise continuous and thus integrable for all \( j \geq 1 \). I can show

**Proposition 4.2**

Assume the bailin declines point-wise via \( \hat{b}_{j+1}(n) \leq \hat{b}_j(n) \), for all \( n \in [n_p, 1] \). Then

(i) the average bailin needs to increase over the withdrawal range \( n \in [n_p, 1] \),

\[
\int_{n_p}^1 \kappa_j(n) \, dn < \int_{n_p}^1 \kappa_{j+1}(n) \, dn \quad (23)
\]

(ii) for all \( n \in (n^*, n_p) \): \( k_{j+1}(n) < k_j(n) \).

(iii) for all \( n \in [\lambda, n_p) \cap [\lambda, n^*) \): \( k_{j+1}(n) > k_j(n) \).

The result (i) in Proposition 4.2 holds because the average net bailout to agents that roll over, \( (\hat{b}(n) - \tau(\hat{b})) \), weakly decreases as \( \hat{b}(\cdot) \) declines, where the average is taken over the interval \([n_p, 1] \). This holds because the bailout provision declines on average faster

\(^5\)We assume one-sided strategic complementarity. Therefore, once the payoff difference between roll-over versus withdrawal becomes negative this difference may increase as long as it remains negative.
than the tax.\textsuperscript{6} Therefore, agents who roll over become on average less happy as the bailout declines. Withdrawing agents, on the other hand, become more happy because they have no claim on the bailout but pay a smaller tax. Stability-equivalence, however, requires that also agents that withdraw suffer from the decline in the bailout. The way to accomplish this is by charging a higher bailin.

For the withdrawal range where the bailout is not provided, \( n \in [\lambda, n_p) \), the bailout reduction impacts all creditors via the tax reduction. Both, agents who roll over and withdrawing agents become happier as the tax declines. Stability-equivalence, however requires that the extent to which both agent groups become happier is exactly the same. With risk-averse agents, the agent group whose happiness grows faster is the agent group with the smaller consumption level. Here, recall that \( n^* \) is the withdrawal level at which a creditor’s optimal response switches from roll over for withdrawals below \( n^* \) to “withdraw” for withdrawals above \( n^* \). For \( n \in [\lambda, n^*) \), the consumption of withdrawing agents is lower than the consumption of agents that roll over. Thus, the bailin needs to increase as the bailout drops in order to reduce the withdrawing agents’ happiness level to the level of agents that roll over. If \( n^* < n_p \) and \( n \in (n^*, n_p) \), then the consumption of withdrawing agents is higher than the consumption of agents that roll over. Thus, the bailin needs to decrease as the bailout declines.

\section{4.2 Special case: risk-neutrality}

In the case where agents are risk-averse, the least costly, alternative, stability-equivalent bailin policy can directly be constructed and the proof is straight forward.

Let again \( b(n) \) the bailout that is granted to agents that roll over whenever withdrawals realize high enough in \([n_p, 1]\). Consumption following the original bailout equals

\[
\begin{align*}
    u_1(b, 0) &= \begin{cases} 
        c_1(n) - \tau(b, \varepsilon), & n \in [n_p, 1] \\
        c_1(n) - \tau(b, \varepsilon), & n \in [0, n_p)
    \end{cases} \\
    u_2(b, 0) &= \begin{cases} 
        c_2(n) + b(n) - \tau(b, \varepsilon), & n \in [n_p, 1] \\
        c_2(n) - \tau(b, \varepsilon), & n \in [0, n_p)
    \end{cases}
\end{align*}
\]

where the lump-sum tax is given as before by \( \tau(b, \varepsilon) = \frac{M_2}{\bar{M}} \int_0^{\theta_p} b(n(\theta, \theta^*(b))) \, d\theta \). For every

\textsuperscript{6}This is however not true for every point \( n \in [n_p, 1] \): for some points the bailout may stay constant while the tax nevertheless declines since the tax takes into account the average bailout reduction over the range \([n_p, 1]\). In that case \((b(n) - \tau(b))\) increases.
$n \in [n_p, 1]$, the payoff difference following the original bailout guarantee $b(n)$ satisfies

$$v(n, \theta) = \begin{cases} (u_2(n) + b(n) - \tau(b, \varepsilon)) - (u_1(n) - \tau(b, \varepsilon)), & \text{(bailout)} \\ u_2(n) - (u_1(n) - b(n)), & \text{(bail-in)} \end{cases}$$

(25)

That is, the stability-equivalent bailin exactly equals the amount of the original bailout, $\kappa(b, n) = b(n)$. For the withdrawal range $n \in [0, n_p)$ the original bailout is not imposed. The payoff difference for $n \in [0, n_p)$ following the original bailout guarantee equals

$$v(n, \theta) = (u_2(n) - \tau(b, \varepsilon)) - (u_1(n) - \tau(b, \varepsilon))$$

(26)

$$= u_2(n) - u_1(n)$$

Because the tax payments cancel out, and in contrast to the risk-averse case, there is no need to impose a bailin for low withdrawals $n \in [0, n_p)$ for keeping the run-likelihood stable. Overall, in the case of risk-neutrality the least costly, feasible, stability-equivalent bailout is given by

$$\kappa(b, n) = \begin{cases} b(n), & n \in [n_p, 1] \\ 0, & n \in [0, n_p) \end{cases}$$

(27)

Consumption following the stability-equivalent bailin equals

$$u_1(0, \kappa) = \begin{cases} u_1(n) - b(n), & n \in [n_p, 1] \\ u_1(n), & n \in [0, n_p) \end{cases}$$

$$u_2(0, \kappa) = \begin{cases} u_2(n), & n \in [n_p, 1] \\ u_2(n), & n \in [0, n_p) \end{cases}$$

(28)

No taxation is required to finance the stability-equivalent bailin. Feasibility requires $u_1(n) - b(n) \geq 0$.

5 Welfare implications

We have just seen that for a given bank creditor bailout we can find cheaper policy alternatives that attain the same ex ante level of bank stability. A question on the welfare implications of this result arises. Some of the bank’s creditors are impatient and have to withdraw early to consume. The bailin that comes with the less costly policy harms their utility directly. On the other hand, also the original bailout affects impatient types adversely via the higher tax. Can welfare be higher when implementing a less costly, stability-equivalent policy rather than the original bailout?
Proposition 5.1 (Bailin versus Bailout)
Welfare when implementing a less costly, stability-equivalent policy is higher than welfare following the original bailout if
(i) either the bank is small relative to the economy, that is, \( M_1 \) is large relative to \( M_2 \),
(ii) or the bank is large relative to the economy but the ex ante probability of a run is below an upper bound, \( \theta_b < B^* \).
(iii) If the bank is negligible relative to the economy, \( M_1 \to \infty \), then welfare following either policy coincides.

The proof of Proposition 5.1 shows that at the limit the welfare difference is given as

\[
\lim_{\varepsilon \to 0} \Delta W = W^{\text{Bailin}}(\hat{b}, \kappa(\hat{b})) - W^{\text{Bailout}}(b, 0) 
\]

\[
= M_1 \left( u(\bar{c} - \hat{\tau}) - u(\bar{c} - \tau) \right) 
\]

\[
+ M_2 \left[ \int_{\theta_p}^{1} \lambda(u(c_1 - \kappa(\hat{b}(n(\theta)))) - \hat{\tau}) - u(c_1 - \tau) + (1 - \lambda)(u(c_2 - \hat{\tau}) - u(c_2 - \tau)) \right] d\theta 
\]

\[
\geq 0 \quad (31)
\]

\[
\geq 0 \quad (32)
\]

where \( \hat{\tau} \leq \tau \) is the reduced tax that comes with the less costly policy \( \hat{b} \) relative to the original bailout policy \( b \) with tax \( \tau \), \( M_1 \) is the measure of regular citizens that are not bank creditors, and measure \( M_2 \) are bank creditors. At the limit \( \varepsilon \to 0 \), the miscoordination range vanishes, and by design no bailout is paid in the full range \( n \in [\lambda, 1] \).

Note, it appears as if the bailout is never needed, and could thus be abandoned. This is however misleading, see the discussion in Diamond and Kashyap (2016) on the last taxi that may never leave the taxi stand. The bailout is credible and ready to be paid, and thus alters the rollover incentives of all patient agents.

To understand the welfare implications (29), all citizens who are not affiliated with the bank prefer the less costly policy since citizens never benefit from the bailout but are

\footnote{All creditors withdraw their deposits for state realizations in \([0, \theta_p]\), thus foregoing a share of the bailout. In the case of the less costly policy, all withdrawing agents are partially bailed in, paying the haircut \( \kappa \), whereas no bailin is imposed following the original bailout policy. For state realizations in \([\theta_p, 1] \), all patient agents roll over their deposits and, since withdrawals are low, no bailout is triggered in the case of either policy.}
required to pay taxes. Thus, the first term in (29) is positive.

The second term considers the creditors’ utility difference inferred from the bailin versus bailout in bad times where a run occurs. Given bad times, \( \theta \in [0, \theta_p] \), the utility to withdrawing agents following the original more costly bailout is higher than utility following the less costly alternative. In bad times all creditors run on the bank. Interestingly, the costly bailout policy during a run is favored even though the bailout is designed such that agents that run are not rewarded for their behavior, and do not receive a share of the (larger) bailout.\(^8\) Rather, the withdrawing agents favor the more expensive policy because the haircut implied by the bailin jointly with its required tax is on average more expensive than the tax to finance the original, larger bailout; see inequality (130). Thus, given bad times, all creditors favor the more expensive bailout.

The third term considers the creditors’ utility difference inferred from the bailin versus bailout in good times when no run occurs. In good times, the bailout and its required taxation turn out to be unnecessary ex post. All creditors favor the cheaper alternative over the costly bailout. Agents that roll over prefer it because it requires a lower ex ante tax, whereas agents that withdraw favor it because, as is shown in (125), the stability-equivalent bailin \( \kappa \) plus the reduced tax \( \hat{\tau} \) are jointly still smaller than the tax required to finance the original bailout \( \tau \).

To understand Proposition 5.1, if the economy is large relative to the bank, \( M_1 >> M_2 \), then the share of the population that suffers from the original costly bailout via taxation is large relative to the share of the population that benefits. Therefore, welfare is higher following the less costly policy since the tax on citizens is lower. In the extreme case though, where the bank becomes negligible in the economy, the welfare difference between the two policies becomes zero because the costs of either bailout are financed by an increasing population of citizens so that the taxes to finance either policy coincide and vanish.

But even if all citizens are simultaneously bank creditors, \( M_1 = 0 \), welfare following the less costly policy can be higher than welfare following the original bailout if the run probability is sufficiently low. The ex ante welfare difference depends on how often the run occurs that triggers the painful bailin versus how often no run occurs but taxes nevertheless apply.

\(^8\) Bailouts that are paid to withdrawing agents is a bad idea since this increase the ex ante run probability, see Schilling (2022).
6 Example: Conditional deposit insurance

Consider a modified version of the Goldstein and Pauzner (2005) model, henceforth GP. As in the benchmark model, there is an agent population of measure $M = M_1 + M_2$. The measure $M_1$ denotes citizens that are not invested whereas bank investors are given by the continuum $[0, 1] \times M_2$. All agents share a common utility function over consumption $u(\cdot)$ that is positive, strictly increasing and weakly concave with $u(0) = 0$. Agents are symmetric in $t = 0$. In $t = 1$ agents privately learn their type: a share $\lambda$ of the investors turns out to be of the impatient type whereas a share $1 - \lambda$ is patient. Impatient types can only consume in $t = 1$ whereas impatient types care for consumption in $t = 1$ and $t = 2$ equally. Types are private information in $t = 1$ and are not observable.

In the GP model, there is no regulator present, and payoffs are solely determined via the investors’ contract with the bank, and the bank’s assets. All investors invest in a demand-deposit contract with a bank at time $t = 0$. The contract promises a payoff when opting to withdraw in $t = 1$ equal to $c_1 = r_1 > 1$, whereas the payoff to roll-over is withdrawal-contingent, and equals $c_2 = \frac{R(\theta)(1 - nr_1)}{1 - n}$ where $R(\theta)$ is the random return of the bank’s asset in $t = 2$, $\theta \sim U[0, 1]$ is the random state of the economy, and $n \in [0, 1]$ is the endogenous share of investors who choose to withdraw in $t = 1$. I assume that $R(\theta)$ is strictly increasing, and differentiable in $\theta$ with $E[R(\theta)] > 1$. Following GP, I make the following assumptions to attain an upper and lower dominance region, which ultimately leads to a unique equilibrium selection: I assume the function $R(\theta)$ is such that there exist boundary states $\theta, \theta \in (0, 1)$ such that for high states $\theta \in [\theta, 1]$ the asset pays already in $t = 1$ and satisfies $R(\theta) > r_1 > 1$. The boundary state $\theta = \theta$ is implicitly defined as

$$\frac{R(\theta)(1 - \lambda r_1)}{1 - \lambda} = r_1$$

For low states, $\theta \in [0, \theta]$ the asset return thus satisfies $\frac{R(\theta)(1 - \lambda r_1)}{1 - \lambda} < r_1$, meaning that “withdraw” is dominant. In $t = 1$, the asset has a liquidation value $L(\theta) = 1$ for all $\theta \in [0, \theta]$, as in GP.

In $t = 1$, before making the roll-over decision, all investors observe a noisy signal $\theta_i = \theta + \varepsilon_i, i \in [0, 1]$ about the unobservable state of the world. After making their roll-over decision, the aggregate withdrawals $n$ realize. The bank observes $n$, and is obliged to pay out $nr_1$ to the withdrawing creditors. The bank finances withdrawals by liquidating assets. However, if the withdrawals realize high, $n \in [n_c, 1]$, such that $nr_1 > 1$, the bank does not have enough assets to repay all withdrawing investors, and is subject to
a run. I call \(n_c \equiv 1/r_1\) the illiquidity point. If the bank is subject to a run, the payoff to roll-over drops to zero, and investors who try to withdraw are not served for sure. Instead, they queue in front of the bank, and are only served if their position in the queue is sufficiently early (sequential service). Positions in the queue are random so that a creditors’ likelihood of being served equals \(n_c/n = 1/(nr_1)\).

From Goldstein and Pauzner (2005), we know that the economic setting above yields a unique equilibrium in the form of a trigger signal \(\theta^*\) such that all agents withdraw when observing a private signal \(\theta_i < \theta^*\), and otherwise roll over.

6.1 Costly government intervention (Bailout)

I now build a costly bailout \(b\) into the GP model. The bailout is paid to agents that roll over whenever the aggregate withdrawals become high, \(n \in [n_p, 1]\).

The model section 2 considers a general bailout provision \(b(n)\) to agents that roll over which may change in the aggregate withdrawals. As an application, I will consider here a bailout in the form of a government deposit guarantee or deposit insurance. The government guarantees a minimum payoff \(b \in (0, r_1)\) to the agents that roll over.

Let \(n_p(b) \in (0, n_c)\) the government’s intervention point at which it starts subsidizing the bank’s payments to agents that roll over. For high withdrawals in \(n \in [n_p(b), 1]\), the bailout is “active”, meaning the government subsidizes payments by the bank, paying the amount

\[
b(n) = b - \frac{R(\theta(n, \theta^*))(1 - n r_1)}{1 - n}
\]

per investor that rolls over. The bailout provision does not affect the illiquidity point \(n_c\) because the guarantee is not extended to agents that withdraw.

The government’s intervention point \(n_p\) is implicitly defined as the measure of withdrawals at which the roll-over payoff drops to the guaranteed payoff:

\[
\frac{R(\theta(n_p))(1 - n_p r_1)}{1 - n_p} = b
\]

where \(\theta(n)\) is the inverse of \(n(\theta, \theta^*_b)\), holding the trigger \(\theta^*_b\) fixed. For withdrawals \(n \in (n_p, 1]\), the roll-over payoff \(\frac{R(\theta)(1 - n r_1)}{1 - n}\) falls short of the guarantee \(b\), because the former strictly declines in the aggregate withdrawals. Thus, the government needs to become active and top up payoffs to creditors that roll over to guarantee level \(b\). From (35) it is straightforward to see that, in contrast to the benchmark model 2 where the intervention
point is fixed in the bailout size, here the government’s intervention point \( n_p(b) \) changes endogenously in the deposit guarantee \( b \). As the regulator guarantees a lower payoff \( b \), the aggregate withdrawals need to be higher to trigger the intervention, \( n_p(b) \) declines in \( b \). See that for a zero guarantee \( b = 0 \), the intervention point and the illiquidity point coincide \( n_p(0) = n_c \), whereas for \( b > 0 \) it holds \( n_p(b) < n_c \). As in the benchmark model, the critical intervention threshold \( n_p(b) \) implicitly pins down the critical state \( \theta_p(b) \) in \( [\theta^*_b - \varepsilon, \theta^*_b + \varepsilon] \) below which the regulator needs to intervene to guarantee \( b \) via

\[
n(\theta_p(b), \theta^*(b)) = n_p(b). \tag{36}
\]

That is, in state \( \theta_p(b) \) the withdrawals realize just equal to the intervention point \( n_p(b) \). Because the intervention point \( n_p(b) \) depends on the guarantee \( b \), so does the critical state. As the guarantee \( b \) declines, the critical state \( \theta_p(b) \) declines since the regulator tolerates lower state realizations without intervention.

**Bailout financing** To finance the bailout, the government levies the symmetric lump-sum tax

\[
\tau(b, \varepsilon) = \frac{M_2}{M} \left( \int_{\theta}^{\theta_p(b)} \left( b - \frac{R(\theta)(1 - n(\theta, \theta^*_b))}{1 - n(\theta, \theta^*_b)} \right) d\theta \right) \tag{37}
\]

on all citizens and creditors before investment in the bank takes place in \( t = 0 \). A larger, credible guarantee \( b \) can only be financed by levying a larger tax \( \tau(b) \) on all agents, see equation (144). See that the tax (37) already preempts the existence of a unique equilibrium in the form of a trigger signal \( \theta^*(b) \), which I show below in Lemma 6.1.

In this application, payoffs, and thus the payoff difference to roll-over versus withdraw are determined via the bank contract and policy intervention. For all possible states and withdrawals \((\theta, n) \in [0, 1]\) the payoff difference is given as

\[
\nu_{BO}(b, 0, n) = \begin{cases} 
  u \left( \frac{R(\theta) - nr_1}{1-n} - \tau(b, \varepsilon) \right) - u(r_1 - \tau(b, \varepsilon)), & \theta \in [\overline{\theta}, 1] \\
  u \left( \frac{R(\theta)(1-nr_1)}{1-n} - \tau(b, \varepsilon) \right) - u(r_1 - \tau(b, \varepsilon)), & n \in [n_p(b), n_c) \text{ and } \theta \in [\lambda, \overline{\theta}] \\
  u(b - \tau(b, \varepsilon)) - u(r_1 - \tau(b, \varepsilon)), & n \in [n_p(b), n_c) \text{ and } \theta \in [\lambda, \overline{\theta}] \\
  u(b - \tau(b, \varepsilon)) - \frac{1}{nr_1} u(r_1 - \tau(b, \varepsilon)), & n \in [n_c, 1] \text{ and } \theta \in [\lambda, \overline{\theta}]
\end{cases} \tag{38}
\]

Recall that for particularly high state realizations \([\overline{\theta}, 1]\) the asset already pays early, and \( R(\theta) > r_1 \) so that no liquidation is required to repay withdrawing creditors.
I choose the guarantee $b$ small enough to satisfy feasibility. This means, the tax to finance $b$ can be paid by all agents in the economy, leaving their consumption levels weakly positive. As in the benchmark model, and without loss of generality, I assume that the bailout is, therefore, not too large such that the run does not vanish altogether. This is a reasonable assumption since the tax that was required to finance a full bailout may not be affordable by all agents in the economy.

I next need to show equilibrium existence and uniqueness. For that purpose, I need to verify that assumptions 2.1 and 2.2 hold. I do so in the appendix, section 9.5. Therefore, I can conclude:

**Lemma 6.1.** For a given feasible deposit guarantee $b$, the creditors’ coordination game has a unique equilibrium given by the trigger signal $\theta^*(b)$.

For the given feasible, costly deposit guarantee $b$, I will next construct a less costly policy that attains the same level of bank stability $\theta^*(b)$.

### 6.2 Construction of stability-equivalent bailin

To derive the stability-equivalent bailin, let $\hat{b} = b - z \in [0, b]$ a smaller, feasible deposit guarantee to agents that roll-over, let $z \in [0, b]$ the cost reduction, and let $\kappa(b, n)$ the bailin of agents that withdraw with $\kappa(b, n) = 0$.

The payoff difference with the less costly bailout and the bailin becomes

$$v_{BI}(\hat{b}, \kappa) = \begin{cases} 
    u\left(\frac{R(\theta)-nr_1}{1-n} - \tau(\hat{b}, \varepsilon)\right) - u(r_1 - \tau(\hat{b}, \varepsilon) - \kappa(z, n)), & \theta \in [\theta, 1] \\
    u\left(\frac{R(\theta)(1-nr_1)}{1-n} - \tau(\hat{b}, \varepsilon)\right) - u(r_1 - \tau(\hat{b}, \varepsilon) - \kappa(z, n)), & n \in [\lambda, n_p(\hat{b})] \text{ and } \theta \in [0, \theta] \\
    u\left(\hat{b} - \tau(\hat{b}, \varepsilon)\right) - u(r_1 - \tau(\hat{b}, \varepsilon) - \kappa(z, n)), & n \in [n_p(\hat{b}), n_c) \text{ and } \theta \in [0, \theta] \\
    u\left(\hat{b} - \tau(\hat{b}, \varepsilon)\right) - \frac{1}{nr_1} u(r_1 - \tau(\hat{b}, \varepsilon) - \kappa(z, n)), & n \in [n_c, 1] \text{ and } \theta \in [0, \theta] 
\end{cases}$$

Our goal is to find the least costly, feasible bailout guarantee $\hat{b}^*$, jointly with the implied cost reduction $z^* = b - \hat{b}$, and the bailin $k(\hat{b}^*)$ such that stability is held constant at level $\theta^*(b) = \theta^*(\hat{b}, k(\hat{b}))$. We hope to set $\hat{b} = 0$ but need to verify whether this can be done in a stability-equivalent way. And indeed, this is possible:

**Proposition 6.1** (Stability-equivalent haircut design)

Consider the costly policy to guarantee the deposit insurance level $b \in (0, r_1)$ to all creditors that roll over whenever withdrawals exceed $n_p(b) \in [\lambda, n_c)$. Assume this policy is financed via taxation in $t = 0$ in a budget-balancing manner. Then there exists a non-costly, feasible policy alternative $(0, \kappa(b))$ that attains the same level of ex ante bank stability $\theta^*(b) = \theta^*(0, \kappa(0))$ (stability-equivalence): The alternative policy reduces the bailout
to zero, requires zero taxation and instead imposes a non-costly withdrawal-contingent bailin \( \kappa \) on all withdrawing agents given by

\[
\kappa(n) = \begin{cases} 
    r_1 - u^{-1} \left( u \left( \frac{R(\hat{\theta})}{1 - n(\theta)} r_1 - (u(r_1 - \tau(b))) \right) - u \left( \frac{R(\hat{\theta})}{1 - n(\theta)} r_1 - \tau(b) \right) \right), & \theta \in [\bar{\theta}, 1] \\
    r_1 - u^{-1} \left( u \left( \frac{R(\hat{\theta})}{1 - n(\theta)} r_1 - (u(r_1 - \tau(b))) \right) - u \left( \frac{R(\hat{\theta})}{1 - n(\theta)} r_1 - \tau(b) \right) \right), & \theta \in (\theta_p(b), \bar{\theta}] \\
    r_1 - u^{-1} \left( u \left( \frac{R(\hat{\theta})}{1 - n(\theta)} r_1 - (u(r_1 - \tau(b))) \right) - u \left( \frac{R(\hat{\theta})}{1 - n(\theta)} r_1 - \tau(b) \right) \right), & \theta \in (\theta_p(b), \theta_p(b)] \\
    r_1 - u^{-1} \left( u(0) - (u(b - \tau(b, \varepsilon)) - u(r_1 - \tau(b, \varepsilon))) \right), & \theta \in [\theta_p(0), \theta_p(b)] \\
    r_1 - u^{-1} \left( u(0) - (u(b - \tau(b, \varepsilon)) - \frac{1}{n(\theta)} u(r_1 - \tau(b, \varepsilon))) \right), & \theta \in [0, \theta_p(0)] 
\end{cases}
\]

where I have used the shortcut \( n(\theta) \equiv n(\theta, \theta^*(b)) \).

Here, recall that for a trigger equilibrium around signal \( \theta^*(b) \), the aggregate withdrawals \( n(\theta, \theta^*(b)) \) are a deterministic function of the state. Therefore, the withdrawals in (40) are determined by (42).

For the construction it is crucial to notice: We design the bailin \( \kappa \) such that the equilibrium trigger \( \theta^*(\hat{b}, \kappa(\hat{b})) \) and thus the range \( [\theta^* - \varepsilon, \theta^* + \varepsilon] = [\theta^*(\hat{b}, \kappa(\hat{b})) - \varepsilon, \theta^*(\hat{b}, \kappa(\hat{b})) + \varepsilon] \) is held constant as \( \hat{b} \) and its required tax \( \tau(\hat{b}) \) decline. However, unlike in the benchmark case, the critical threshold at which the regulator intervenes, \( n_p(\hat{b}) \), is not constant in the size of the guarantee \( \hat{b} \) but shifts within \( [\theta^* - \varepsilon, \theta^* + \varepsilon] \), see the discussion below (35).\(^9\)

Because the original and the less costly bailout have different critical states, the regions of intervention diverge as the bailout becomes smaller. Therefore, unlike in the benchmark model, I need to construct the bailin for an additional withdrawal range \( [n_p(b), n_p(\hat{b})] \). In this withdrawal range the regulator is inactive when granting the less costly deposit guarantee \( \hat{b} \) but is actively providing a bailout, and topping up creditor payoffs following the original, higher guarantee \( b \).

The construction of the stability-equivalent bailin and feasibility are discussed in section 9.6 of the appendix.

7 Moral Hazard

The main analysis assumes that as policy transitions from the bailout \( b(n) \) to the less costly policy \( (\hat{b}, \kappa) \) the bank does not react. In this section, I briefly sketch how moral

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\(^9\)As the guarantee \( \hat{b} \) declines in \( [0, b] \), the regulator tolerates more withdrawals until intervention, the intervention point \( n_p(\hat{b}) \) increases, and the critical state \( \theta_p(\hat{b}) \) at which the withdrawals realize to trigger the intervention declines. Further recall that for \( \hat{b} = 0 \), the illiquidity point is attained, that is, the aggregate withdrawals realize such that the bank needs to ration withdrawals. Therefore, \( \theta_p(0) \) is the critical state at which \( n_p(0) = n_c = 1/r_1 \).
hazard from the side of the bank can be incorporated into the derivation of stability-
equivalent bailins.

Assume, as before, the regulator grants bailout \( b(n) \) for withdrawals in \( n \in [n_p, 1] \),
and assume that the bank’s asset choice in anticipation of the bailout is given by the
asset payoff \( R(\theta) \equiv R(\hat{b}, \kappa)(\theta) \). That is, \( v_{BO}(n, \theta) \) is given as before in (5).

As the regulator now announces to apply the less costly policy \((\hat{b}, \kappa)\), the bank changes
its asset side and shifts its risk by investing in the asset \( R(\hat{b}, \kappa)(\theta) \) instead of \( R(\theta) \). I assume
that \( R(\hat{b}, \kappa)(\theta) \) has the same mean as \( R(\theta) \) but \( R(\hat{b}, \kappa)(\theta) \) is less risky.\(^{10}\)

Pin down the function that maps a regulator’s policy choice to the bank’s asset choice
\( (\hat{b}, \kappa) \rightarrow R(\hat{b}, \kappa)(\theta) \). Thus, for given \( (\hat{b}, \kappa) \), we can now pin down \( v_{BI}(\hat{b}, \kappa)(n, \theta) \), taking into
account that the policy choice \( (\hat{b}, \kappa) \) impacts \( v_{BI}(\hat{b}, \kappa)(n, \theta) \) also via a change in the asset
side. In the application of section 6, the payoff difference with the less costly alternative bailin becomes

\[
v_{BI}(\hat{b}, \kappa) = \begin{cases}
u \left( \frac{R(\hat{b}, \kappa)(\tau(n, 1-n)) - \tau(b, \varepsilon)}{1-n} - u(r_1 - \tau(b, \varepsilon) - \kappa(z, n)), \quad \theta \in [\overline{\theta}, 1] \right) \\
\frac{R(b, \varepsilon)(\tau(1-n)) - \tau(b, \varepsilon)}{1-n} - u(r_1 - \tau(b, \varepsilon) - \kappa(z, n)), \quad n \in [\lambda, n_p(b)) \text{ and } \theta \in [0, \overline{\theta}) \\
u \left( b - \tau(b, \varepsilon) - u(r_1 - \tau(b, \varepsilon) - \kappa(z, n)), \quad n \in [n_p(b), 1] \text{ and } \theta \in [0, \overline{\theta}) \\
\left( b - \tau(b, \varepsilon) - \frac{1}{nr_1} u(r_1 - \tau(b, \varepsilon) - \kappa(z, n)), \quad n \in [n_c, 1] \text{ and } \theta \in [0, \overline{\theta}) \right)
\end{cases}
\]

(41)

To derive the stability-equivalent bailin for a less costly bailout \( \hat{b}(n) \in [0, b(n)) \), one proceeds as before: For every \( (n, \theta) \) fix the value of the payoff difference following the
original bailout \( v_{BO}(b, 0, n) \), set this value equal to \( v_{BI}(\hat{b}, \kappa) \), and then solve this equation
for \( \kappa \), verifying feasibility on the go. A challenge in the case of moral hazard is, that
the bailin \( \kappa \) enters \( v_{BI}(\hat{b}, \kappa) \) in two ways, via the utility to agents that withdraw that
encounter the bailin, and indirectly via the utility to agents that roll over who earn a
pro-rata share of a different asset. A closed form solution for the stability-equivalent bailin is tricky to derive due to the application of a general utility function.

8 Conclusion

Costly government intervention ("bailouts") such as deposit insurance, government guarantees or equity injections are a common tool to stabilize banks against runs. This paper
shows that a creditor bailin can have the same stabilizing effect on a bank as a creditor bailout. For a given, arbitrary bailout I construct the feasible, stability-equivalent creditor bailin that leaves the ex ante run probability on the bank constant. Such feasible,
stability-equivalent bailins can always be found, even for risk-averse agents. The result has consequences for welfare. Even though there are impatient types among the creditors with early consumption needs that suffer from a bailin, I show that a bailin can increase welfare relative to a creditor bailout. The reason is that credible bailouts impose indirect costs on all agents in the economy because they require higher taxation for financing, thus reducing consumption. In addition, bailouts do not bring a stability advantage. Therefore, bailins are optimal relative to bailouts whenever the bank is small in relation to the economy as a whole or when the ex ante run probability is small since absent runs all creditors prefer the bailin over the costly bailout.

These results shed doubt on the effectiveness of bailout policies.

References


9 Appendix

9.1 Proof: Existence and Uniqueness of Equilibria

Proof. [Proposition 3.1] The proof largely follows Goldstein and Pauzner (2005). I first show existence and uniqueness of a trigger equilibrium: Fix the bailout $b(n) \geq 0$. Assume all investors follow the same strategy that maps signals to actions. Moreover, assume the investors follow a threshold strategy around $\theta^*$ (for sake of brevity, in this proof I suppress the dependence of $\theta^*$ on the bailout $b$). Then the measure of agents that run at each state $\theta$ and threshold $\theta^*$ is deterministic and continuous in either argument,

$$n(\theta, \theta^*(p)) = \Pr(\theta_i < \theta^*|\theta) = \begin{cases} 
\lambda + (1 - \lambda) \left( \frac{1}{2} + \frac{\theta^*(p) - \theta}{2\varepsilon} \right), & \theta \in [\theta^*(p) - \varepsilon, \theta^*(p) + \varepsilon] \\
1, & \theta < \theta^*(p) - \varepsilon \\
\lambda, & \theta > \theta^*(p) + \varepsilon
\end{cases} \quad (42)$$

Given a signal $\theta_i$ and threshold signal $\theta^*$, an agent holds the following expectation over the payoff difference

$$H(\theta_i, n(\cdot)) = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(n(\theta, \theta^*), \theta) \, d\theta \quad (43)$$

The function $H(\theta_i, n(\cdot))$ is continuous in signal $\theta_i$ because by assumption 2.2, the payoff difference is Lebesgue integrable, because the functions $g_1(\theta_i) = \theta_i + \varepsilon$ and $g_2(\theta_i) = \theta_i - \varepsilon$ are continuous, because compositions of continuous functions are continuous, and because continuous functions on bounded intervals are bounded. By the same argument, an agent’s expected payoff difference when observing the trigger signal $\theta^*$,

$$H(\theta^*, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} v(n(\theta, \theta^*), \theta) \, d\theta, \quad (44)$$

is continuous in $\theta^*$. Also, $H(\theta^*, n(\cdot, \theta^*))$ is strictly increasing in signal $\theta^*$, as long as $\theta^* < \overline{\theta}_p + \varepsilon$, because for larger signals $\theta^*$ the expectation is taken over a higher range of fundamentals $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$. By assumption 2.1, the payoff difference is strictly increasing in $\theta$ for all $\theta \in [\overline{\theta}_p, \overline{\theta}_p]$, whereas the function $n(\theta, \theta^*)$ is evaluated at the same values due to a shift in the argument $\theta^*$ and the range of fundamentals. By the existence of an upper and lower dominance region, assumption 2.1, we know that $v(n(\theta, \theta^*), \theta^*) < 0$ for all $\theta \in [0, \overline{\theta}_p]$, whereas $v(n(\theta, \theta^*), \theta^*) > 0$ for all $\theta \in [\overline{\theta}_p, 1]$. Thus, $H(\theta^*, n(\cdot, \theta^*)) < 0$ for all $\theta^* \in [0, \overline{\theta}_p - \varepsilon]$ and $H(\theta^*, n(\cdot, \theta^*)) > 0$ for all $\theta^* \in [\overline{\theta}_p + \varepsilon, 1]$. Allover, because $H(\theta^*, n(\cdot, \theta^*))$ is continuous and strictly increasing in $\theta^*$, is positive for high and negative
for low values of $\theta^*$, there must exist a unique threshold signal $\theta^*$ that satisfies

$$H(\theta^*, n(\cdot, \theta^*)) = 0$$

(45)

To show that $\theta^*$ is an equilibrium, that is, $H(\theta_i, n(\cdot, \theta^*)) < 0$ for $\theta_i < \theta^*$ and $H(\theta_i, n(\cdot, \theta^*)) > 0$ for $\theta_i > \theta^*$, the proof in Goldstein and Pauzner (2005) applies. Using an interval decomposition, they show that for $\theta_i < \theta^*$, it must follow $H(\theta_i, n(\cdot, \theta^*)) < 0$ because this expected value is taken over a lower range of fundamentals than the expected value $H(\theta^*, n(\cdot, \theta^*))$, and because the payoff difference function satisfies single-crossing in $n$ by assumption 2.1. Last, it remains to show that there exist no non-threshold equilibria. By assumption 2.1 and 2.2, $\nu(n, \theta)$ is strictly increasing in $\theta \in [\theta_p, \theta]\), is strictly decreasing in $n$ whenever positive, and satisfies single-crossing. Therefore, the proof in (Goldstein and Pauzner, 2005) applies.

9.2 General proof: Construction of stability-equivalent bailin

Proof. [Proposition 4.1]

I next construct a less costly, feasible, stability-equivalent policy involving a smaller bailout $\hat{b} \in [0, b(n)]$ to agents that roll over and a stabilizing partial bailin $\kappa(\hat{b}, n)$ of depositors that withdraw. When designing the bailin $\kappa(b(n))$ to attain stability-equivalence: By section 3 the equilibrium trigger following the original bailout $\theta^*(b)$ is determined as the zero of the expected payoff difference in equation (46),

$$0 = H(b, \theta^*(b)) = \int_{\lambda}^{1} v_{BO}(n, \theta(n, \theta^*(b))) \, dn.$$  

(46)

For a given bailout reduction $\hat{b} \in [0, b(n)]$ the bailin $\kappa(\hat{b}, n)$ grants stability-equivalence, if the bailin is such that for all $(n, \theta) \in [0, 1]^2$ the payoff difference functions coincide,

$$v_{BO}(n, \theta) = v_{BI}(n, \theta)$$

(47)

where $v_{BO}(n, \theta)$ is given in (5) and $v_{BI}(n, \theta)$ is given in (18) and is repeated here for clarity:

$$v_{BI}(n, \theta) = \begin{cases} u(c_2(n, \theta) + \hat{b}(n) - \tau(\hat{b})) - u(c_1(n, \theta) - \tau(\hat{b}) - \kappa(\hat{b}, n)), & n \in [n_p, 1] \\
(u(c_2(n, \theta) - \tau(\hat{b})) - u(c_1(n, \theta) - \tau(\hat{b}) - \kappa(\hat{b}, n)), & n \in [0, n_p]
\end{cases}$$

(48)
Moreover, because we design the bailin \( \kappa \) such that the payoff difference functions coincide for every \((n, \theta)\), and because \( v_{BO}(n, \theta) \) satisfied the monotonicity condition in assumptions 2.1 and 2.2, so does \( v_{BI}(n, \theta) \). As a consequence, the equilibrium trigger signal \( \theta^*(\hat{b}, \kappa(\hat{b})) \) in the setting when imposing the stability-equivalent less costly policy \((\hat{b}, \kappa(\hat{b}))\) exists, it is indeed the unique equilibrium of the creditors’ coordination game, and by construction equals the unique equilibrium trigger of the original setting where bailout \( b \) is granted, \( \theta^*(\hat{b}, \kappa(\hat{b})) = \theta^*(b, 0) \).

As the second requirement, the bailin needs to be feasible. That is, consumption has to remain weakly positive to all agents. This is not an issue for investors that roll over and for citizens that are not affiliated with the bank: The original bailout is feasible, meaning that the bailout guarantee \( b(n) \) is small enough such that the tax satisfies

\[
\tau(b, \varepsilon) \leq \begin{cases} 
\min (c_2(n) + b(n), c_1(n)), & \text{for all } n \in [n_p, 1] \\
\min (c_2(n), c_1(n)), & \text{for all } n \in [0, n_p]
\end{cases}
\]

where \( \bar{c} \) is the optimal consumption level of citizens that are not invested in the bank.

Because the alternative policy requires a lower bailout, the tax \( \tau(\hat{b}) \) is smaller than \( \tau(b) \), and thus \( 0 < \bar{c} - \tau(\hat{b}) < \bar{c} - \tau(\hat{b}) \) holds. In addition, \( \tau(b, \varepsilon) = \frac{M_2}{M} \int_0^{\theta_p} b(n(\theta, \theta^*(b'))) d\theta < b(n) \) and \( \tau(\hat{b}, \varepsilon) = \frac{M_2}{M} \int_0^{\theta_p} \hat{b}(n(\theta, \theta^*(b'))) d\theta < \hat{b}(n) \) by \( M_2 < M \) and \([\theta^* - \varepsilon, \theta_p] \subset [0, 1] \), implying \( c_2(n) + \hat{b}(n) - \tau(\hat{b}) > c_2(n) \geq 0 \) and \( c_2(n) - \tau(\hat{b}) > c_2(n) - \tau(b) \geq 0 \). It remains to verify feasibility of the bailin to agents that withdraw. I do so while explicitly constructing the bailin:

I need to construct the least costly, alternative policy for every pair \((\theta, n) \in [0, 1]^2 \). It turns out that it is sufficient to consider the withdrawal intervals \([0, 1] = [0, n_p] \cup [n_p, 1] \). The reason for this is by the nature of a trigger equilibrium, the aggregate withdrawals are a deterministic function of the state \( \theta \), given by (42). That is, it is sufficient to consider all states \( \theta \in [0, 1] \) jointly with the implied withdrawals given via (42).

Case A For states \( \theta < \theta^* - \varepsilon \), all agents receive a signal below the trigger and withdraw, \( n = 1 \). Withdrawing agents receive no bailout payment. For states \( \theta \in [\theta^* - \varepsilon, \theta_p] \) some agents roll over and receive a bailout. In particular, in \( \theta = \theta_p \), the withdrawals realize equal to \( n_p \). Thus, considering the states \( \theta \in [0, \theta_p] \) is equivalent to considering withdrawals in \([n_p, 1] \). Fix \( n \in [n_p, 1] \). The payoff difference following the
original, costly bailout equals

\[ v_{BO}(b, 0, n) = u(c_2(n) + b(n) - \tau(b, \varepsilon)) - u(c_1(n) - \tau(b, \varepsilon)) \] (50)

Set \( \bar{b} = b(n) \) as the upper cost bound for the alternate policy, and fix \( c_1 = c_1(n), c_2 = c_2(n) \).

The goal is to maximally lower the bailout to agents that roll over away from \( \bar{b} \) to some level \( \hat{b} \in [\bar{b}, \bar{b}] \) while simultaneously keeping the PI constant at the level \( v(b, 0, n) \). Generically, the bailout reduction \( \hat{b} \) can depend on \( n \), that is, can be designed separately for every \( n \). I, henceforth, suppress the dependence on \( n \) for an easier exposition.

To find the least costly, feasible, stability-equivalent bailout \( \hat{b} \), I need to compensate roll-over incentives for the payoff reduction by likewise reducing the payoffs to agents that withdraw by a sufficient amount \( \kappa(\hat{b}) > 0 \). The required reduction (haircut) \( \kappa \) for keeping the PI constant at level \( v_{BO}(b, 0, n) \) is described by the implicit function theorem and can be understood as an alternative, stability-equivalent (partial) bailout policy which is implicitly described via the function

\[ v_{BO}(b, 0, n) \equiv v_{BI}(\hat{b}, \kappa(\hat{b}), n) = u(c_2 + \hat{b} - \tau(\hat{b})) - u(c_1 - \kappa - \tau(\hat{b})), \hat{b} \in [0, \bar{b}], \] (51)

where the ex ante tax \( \tau(\hat{b}) \leq \tau(b) \) adjusts to the lower bailout guarantee \( \hat{b} \leq b \). A marginal reduction in the guarantee \( \hat{b} \in [0, \bar{b}] \) and thus tax \( \tau(\hat{b}) \) is compensated by an adequate increase in the haircut of agents that withdraw, \( \kappa(\hat{b}) \), to hold the LHS of (51) exactly constant at the level \( v(b, 0, n) \). Clearly \( \kappa(\bar{b}) = 0 \).

For feasibility we distinguish two cases.

**A1** Assume for the considered \( n \in [n_p, 1] \), the payoff difference following the original bailout is such that \( v_{BO}(b, 0, n) > 0 \). Via equation (51), we can thus follow that the stability-equivalent bailout \( \kappa \) is such that \( u(c_2 + \hat{b} - \tau(\hat{b}, \varepsilon)) > u(c_1 - \tau(\hat{b}, \varepsilon) - \kappa) \) and \( u(c_2 + \hat{b} - \tau(\hat{b}, \varepsilon)) > v_{BO}(b, 0, n) \). Therefore, rearranging (51), it holds for all \( \hat{b} \in [0, \bar{b}] \),

\[ u(c_1 - \kappa - \tau(\hat{b}, \varepsilon)) = u(c_2 + \hat{b} - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n) > 0. \] (52)

But then also the inverse of utility must be positive for all \( \hat{b} \in [0, \bar{b}] \),

\[ c_1 - \kappa - \tau(\hat{b}, \varepsilon) = u^{-1} \left( u(c_2 + \hat{b} - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n) \right) > 0 \] (53)
From (53), we can follow that the stability-equivalent bailin is feasible, and positive for all \( \hat{b} \in [0, \bar{b}] \), \( c_1 - \tau(\hat{b}) > \kappa \), and \( c_1 - \tau(\hat{b}, \varepsilon) > u^{-1}\left(u(c_2 + \hat{b} - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n)\right) \).

For every bailout \( \hat{b} \in [0, b(n)] \), the stability-equivalent bailin is given as the withdrawal-contingent function

\[
\kappa(\hat{b}, n) = c_1 - \tau(\hat{b}, \varepsilon) - u^{-1}\left(u(c_2 + \hat{b} - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n)\right) > 0, \text{ for all } n \in [n_p, 1], \text{ and } \hat{b} \in [0, \bar{b}]
\]  

(54)

The least costly, feasible, stability-equivalent bailout is, thus, given by a zero bailout, \( \hat{b} = 0 \), implying a zero tax \( \tau(0, \varepsilon) = 0 \), and the stability-equivalent, feasible bailin

\[
\kappa(0, n) = c_1 - u^{-1}(u(c_2) - v_{BO}(b, 0, n)) > 0, \text{ for all } n \in [n_p, 1]
\]  

(55)

**A2** Assume for the considered \( n \in [n_p, 1] \), the payoff difference following the original bailout is such that \( v_{BO}(b, 0, n) \leq 0 \). Via equation (51), we can thus follow that the stability-equivalent bailin satisfies \( 0 \leq u(c_2 + \hat{b} - \tau(\hat{b}, \varepsilon)) \leq u(c_1 - \tau(\hat{b}, \varepsilon) - \kappa) \) and \( u(c_2 + \hat{b} - \tau(\hat{b}, \varepsilon)) \geq v_{BO}(b, 0, n) \). Nevertheless, as before, rearranging (51), it holds for all less costly bailouts \( \hat{b} \in [0, \bar{b}] \),

\[
u(c_1 - \kappa - \tau(\hat{b}, \varepsilon)) = u(c_2 + \hat{b} - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n) \geq 0
\]  

(56)

Thus, following the same steps as in the case of A1, the stability-equivalent bailin \( \kappa \) is positive, feasible for all \( \hat{b} \in [0, \bar{b}] \), and given by (54). The least costly, feasible, stability-equivalent bailout is, again given by \( \hat{b} = 0 \) with a simultaneous bailin given by (55).

**Case B** For states \( \theta > \theta^* + \varepsilon \), all agents receive a signal above the trigger and all patient creditors roll over, \( n = \lambda \). Since the bailout is only granted for high withdrawals in the range \( [n_p, 1] \), the bailout is not triggered. For state realizations \( \theta \in (\theta_p, \theta^* + \varepsilon] \), some agents receive signals below the trigger and withdraw but the aggregate withdrawals remain too low to trigger the payment of the bailout. Thus, for all states \( \theta \in (\theta_p, 1] \) no bailout is paid since the aggregate withdrawals realize in \( [\lambda, n_p] \).

Fix \( n \in [\lambda, n_p] \). The payoff difference following the original, costly bailout equals

\[
v_{BO}(b, 0, n) = u(c_2(n) - \tau(b, \varepsilon)) - u(c_1(n) - \tau(b, \varepsilon)), \text{ } n \in [0, n_p]
\]  

(57)
While for this withdrawal range \([\lambda, n_p]\) no bailout is granted, we yet need to impose a bailout to agents that withdraw as in case A. The reasons is that a bailout reduction reduces the ex ante tax, and thus changes relative payoffs also in the range \(n \in [0, n_p]\) unless a bailout \(\kappa\) is designed to keep these relative payoffs unchanged at the level of the original bailout. As before, I employ the implicit function theorem to determine the haircut that keeps the PI constant when reducing the bailout allocation and thus the tax. The payoff difference under the less costly policy alternative is given by

\[
v_{BI}(\hat{b}, \kappa(\hat{b}), n) = u(c_2 - \tau(\hat{b})) - u(c_1 - \kappa - \tau(\hat{b})), \hat{b} \in [0, \bar{b}]. \tag{58}
\]

For every \(n \in [0, n_p]\) and \(\hat{b} \in [0, \bar{b}]\) the stability-equivalent bailout \(\kappa\) is such that \(v_{BI}(\hat{b}, \kappa(\hat{b}), n) = v_{BO}(b, 0, n)\), as given in (57). For feasibility I again distinguish two cases.

**B1** Assume for the considered \(n \in [0, n_p]\), the payoff difference following the original bailout is such that \(v_{BO}(b, 0, n) > 0\). Via equation (58), it thus follows that \(u(c_2 - \tau(\hat{b}, \varepsilon)) > u(c_1 - \tau(\hat{b}, \varepsilon) - \kappa)\) and \(u(c_2 - \tau(\hat{b}, \varepsilon)) > v_{BO}(b, 0, n)\). Therefore, rearranging (51), it holds for all \(\hat{b} \in [0, \bar{b}]\),

\[
u(c_1 - \kappa - \tau(\hat{b}, \varepsilon)) = u(c_2 - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n) > 0 \tag{59}
\]

Thus, following the same steps as in the case of A1, the stability-equivalent bailout \(\kappa\) is positive and feasible for all \(\hat{b} \in [0, \bar{b}]\), given by

\[
\kappa(\hat{b}, n) = c_1 - \tau(\hat{b}, \varepsilon) - u^{-1}\left(u(c_2 - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n)\right) > 0, \text{ for all } n \in [0, n_p], \text{ and } \hat{b} \in [0, \bar{b}] \tag{60}
\]

The least costly, feasible, stability-equivalent bailout in this withdrawal range is again given by a zero bailout, \(\hat{b} = 0\), implying a zero tax \(\tau(0, \varepsilon) = 0\), and the stability-equivalent, feasible bailout

\[
\kappa(0, n) = c_1 - u^{-1}(u(c_2) - v_{BO}(b, 0, n)) > 0, \text{ for all } n \in [0, n_p] \tag{61}
\]

**B2** Assume for the considered \(n \in [0, n_p]\), the payoff difference following the original bailout is such that \(v_{BO}(b, 0, n) \leq 0\). Via equation (58), it follows \(0 \leq u(c_2 - \tau(\hat{b}, \varepsilon)) \leq u(c_1 - \tau(\hat{b}, \varepsilon) - \kappa)\) and \(u(c_2 - \tau(\hat{b}, \varepsilon)) \geq v_{BO}(b, 0, n)\). As before, rearranging (58) yields
that for all $\hat{b} \in [0, \bar{b}]$, $u(c_1 - \kappa - \tau(\hat{b}, \varepsilon)) = u(c_2 - \tau(\hat{b}, \varepsilon)) - v_{BO}(b, 0, n) \geq 0$. Thus, following the same steps as in the case of A1, and B1, the stability-equivalent bailin $\kappa$ is positive and feasible for all $\hat{b} \in [0, \bar{b}]$, given by (60). The least costly, feasible, stability-equivalent bailout is given by $\hat{b} = 0$ with a simultaneous bailin given by (61).

\section*{9.3 Proof: Comparative Statics}

\textbf{Proof. [Proposition 4.2]}

Assume $\hat{b}_{j+1}(n) \leq \hat{b}_j(n)$ for all $n \in [n_p, 1]$. A pointwise decline of $\hat{b}_j(n)$ in $j$ prompts a decline in the tax $\tau(\hat{b}_j)$ via (15). Therefore, agents that roll-over receive a smaller share of the bailout but also pay a reduced tax.

(i) Generically, $(\hat{b}_j(n) - \tau(\hat{b}_j))$ does not decline in $j$ for all $n \in [n_p, 1]$. This because the bailout declines over the range $[n_p, 1]$, thus lowering the tax, but may stay constant for some points $n$. In that case, $(\hat{b}_j(n) - \tau(\hat{b}_j))$ would be increasing.

However, $(\hat{b}_j(n) - \tau(\hat{b}_j))$ declines in $j$ on average over the interval $[n_p, 1]$:

\begin{align*}
\int_{n_p}^{1} (\hat{b}_j(n) - \tau(\hat{b}_j)) \, dn & = \int_{n_p}^{1} \left( \hat{b}_j(n) - \frac{M_2}{M} \int_{n_p}^{1} \hat{b}_j(k) \, dk \right) \, dn \\
& = \int_{n_p}^{1} \hat{b}_j(n) \left( 1 - \frac{M_2}{M} (1 - n_p) \right) \, dn \\
& \geq \int_{n_p}^{1} \hat{b}_{j+1}(n) \left( 1 - \frac{M_2}{M} (1 - n_p) \right) \, dn \\
& = \int_{n_p}^{1} (\hat{b}_{j+1}(n) - \tau(\hat{b}_{j+1})) \, dn
\end{align*}

where the inequality holds by $\frac{M_2}{M} (1 - n_p) \in (0, 1)$. Therefore, the average utility of agents...
that roll over declines as the bailout allocation becomes smaller,

\[
\int_{n_p}^{1} \left( u(c_2(n) + \hat{b}_{j+1}(n) - \tau(\hat{b}_{j+1})) - u(c_2(n) + \hat{b}_j(n) - \tau(\hat{b}_j)) \right) \, dn
\]

(66)

\[
\leq \int_{n_p}^{1} u'(c_2(n) + \hat{b}_j(n) - \tau(\hat{b}_j)) \left( \hat{b}_{j+1}(n) - \tau(\hat{b}_{j+1}) - (\hat{b}_j(n) - \tau(\hat{b}_j)) \right) \, dn
\]

(67)

\[
< u'(0) \int_{n_p}^{1} \left( (\hat{b}_{j+1}(n) - \tau(\hat{b}_{j+1})) - (\hat{b}_j(n) - \tau(\hat{b}_j)) \right) \, dn
\]

(68)

\[
\leq 0
\]

(69)

where in the first and second inequality I use the concavity of utility, and the last inequality follows from (62). We want to figure out how the stability-equivalent bailin changes as the bailout allocation drops. Fix a particular value \( n \in [n_p, 1] \). We know that for the stability-equivalent bailins \( \kappa_j(\hat{b}_j, n) \) and \( \kappa_{j+1}(\hat{b}_{j+1}, n) \) it must hold

\[
v_{BO}(\hat{b}_j, 0, n) = v_{BI}(\hat{b}_j, \kappa_j, n)
\]

(70)

\[
= u(c_2(n) + \hat{b}_j(n) - \tau(\hat{b}_j)) - u(c_1(n) - \tau(\hat{b}_j) - \kappa_j), \ n \in [n_p, 1]
\]

(71)

\[
= u(c_2(n) + \hat{b}_{j+1}(n) - \tau(\hat{b}_{j+1})) - u(c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1}), \ n \in [n_p, 1]
\]

(72)

\[
= v_{BI}(\hat{b}_{j+1}, \kappa_{j+1}, n)
\]

(73)

That is, as \( j \) increases (as the bailout and the tax decline), the stability-equivalent bailin function \( \kappa_j(n) \) changes with the bailout function \( \hat{b}_j \) such that the left hand side of (70) stays constant at that \( n \in [n_p, 1] \). But then the following inequality holds with (66) and (70)

\[
\int_{n_p}^{1} \left( u(c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1}(n)) - u(c_1(n) - \tau(\hat{b}_j) - \kappa_j(n)) \right) \, dn
\]

(74)

\[
= \int_{n_p}^{1} \left( u(c_2(n) + \hat{b}_{j+1}(n) - \tau(\hat{b}_{j+1})) - u(c_2(n) + \hat{b}_j(n) - \tau(\hat{b}_j)) \right) \, dn
\]

(75)

\[
\leq 0
\]

(76)

We have shown, because the average utility of agents that roll-over declines in \( j \) on the interval \( n \in [n_p, 1] \) the stability-equivalent bailin needs to adjust such that also the average utility of agents that withdraw declines in \( j \).
\[ \int_{n_p}^{1} \left( u(c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1}(n)) \right) \, dn \leq \int_{n_p}^{1} u(c_1(n) - \tau(\hat{b}_j) - \kappa_j(n)) \, dn \]  

(77)

As the last step, because the tax is decreasing in \( j \), and assuming \( k_j(n) \) remained constant as \( \hat{b} \) declines, it holds

\[ u(c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_j(n)) > u(c_1(n) - \tau(\hat{b}_j) - \kappa_j(n)), \quad \text{for all } n \in [n_p, 1] \]  

(78)

But this contradicts (77). For (77) to hold, the average bailin \( \kappa_{j+1}(n) \) over \([n_p, 1]\) has to increase as the bailout declines with \( j \),

\[ 0 \geq \int_{n_p}^{1} \left( u(c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1}(n)) \right) - \left( u(c_1(n) - \tau(\hat{b}_j) - \kappa_j(n)) \right) \, dn \]  

\[ \geq \int_{n_p}^{1} \left( u'(c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1}(n)) \right) \left( \frac{\tau(\hat{b}_j) - \tau(\hat{b}_{j+1})}{0} + \frac{(\kappa_j(n) - \kappa_{j+1}(n))}{0} \right) \, dn \]

(80)

requiring \( \int_{n_p}^{1} \kappa_j(n) \, dn < \int_{n_p}^{1} \kappa_{j+1}(n) \, dn \) because \( u'(c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1}(n)) \) is bounded from below on \( n \in [n_p, 1] \) via \( u'(c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1}(n)) > \max_{n \in [n_p, 1]} u'(c_1(n)). \)

(iii)

Now consider the range \( n \in [\lambda, n_p]. \) In addition assume \( n_p < n^* \) so that for all \( n \in [\lambda, n_p] \) it holds \( v_{BO}(b, 0, n) > 0. \) Next, because of stability-equivalence of the bailins and \( v_{BO}(b, 0, n) > 0 \) it follows: \( \kappa_j \) and \( \kappa_{j+1} \) are such that

\[ 0 < v_{BO}(b, 0, n) = v_{BI}(\hat{b}_j, \kappa_j, n) = v_{BI}(\hat{b}_{j+1}, \kappa_{j+1}, n) \]  

(81)

which implies the inequalities

\[ c_2(n) - \tau(\hat{b}_{j+1}) > c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1} \]  

(82)

and

\[ c_2(n) - \tau(\hat{b}_j) > c_1(n) - \tau(\hat{b}_j) - \kappa_j \]  

(83)

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Because \( \hat{b}_{j+1}(n) \leq \hat{b}_j(n) \) for all \( n \in [\lambda, n_p) \), we know that the tax to finance the smaller bailout \( \hat{b}_{j+1} \) is smaller \( \tau(\hat{b}_{j+1}) \leq \tau(\hat{b}_j) \). We can thus follow

\[
c_2(n) - \tau(\hat{b}_{j+1}) > c_2(n) - \tau(\hat{b}_j) > c_1(n) - \tau(\hat{b}_j) - \kappa_j, \quad \text{for all } n \in [\lambda, n_p)
\]  

(84)

We want to determined whether \( k_{j+1} \) exceeds or undercuts \( k_j \). For that purpose, stability-equivalence and (81) imply the equality

\[
u(\tau(\hat{b}_{j+1})) - u(c_2(n) - \tau(\hat{b}_j)) = u(c_1(n) - \tau(\hat{b}_j) - \kappa_j) - u(c_1(n) - \tau(\hat{b}_j) - \kappa_j) \tag{85}
\]

From (85), and since \( c_2(n) - \tau(\hat{b}_{j+1}) > c_2(n) - \tau(\hat{b}_j) \), it follows \( 0 < u(\tau(\hat{b}_{j+1})) - u(c_2(n) - \tau(\hat{b}_j)) = u(c_1(n) - \tau(\hat{b}_j) - \kappa_j) - u(c_1(n) - \tau(\hat{b}_j) - \kappa_j) \), and thus

\[
\tau(\hat{b}_{j+1}) - k_{j+1} > c_1(n) - \tau(\hat{b}_j) - \kappa_j
\]  

(86)

Considering (85), we know that both agents that roll over and withdrawing agents become happy as the tax declines due to the reduced bailout. For pinning down the change in the stability-equivalent bailin, we need to determine whose happiness increases faster. By \( \nu_{BO}(b, 0, n) > 0 \), we will now show that the utility of agents that withdraw increases faster which is why the bailin needs to increase as the bailout drops from \( \hat{b}_j \) to \( \hat{b}_{j+1} \).

By the fundamental theorem of calculus and the intermediate value theorem for integrals, we can write

\[
u(c_2(n) - \tau(\hat{b}_{j+1})) - u(c_2(n) - \tau(\hat{b}_j))\]

\[
= \int_{c_2(n) - \tau(\hat{b}_j)}^{c_2(n) - \tau(\hat{b}_{j+1})} u'(x) \, dx \tag{87}
\]

\[
= u'(c_A(n)) \left( \tau(\hat{b}_j) - \tau(\hat{b}_{j+1}) \right) \tag{88}
\]

for some \( c_A(n) \in I_A \equiv [c_2(n) - \tau(\hat{b}_j), c_2(n) - \tau(\hat{b}_{j+1})] \). For the same reason,

\[
u(c_1(n) - \tau(\hat{b}_{j+1}) - k_{j+1}) - u(c_1(n) - \tau(\hat{b}_j) - k_j)\]

\[
= \int_{c_1(n) - \tau(\hat{b}_j) - k_j}^{c_1(n) - \tau(\hat{b}_{j+1}) - k_{j+1}} u'(x) \, dx \tag{90}
\]

\[
= u'(c_B(n)) \left( \left( \tau(\hat{b}_j) - \tau(\hat{b}_{j+1}) \right) + (k_j - k_{j+1}) \right) \tag{92}
\]

for some \( c_B(n) \in I_B \equiv [c_1(n) - \tau(\hat{b}_j) - k_j, c_1(n) - \tau(\hat{b}_{j+1}) - k_{j+1}] \). Plugging (89) and (92)
into (85), stability-equivalence demands

\[
u'(c_A(n)) (\tau(\hat{b}_j) - \tau(\hat{b}_{j+1})) = u'(c_B(n)) \left( (\tau(\hat{b}_j) - \tau(\hat{b}_{j+1})) + (k_j - k_{j+1}) \right) \tag{93}\]

We know \(u'(c_A), u'(c_B) > 0\), and \((\tau(\hat{b}_j) - \tau(\hat{b}_{j+1})) > 0\). Thus, we can follow \(k_{j+1} > k_j\) only if \(u'(c_B) > u'(c_A)\), that is, if \(c_A > c_B\).

To show that \(c_A > c_B\), we need to analyse the intervals \(I_A\) and \(I_B\). Given (82), (86) and (84), the size ordering of the integral boundaries is fully determined except for the upper bound of \(I_B\). There can only be two cases:

**Case A** \(c_1(n) - \tau(\hat{b}_{j+1}) - k_{j+1} = (c_2(n) - \tau(\hat{b}_j), c_2(n) - \tau(\hat{b}_{j+1}))\) or

**Case B** \(c_1(n) - \tau(\hat{b}_{j+1}) - k_{j+1} = (c_1(n) - \tau(\hat{b}_j) - k_j, c_2(n) - \tau(\hat{b}_j))\).

Consider case B. Then \(I_A \cap I_B = \emptyset\), and interval A is positioned higher than interval B. Thus, the function \(u'(x)\) when integrated over \(I_A\) is evaluated at higher points, and thus takes lower values than when integrated over \(I_B\). Thus \(c_A(n) > c_B(n)\).

In case A, the integrals overlap but interval \(I_B\) still contains on average lower values. Again by the intermediate value theorem, there must exist a value \(c \in [a, b]\) (for \(a, b\) general here) such that the following equation holds

\[
\frac{1}{b-a} \int_a^b u'(x) \, dx = u'(c) \tag{94}\]

I next show that \(c\) is increasing in both \(a\) and \(b\), which then implies \(c_A > c_B\): Consider the implicit function

\[
F(a, b, c) = u'(c) - \frac{1}{b-a} \int_a^b u'(x) \, dx \tag{95}\]

then \(\frac{\partial F}{\partial c} < 0\), \(\frac{\partial F}{\partial a} \geq 0\) and \(\frac{\partial F}{\partial b} \geq 0\) by the concavity of utility. Thus \(\frac{\partial c}{\partial a} > 0\) and \(\frac{\partial c}{\partial b} > 0\), implying \(c_A > c_B\) and thus \(k_{j+1}(n) > k_j(n)\) for all \(n \in [\lambda, n^*) \cap [\lambda, n^*]\).

(ii)

Now assume \(n^* < n_p\) and fix \(n \in (n^*, n_p)\). For such \(n\), \(0 > v_{BO}(n) = v_{BI}(\hat{b}_j, \kappa_j, n) = v_{BI}(\hat{b}_{j+1}, \kappa_{j+1}, n)\). It therefore holds

\[
c_2(n) - \tau(\hat{b}_{j+1}) < c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1} \tag{96}\]
and

\[ c_2(n) - \tau(\hat{b}_j) < c_1(n) - \tau(\hat{b}_j) - \kappa_j \]  

(97)

Because the tax still declines, \( \tau(\hat{b}_{j+1}) \leq \tau(\hat{b}_j) \), and with (96),

\[ c_1(n) - \tau(\hat{b}_{j+1}) - \kappa_{j+1} > c_2(n) - \tau(\hat{b}_{j+1}) > c_2(n) - \tau(\hat{b}_j). \]  

(98)

Equality (85) and, with the decline in the tax, inequality (86) continue to hold. Likewise (89), (92) and thus (93) still hold.

To determine the ordering of \( c_A \) and \( c_B \), see that the intervals \( I_A \) and \( I_B \) are now shifted. The relation of the lower bound of \( I_B \) relative to the upper bound of \( I_A \) is not determined via the inequalities. But again, there can only be two cases:

**Case A** \( c_1(n) - \tau(\hat{b}_j) - k_j \in (c_2(n) - \tau(\hat{b}_j), c_2(n) - \tau(\hat{b}_{j+1})) \) or

**Case B** \( c_1(n) - \tau(\hat{b}_j) - k_j \in (c_2(n) - \tau(\hat{b}_{j+1}), c_1(n) - \tau(\hat{b}_{j+1}) - k_{j+1}). \)

In case A, the intervals \( I_A \) and \( I_B \) overlap, but interval \( I_A \) is now positioned low relative to \( I_B \). The lower integral boundary of \( I_B \) exceeds the lower integral boundary of \( I_A \). Likewise, the upper integral boundary of \( I_B \) exceeds the upper integral boundary of \( I_A \). Thus, by the arguments above, \( c_B > c_A \). In the case B, the intervals are disjoint and interval B is positioned higher, thus by the same reasoning, \( c_B > c_A \). In either case, \( k_{j+1}(n) < k_j(n) \) for all \( n \in (n^*, n_p) \).

\[ \square \]

### 9.4 Proof: Welfare

**Proof.** [Proposition 5.1] Let \((b, 0)\) denote the original bailout requiring a budget-balancing lump-sum tax \( \tau = \tau(b, 0) \) that is levied on all agents in the economy

\[ \tau(b, \epsilon)M = M_2 \int_0^{\theta^*_b} b(n(\theta, \theta^*(b, 0))) \, d\theta \]  

(99)

Let \((\hat{b}, \kappa(\hat{b}))\) denote the partial bailin requiring tax \( \hat{\tau} = \tau(\hat{b}, \epsilon) \). Let \( \bar{c} \) the optimal consumption level of citizens in the economy that are not affiliated with the bank before the tax is applied. For sake of brevity, I write \( n(\theta) \) instead of \( n(\theta, \theta^*(b, 0)) \), and likewise \( c(n(\theta)) \).
Welfare in the case of the original, costly bailout equals

\[ W^{\text{Bailout}}(b, 0) = M_1 u(c - \tau) \]

\[ + M_2 \left( \int_{0}^{\theta^* - \epsilon} u(c_1(n(\theta)) - \tau) \, d\theta \right) \]

\[ + \int_{\theta^* - \epsilon}^{\theta_p} n(\theta)u(c_1(n(\theta)) - \tau) + (1 - n(\theta))u(c_2(n(\theta)) + b(n(\theta)) - \tau) \, d\theta \]

\[ + \int_{\theta_p}^{\theta^* + \epsilon} n(\theta)u(c_1(n(\theta)) - \tau) + (1 - n(\theta))u(c_2(n(\theta)) - \tau) \, d\theta \]

\[ + \int_{\theta^* - \epsilon}^{1} \lambda u(c_1(n(\theta)) - \tau) + (1 - \lambda)u(c_2(n(\theta)) - \tau) \, d\theta \]

To understand this equation, recall that the bailout is only paid to agents that roll-over in the case where the withdrawals are large enough to trigger the regulator’s intervention, \( n \in [n_p, 1] \), that is, if the state is low enough, in \( \theta \in [0, \theta_p] \). Moreover, for states below \( \theta^* - \epsilon \) all agents observe signals below the trigger and thus run on the bank, foregoing their share of the bailout. Therefore, the bailout is only paid to agents that “make a coordination mistake” and rollover despite high withdrawals above \( n_p \), that is, for states in \([\theta^* - \epsilon, \theta_p] \). For states in \([\theta_p, \theta^* + \epsilon] \) the aggregate withdrawals realized below \( n_p \) and the bailout intervention is not triggered. For states above \( \theta^* + \epsilon \) all agents observe signals above the trigger signal and thus all patient agents roll over. Impatient agents always withdraw because they can only consume in \( t = 1 \).
Welfare in the case of the less costly, partial bailin equals

\[ W^{\text{Bailin}}(\hat{b}, \kappa(\hat{b})) = M_1 u(\bar{c} - \hat{\tau}) \] (105)

\[ + M_2 \left( \int_{0}^{\theta^*(\hat{b}, \kappa(\hat{b})) - \varepsilon} u(c_1(n(\theta))) - \kappa(\hat{b}(n(\theta))) - \hat{\tau}) d\theta \right) \] (106)

\[ + \int_{\theta^*(\hat{b}, \kappa(\hat{b})) - \varepsilon}^{\theta^*(\hat{b}, \kappa(\hat{b})) + \varepsilon} n(\theta)u(c_1(n(\theta))) - \kappa(\hat{b}(n(\theta))) - \hat{\tau}) + (1 - n(\theta))u(c_2(n(\theta))) + \bar{b}(n(\theta)) - \hat{\tau}) d\theta \] (107)

\[ + \int_{\theta^*(\hat{b}, \kappa(\hat{b})) + \varepsilon}^{1} \lambda u(c_1(n(\theta))) - \kappa(\hat{b}(n(\theta))) - \hat{\tau}) + (1 - \lambda)u(c_2(n(\theta))) - \hat{\tau}) d\theta \] (108)

where \( \hat{b}(n) \in [0, b(n)) \) is the reduced bailout that is granted to agents that roll-over if the withdrawals are high enough to cause regulatory intervention (states below \( \theta_p \)), \( \kappa(\hat{b}(n)) \) is the bailin\(^{11} \) imposed on agents that withdraw for the full range of withdrawals \( n \in [0, 1] \), and \( \hat{\tau} = \tau(\hat{b}, \kappa(\hat{b})) \) is the reduced budget balancing tax given by

\[ \hat{\tau} = \tau(\hat{b}, \kappa(\hat{b})) = \frac{M_2}{M} \int_{0}^{\theta_p} \hat{b}(n(\theta, \theta^*)) d\theta \] (110)

By design, the bailin and the bailout are stability-equivalent, meaning their equilibrium triggers and critical states are identical

\[ \theta^* = \theta^*(b, 0) = \theta^*(\hat{b}, \kappa(\hat{b})) \] (111)

\[ \theta_p = \theta_p(b, 0) = \theta_p(\hat{b}, \kappa(\hat{b})) \] (112)

because both policies imply the exact same payoff difference function on the full withdrawal range \( n \in [0, 1] \). Here, the critical state is the state at which the aggregate withdrawals realize such that the regulator grants the bailin, \( n = n_p \). In section 6, we see a variation of this feature where the critical state changes with the bailout provision

\(^{11}\)Recall, in case of risk-aversion we need to impose the bailin on the full range even where the original bailout is not granted to guarantee stability-equivalence. This is, because the lower bailout causes a tax reduction. Without the bailin, the tax reduction would affect relative payoffs in the withdrawal range where the bailout is not granted, which then would cause a change in ex ante stability.

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because the regulator does not hold the intervention threshold fixed but gives a payoff guarantee.

The welfare difference between the setting that imposes a stability-equivalent bailin versus the original bailout equals

\[
\Delta W = W^{\text{Bailin}}(\tilde{b}, \kappa(\tilde{b})) - W^{\text{Bailout}}(b, 0)
\]

\[
= M_1 (u(\tilde{c} - \hat{\tau}) - u(\tilde{c} - \tau))
\]

\[
+ M_2 \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} n(\theta, \theta^*) \left( u(c_1 - \kappa(\tilde{b}(n(\theta))) - \hat{\tau}) - u(c_1 - \tau) \right) + (1 - n) (u(c_2 + \tilde{b}(n) - \hat{\tau}) - u(c_2 + b(n) - \tau)) d\theta
\]

\[
+ M_2 \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} n(\theta, \theta^*) \left( u(c_1 - \kappa(\tilde{b}(n(\theta))) - \hat{\tau}) - u(c_1 - \tau) \right) + (1 - n) (u(c_2 - \hat{\tau}) - u(c_2 - \tau)) d\theta
\]

\[
+ M_2 \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} \lambda(u(c_1 - \kappa(\tilde{b}(n(\theta))) - \hat{\tau}) - u(c_1 - \tau)) + (1 - \lambda)(u(c_2 - \hat{\tau}) - u(c_2 - \tau)) d\theta
\]

At the limit as noise vanishes, \( \varepsilon \to 0 \), it holds \( \theta^* - \varepsilon \to \theta_p \), \( \theta^* + \varepsilon \to \theta_p \). Moreover, all integrands are bounded, thus, by Lebesques bounded convergence theorem

\[
M_2 \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} n(\theta, \theta^*) \left( u(c_1 - \kappa(\tilde{b}(n(\theta))) - \hat{\tau}) - u(c_1 - \tau) \right) + (1 - n) (u(c_2 + \tilde{b}(n) - \hat{\tau}) - u(c_2 + b(n) - \tau)) d\theta
\]

\[
\to 0, \text{ as } \varepsilon \to 0
\]

Therefore, at the limit the welfare difference simplifies to

\[
\lim_{\varepsilon \to 0} \Delta W = W^{\text{Bailin}}(\tilde{b}, \kappa(\tilde{b})) - W^{\text{Bailout}}(b, 0)
\]

\[
= M_1 (u(\tilde{c} - \hat{\tau}) - u(\tilde{c} - \tau))
\]

\[
+ M_2 \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} u(c_1 - \kappa(\tilde{b}(n(\theta))) - \hat{\tau}) - u(c_1 - \tau) \ d\theta
\]

\[
+ M_2 \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} \lambda(u(c_1 - \kappa(\tilde{b}(n(\theta))) - \hat{\tau}) - u(c_1 - \tau)) + (1 - \lambda)(u(c_2 - \hat{\tau}) - u(c_2 - \tau)) d\theta
\]
Since \( \hat{\tau} < \tau \), it holds \( u(\bar{c} - \hat{\tau}) - u(\bar{c} - \tau) > 0 \) and \( u(c_2 - \hat{\tau}) - u(c_2 - \tau) > 0 \). Thus, the first term is positive. Citizens that are not affiliated with the bank always favor the cheaper policy since they never benefit from the policy but suffer from taxation. Moreover, in good times when no intervention is triggered, \( \theta \in [\theta_p, 1] \), agents that roll over likewise favor the cheaper alternative because it requires a lower ex ante tax.

To analyze the remaining terms, see that by construction of the bailin and stability equivalence, it holds

\[
\begin{aligned}
 u^{BO}_2(n) - u^{BO}_1(n) &\equiv v_{BO}(n) = v_{BI}(n) \equiv u^{BI}_2(n) - u^{BI}_1(n), \text{ for all } n \in [0, 1]
\end{aligned}
\]

which is equivalent to

\[
\begin{aligned}
 u^{BI}_1(n) - u^{BO}_1(n) &\equiv u^{BI}_2(n) - u^{BO}_2(n), \text{ for all } n \in [0, 1]
\end{aligned}
\]

Consider the last term in (119). It considers high states in \( [\theta_p, 1] \) for which all patient agents roll over, \( n = \lambda \), and no bailout is granted by \( \lambda < n_p \). For the first part of the integrand of that term, case B and equations (57), (58) and (124) imply

\[
\begin{aligned}
 u(c_1 - \kappa(\hat{b}(n(\theta)))) - \hat{\tau}) - u(c_1 - \tau) = u(c_2 - \hat{\tau}) - u(c_2 - \tau) > 0
\end{aligned}
\]

by \( \hat{\tau} < \tau \). That is, for \( \theta \in [\theta_p, 1] \), it holds

\[
\begin{aligned}
 \kappa(\hat{b}(n(\theta))) + \hat{\tau} < \tau
\end{aligned}
\]

meaning the stability-equivalent bailin \( \kappa \) plus the reduced tax \( \hat{\tau} \) are jointly still smaller than the tax required to finance the original bailout \( \tau \).

I can thus rewrite

\[
\begin{aligned}
 M_2 \int_{\theta_p}^{1} \lambda(u(c_1 - \kappa(\hat{b}(n(\theta)))) - \hat{\tau}) - u(c_1 - \tau)) + (1 - \lambda)(u(c_2 - \hat{\tau}) - u(c_2 - \tau)) d\theta
\end{aligned}
\]

\[
\begin{aligned}
 = M_2 \int_{\theta_p}^{1} (u(c_2 - \hat{\tau}) - u(c_2 - \tau)) d\theta > 0
\end{aligned}
\]

and the last term of (119) is hence positive. That is, in good times when no intervention is triggered, \( \theta \in [\theta_p, 1] \), all creditors favor the cheaper alternative. Agents that roll over favor it because it requires a lower ex ante tax, whereas agents that withdraw favor it because, as we have just shown in (125), the stability-equivalent bailin \( \kappa \) plus the reduced
tax $\hat{\tau}$ are jointly still smaller than the tax required to finance the original bailout $\tau$.

For the second term of (119), see that for states in $[0, \theta_p)$ all agents withdraw, thus, no agent receives a share of the bailout and all agents are bailed in. Via case A and equations (124),(50) and (51), for the integrand this implies

$$\left( u(c_1 - \kappa(\hat{b}(n(\theta))) - \hat{\tau}) - u(c_1 - \tau) \right) = u(c_2 + \hat{b} - \hat{\tau}) - u(c_2 + b(n) - \tau).$$

(129)

Now using inequality (69), we can infer

$$\int_0^{\theta_p} \left( u(c_1(n(\theta)) - \kappa(\hat{b}(n(\theta))) - \hat{\tau}) - u(c_1(n(\theta)) - \tau) \right) d\theta$$

(130)

$$= \frac{1 - \lambda}{2\varepsilon} \int_{n_p}^1 u(c_2(n) + \hat{b}(n) - \hat{\tau}) - u(c_2(n) + b(n) - \tau) \, dn$$

(131)

$$\leq 0$$

(132)

where I have substituted at the equality using (42). The second term of (119) is, thus, negative. Given bad times, $\theta \in [0, \theta_p]$, utility to withdrawing agents following the original more costly bailout is higher than utility following the less costly alternative. Intuitively, in bad times $\theta \in [0, \theta_p]$ all creditors run on the bank whenever a bailout is triggered, and are bailed in when the regulator follows the less costly policy. Inequality (130) shows that given a run, the haircut implied by the bailin plus the reduced tax are jointly larger than the tax implied by the original bailout.

For (i): Considering the overall sign of (119), clearly, when $M_1$ is large relative to $M_2 << M_1$, then the bailin yields higher welfare than the bailout, $\lim_{\varepsilon \to 0} \Delta W > 0$ since the first term in (69) dominates.

For (iii): As a special case, though, if the bank becomes negligible in the economy, then the welfare difference is zero because the costs of either bailout are financed by an increasing population so that the tax vanishes. This holds because for $M_2$ fixed and $M_1 \to \infty$, $\tau \to \hat{\tau} \to 0$ by

$$\lim_{M_1 \to \infty} ||\hat{\tau}(\hat{b}, \varepsilon) - \tau(b, \varepsilon)|| \leq \lim_{M_1 \to \infty} \frac{M_2}{M_1 + M_2} \int_0^{\theta_p} ||\hat{b}(n(\theta, \theta^*)) - b(n(\theta, \theta^*))|| \, d\theta = 0$$

(133)

Therefore, $u(\bar{c} - \tau) \to u(\bar{c} - \hat{\tau})$ and $\lim_{M_1 \to \infty} \Delta W = \lim_{M_1 \to \infty} M_1(u(\bar{c} - \hat{\tau}) - u(\bar{c} - \tau)) = 0$.

For (ii): But even for $M_2$ large relative to $M_1$: Consider the extreme $M_1 = 0$ and
$M_2 = M$. Recall that for $\varepsilon \to 0$, all agents withdraw for low states in $[0, \theta_p]$, $n(\theta) = 1$, and all patient types roll over for high states in $\theta \in [\theta_p, 1]$, $n(\theta) = \lambda$. Recall that $b(n(\theta))$ and $\hat{b}(n(\theta))$ are only withdrawal contingent in the range $[\theta^* - \varepsilon, \theta^* + \varepsilon]$. For $\varepsilon \to 0$, though, $\theta^* - \varepsilon \to \theta_p$, $\theta^* + \varepsilon \to \theta_p$, implying that $b(n(\theta))$ and $\hat{b}(n(\theta))$ are constant over $[0, \theta_p]$ respectively $\theta \in [\theta_p, 1]$.

With (129) and (127), we can rewrite (119) as

\[
\lim_{\varepsilon \to 0} \Delta W = M_2 \int_{0}^{\theta_p} u(c_2 + \hat{b} - \hat{\tau}) - u(c_2 + b - \tau) d\theta + M_2 \int_{\theta_p}^{1} (u(c_2 - \hat{\tau}) - u(c_2 - \tau)) d\theta
\]

By the mean value theorem for integrals, there exist states $\theta_L \in [0, \theta_p)$ and $\theta_H \in (\theta_p, 1]$ such that

\[
\int_{0}^{\theta_p} u(c_2(n(\theta)) + \hat{b} - \hat{\tau}) - u(c_2(n(\theta)) + b - \tau) d\theta = \theta_p \left( u(c_2(n(\theta_L)) + \hat{b} - \hat{\tau}) - u(c_2(n(\theta_L)) + b - \tau) \right)
\]

and

\[
\int_{\theta_p}^{1} (u(c_2(n(\theta)) - \hat{\tau}) - u(c_2(n(\theta)) - \tau)) d\theta = (1 - \theta_p) \left( u(c_2(n(\theta_H)) - \hat{\tau}) - u(c_2(n(\theta_H)) - \tau) \right)
\]

Then, welfare following the less costly policy exceeds welfare following the original costly bailout, $\lim_{\varepsilon \to 0} \Delta W > 0$, if and only if

\[
(1 - \theta_p) \left( u(c_2(\theta_H) - \hat{\tau}) - u(c_2(\theta_H) - \tau) \right) > \theta_p \left( u(c_2(\theta_L) + b - \tau) - u(c_2(\theta_L) + \hat{b} - \hat{\tau}) \right)
\]

that is, if and only if the ex ante likelihood of a run is sufficiently small:

\[
\theta_p < B^* \equiv \frac{(u(c_2(\theta_H) - \hat{\tau}) - u(c_2(\theta_H) - \tau)))}{\left( (u(c_2(\theta_L) + b - \tau) - u(c_2(\theta_L) + \hat{b} - \hat{\tau}) + (u(c_2(\theta_H) - \hat{\tau}) - u(c_2(\theta_H) - \tau))) \right)}
\]

\[\square\]

### 9.5 Application: Proof of monotonicity properties

**Proof.** [Lemma 6.1]
Verifying monotonicity  

I need to assure that the payoffs satisfy assumptions 2.1. I assume that \( b \) is small enough to satisfy \( b < u^{-1}(\frac{1}{r_1}u(r_1 - \tau(b))) + \tau(b) \). Such \( b \) can always be found since the condition is met for \( b \to 0 \) and thus \( \tau(0) = 0 \). If the condition holds, then \( u(b - \tau(b, \varepsilon)) < \frac{1}{r_1}u(r_1 - \tau(b, \varepsilon)) \), and thus \( u(b - \tau(b, \varepsilon)) < \frac{1}{nr_1}u(r_1 - \tau(b, \varepsilon)) \) for all \( n \in [n_c, 1] \). Such choice of \( b \) further implies that for all \( n \in [n_p(b), 1] \), \( w_2 < u_1 \).

Further, we know that for all states \( \theta \in (\overline{\theta}, 1] \) it holds \( \frac{R(\theta)(1-\lambda r_1)}{1-\lambda} > r_1 \), and thus \( u\left(\frac{R(\theta)(1-\lambda r_1)}{1-\lambda} - \tau(b, \varepsilon)\right) > u(r_1 - \tau(b, \varepsilon)) \). The pro rata share to creditors that roll over strictly declines in \( n \) and is continuous. Thus, by the intermediate value theorem there exists a unique \( n^* \in (\lambda, 1) \) such that

\[
u \left( \frac{R(\theta(n^*, \theta^*_\theta))}{1-n^*}(1-n^*r_1) - \tau(b, \varepsilon) \right) = u(r_1 - \tau(b, \varepsilon)) \tag{140}\]

at which the optimal response switches from roll over to withdraw. Further,

\[
u \left( \frac{R(\theta(n^*, \theta^*_\theta))}{1-n^*}(1-n^*r_1) - \tau(b, \varepsilon) \right) > u(r_1 - \tau(b, \varepsilon)) \]

for all \( n \in [\lambda, n^*] \) and \( u\left(\frac{R(\theta(n^*, \theta^*_\theta))}{1-n^*}(1-n^*r_1)}{1-\lambda} - \tau(b, \varepsilon)\right) < u(r_1 - \tau(b, \varepsilon)) \) for \( n \in [n^*, n_p(b)] \).

Thus, single-crossing and one-sided strategic complementarity holds. Further, the return of the asset \( R(\theta) \) strictly increases in the state, thus, state-monotonicity holds.

Dominance regions  

The dominance regions given in the GP benchmark model likewise apply in the setting with the bailout: By (33), for all states in \([0, \overline{\theta}] \) it holds

\[
u \left( \frac{R(\theta)}{1-\lambda}(1-\lambda r_1) - \tau(b, \varepsilon) \right) \leq u(r_1 - \tau(b, \varepsilon)) \tag{141}\]

and “withdraw” is a dominant action. For states in \([\overline{\theta}, 1] \) the asset pays already in \( t = 1 \), and the return satisfied \( R(\theta) > r_1 > 1 \). Therefore, the bank can finance all withdrawals without costly liquidation. The pro rata share to agents that roll over becomes \( \frac{R(\theta)-nr_1}{1-n} \) which is now an increasing function of the withdrawals \( n \). This pro rata share takes its smallest value in \( n = \lambda \) which already exceeds the value \( R(\theta) \) and thus \( r_1 \). Moreover, for states in \([\overline{\theta}, 1] \) the regulator never intervenes because the pro rata share never drops down to the value \( b < r_1 < R(\theta) \). Thus, roll over is dominant, for all \( \theta \in [\overline{\theta}, 1] \):

\[
u \left( \frac{R(\theta)-nr_1}{1-n} - \tau(b, \varepsilon) \right) > u(r_1 - \tau(b, \varepsilon)) \tag{142}\]

I can thus conclude that assumption 2.1 holds. Last, the payoff difference is continuous in

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the withdrawals \( n \) and the state \( \theta \), thus the continuity assumption 2.2 holds.

Finally, jointly with Proposition 3.1, it follows existence and uniqueness of equilibrium in the form of a trigger signal \( \theta^*(b) \).

\[ \square \]

### 9.6 Application: Construction of stability-equivalent bailin

**Proof.** [Proposition 6.1] I discuss feasibility first, and then construct the bailin.

#### Feasibility

Feasibility requires that all consumption levels are positive. As the guarantee \( \hat{b} = b - z \) declines, so does the tax:

\[
\tau(b - z, \varepsilon) = \frac{M_2}{M} \left( \int_0^{\theta_p(b-z)} \left( b - z - \frac{R(\theta)(1 - n(\theta, \theta^*_p)r_1)}{1 - n(\theta, \theta^*_p)} \right) d\theta \right)
\]

which declines monotonically in the cost reduction \( z \) by

\[
\frac{\partial}{\partial z} \tau(b - z, \varepsilon) = -\frac{M_2}{M} \int_0^{\theta_p(b-z)} d\theta + \frac{\partial \theta_p(b - z)}{\partial z} \times 0 < 0
\]

where we take into account that the critical state, \( \theta_p(\hat{b}) \), changes in \( \hat{b} \). If the deposit guarantee \( \hat{b} = b - z \) is small enough, the critical state falls below the trigger \( \theta_p(b - z) \leq \theta^*(b) \), and the tax becomes zero.\(^{12}\)

A zero bailout \( \hat{b} = 0 \) with tax \( \tau(0) = 0 \) is feasible: See that by \( M_2 < M, (1 - n) < 1 \) and \([\theta^* - \varepsilon, \theta_p] \subset [0, 1] \) it holds

\[
\tau(b - z, \varepsilon) < b - z \leq b < r_1
\]

Therefore, the guarantee \( b - z \) less the required tax \( \tau(b - z) \) to finance the guarantee is feasible to agents that roll over, \( c_2 = b - z - \tau(b - z, \varepsilon) > 0 \). This holds in particular for the cheapest bailout, \( \hat{b} = b - z = 0 \), \( z^* = b \) with \( c_2 = 0 \). This zero bailout implies a zero tax \( \tau(0, \varepsilon) = 0 \). Moreover, the pro rata share to agents that roll over exceeds \( b \) for all \( n \in [\lambda, n_p(b)] \) implying feasibility of the smaller deposit guarantee also for lower withdrawals. Furthermore, citizens that are not creditors can pay the reduced tax \( \tau(\hat{b}) \leq \tau(b) \) because the initial deposit guarantee \( b \) is feasible by assumption, thus \( \bar{c} - \tau(\hat{b}) > \bar{c} - \tau(b) \geq 0 \). It remains to verify that a zero bailout is feasible to agents that

\[ \text{For } b - z \rightarrow 0, \quad \frac{R(\theta)(n_p, \theta^*_p)}{1 - n_p} = 0 \text{ has the solution } n = n_p(0) = n_c = 1/r_1. \]

\[ 50 \]

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withdraw. For that purpose, I solve for \( \kappa(z^*) = \kappa(b) \) and check whether \( c_1 = r_1 - \kappa(b) > 0 \).

**Stability-equivalence**

I need to construct the stability-equivalent bailin \((\hat{b}, \kappa(\hat{b}))\) with the according tax \( \tau(\hat{b}) \) for the entire state range \( \theta \in [0, 1] \), taking into account that for a trigger equilibrium state realizations deterministically determine the aggregate withdrawals in the range \( n \in [0, 1] \).

Because the creditor payoffs differ depending on the state realization, I pin down the stability-equivalent bailin \( \kappa \) for the following ranges of states

\[
[0, \theta^* - \varepsilon] \cup [\theta^* - \varepsilon, \theta_p(0)] \cup [\theta_p(0), \theta_p(\hat{b})] \cup [\theta_p(\hat{b}), \theta_p(b)] \cup [\theta_p(b), \theta^*] \cup [\theta^* + \varepsilon, \overline{\theta}] \cup [\overline{\theta}, 1].
\]

**Case A** Consider state realizations in \([\theta_p(0), \theta_p(\hat{b})]\). Via equation (42) and (36) such state realizations correspond to withdrawals in the range \([n_p(\hat{b}), n_c]\). Fix any withdrawal \( n \in [n_p(\hat{b}), n_c] \). In this withdrawal range the intervention is active following the original and also the less costly bailout. Consider the payoff difference following the original bailout guarantee \( b \) with zero bailin, \((b, 0)\)

\[
v_{BO}(b, 0) \equiv u(b - \tau(b, \varepsilon)) - u(r_1 - \tau(b, \varepsilon)),
\]

which is constant in the withdrawals \( n \) for all \( n \in [n_p(\hat{b}), n_c] \). The payoff difference when granting a smaller bailout \( \hat{b} = b - \varepsilon \leq b \) instead and a simultaneous bailin \( \kappa(z) \) equals

\[
v_{BI}(\hat{b}, \kappa(z)) = u(\hat{b} - \tau(\hat{b}, \varepsilon)) - u(r_1 - \tau(\hat{b}, \varepsilon) - \kappa(z))
\]

Now applying that a zero bailout is feasible, I plug in \( z^* = b \) and \( \kappa(z^*) = \kappa(b) \). The payoff difference following the less costly policy with a zero bailin becomes

\[
v(0, \kappa(b)) = u(0) - u(r_1 - \kappa(b))
\]

We next solve for the required bailin to keep the payoff difference constant at the level of the original bailout, \( v_{BO}(b, 0) \). Stability-equivalence requires that \( \kappa \) is such that \( v_{BI}(\hat{b}, \kappa(z)) = v_{BO}(b, 0) \) for all \( n \in [n_p(\hat{b}), n_c] \).

**Case A1** Assume that for withdrawals in the range \( n \in [n_p(\hat{b}), n_c] \), it holds \( v_{BO}(b, 0, n) < 0 \). Therefore, solving (148) for \( \kappa(b) \) under the imposition that \( v_{BI}(0, \kappa(b)) = v_{BO}(b, 0) \), I obtain

\[
u(r_1 - \kappa(b)) = u(0) - v_{BO}(b, 0) > 0.
\]
where \( v_{BO}(b, 0) \) is given in (146). Now the steps are the same as in the main proof of Proposition 4.1, but are given here for expositional purpose. Also the inverse must, thus, be positive, \( r_1 - \kappa(b) = u^{-1}(u(0) - v_{BO}(b, 0)) > 0 \), implying \( r_1 > \kappa(b) \) and \( r_1 > u^{-1}(u(0) - v_{BO}(b, 0)) \). Last, solving for the bailin,

\[
\kappa(b) = r_1 - u^{-1}(u(0) - v_{BO}(b, 0))
\]

(150)

where I have plugged in for \( v_{BO}(b, 0) \) using (146).

CASE A2 If for \( n \in [n_p(\hat{b}), n_c] \), it holds \( v_{BO}(b, 0) < 0 \) then the same calculations and formulae hold, and one can likewise show that the bailin is feasible, see the proof in the benchmark model. Thus (150) likewise obtains and is feasible.

CASE B Consider state realizations in \([0, \theta_p(0)]\). Via equation (42) such state realizations correspond to withdrawals in the range \( n \in [n_c, 1] \). Fix any withdrawal \( n \in [n_c, 1] \): For any \( n \) in that range, the original deposit guarantee \( b \) yields the withdrawal-contingent payoff difference

\[
v_{BO}(b, 0, n) \equiv u(b - \tau(b, \varepsilon)) - \frac{1}{nr_1}u(r_1 - \tau(b, \varepsilon)),
\]

(151)

To find the least-costly feasible bailout \( \hat{b} = b - z \) jointly with the required, stability-equivalent bailin \( \kappa(\hat{b}) \) of agents that withdraw, the payoff difference under the bailin must attain that same value \( v_{BO}(b, 0) \): \( v_{BO}(b, 0, n) = v_{BI}(z, \kappa(z)) \), where

\[
v_{BI}(z, \kappa(z)) = u(b - z - \tau(b - z, \varepsilon)) - \frac{1}{nr_1}u(r_1 - \tau(b - z, \varepsilon) - \kappa(z))
\]

(152)

As opposed to the range \( n \in [n_p, n_c) \), the stability equivalent bailin \( \kappa \) will now depend on the aggregate withdrawals. See that in the range \( n \in [n_c, 1] \) it holds \( v_{BO}(b, 0) < 0 \) by our assumption \( b < u^{-1}(\frac{1}{r_1}u(r_1 - \tau)) + \tau \) made in section 9.5. From this one can infer (see main proof) that a zero bailout \( \hat{b} = 0 \), \( z^* = b \), is feasible in this withdrawal range, implying a zero tax \( \tau(\hat{b}) = 0 \). Plugging in \( z^* = b \) and \( \kappa(z^*) = \kappa(b) \) we obtain

\[
v_{BI}(z, \kappa(z)) = u(0) - \frac{1}{nr_1}u(r_1 - \kappa(b, n))
\]

(153)
To calculate the bailin at which the payoff difference $v_{BI}(z, \kappa(z))$ equals that of the original bailout $v_{BO}(b,0)$, I solve (153) for $\kappa(b)$ under the imposition that $v_{BI}(z, \kappa(z)) = v_{BO}(b,0,n) < 0$, and obtain

$$\kappa(z^*, n) = \kappa(b, n) = r_1 - u^{-1} (nr_1 (u(0) - v_{BO}(b,0,n)))$$

(154)

$$= r_1 - u^{-1} \left( u(0) - \left( u(b - \tau(b,\varepsilon)) - \frac{1}{nr_1}u(r_1 - \tau(b,\varepsilon)) \right) \right) > 0, \text{ for all } n \in [n_c, 1]$$

(155)

where I have plugged in for $v_{BO}(b,0,n)$ using (151).

**CASE C**  Consider state realizations in $[\theta_p(b), \theta]$. Via equation (42) and (36), and since $\theta > \theta^* + \varepsilon$ such state realizations correspond to withdrawals in the range $n \in [\lambda, n_p(b)]$. Fix any withdrawal level $n \in [\lambda, n_p(b)]$, and consider the payoff difference following the original bailout

$$v_{BO}(b,0,n) = u \left( \frac{R(\theta(n,\theta^*))(1 - nr_1)}{1 - n} - \tau(b,\varepsilon) \right) - u(r_1 - \tau(b,\varepsilon))$$

(156)

For the alternate policy $(\hat{b}, \kappa(z))$ the payoff difference equals

$$v_{BI}(b - z, \kappa(z)) = u \left( \frac{R(\theta(n,\theta^*))(1 - nr_1)}{1 - n} - \tau(b - z) \right) - u(r_1 - \tau(b - z) - \kappa(b, n))$$

(157)

See that when setting the least costly bailout, $\hat{b} = 0$, and $z^* = b$, the tax becomes zero. Thus, if we did not impose a bailin, $\kappa = 0$, it holds $v_{BI} \neq v_{BO}$, so that stability-equivalence would fail.\(^{13}\) Therefore, to keep stability constant, and in particular to not lower stability, investors that withdraw cannot be granted a tax reduction of zero. Instead, they need to pay some small tax in the form of the bailin even though no government intervention is triggered in the range $n \in [\lambda, n_p(b)]$.

**CASE C1** Assume that for $n \in [\lambda, n_p(b))$ roll-over is optimal following the original bailout, that is, $v_{BO}(b,0,n) > 0$. Note, we know for sure that this holds for small enough

\(^{13}\)This is because the function $G(z) = u(A - z) - u(B - z)$ is increasing in $z$ for $\frac{R(\theta(1-nr_1))}{1-n} = A > B = r_1$. Thus,

$$v_{BO} = u \left( \frac{R(\theta(n,\theta^*))(1 - nr_1)}{1 - n} - \tau(b) \right) - u(r_1 - \tau(b)) > u \left( \frac{R(\theta(n,\theta^*))(1 - nr_1)}{1 - n} \right) - u(r_1)$$

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values of $n \in [\lambda, n^*] \subset [\lambda, n_p(b)]$. However, there exist values $n \in (n^*, n_p(b))$ for which $v_{BO}(b, 0, n) \leq 0$, see the appendix section 9.5.

To construct the stability-equivalent bailin $\kappa(n)$, we need to solve (157) for $\kappa$ under the imposition $v_{BI}(b - z, \kappa(z)) = v_{BO}(b, 0, n)$: We obtain

$$u(r_1 - \kappa(n)) = u \left( \frac{R(\theta(n, \theta^*))(1 - nr_1)}{1 - n} \right) - v_{BO}(b, 0, n) > 0$$

(158)

Therefore, also the inverse of utility must be positive and I obtain feasibility of the bailin

$$\kappa(n) = r_1 - u^{-1} \left( u \left( \frac{R(\theta(n, \theta^*))(1 - nr_1)}{1 - n} \right) - v_{BO}(b, 0, n) \right)$$

$$= r_1 - u^{-1} \left( u \left( \frac{R(\theta(n, \theta^*))(1 - nr_1)}{1 - n} \right) - \left( u \left( \frac{R(\theta(n, \theta^*))(1 - nr_1)}{1 - n} - \tau(b) \right) - u(r_1 - \tau(b)) \right) \right) > 0$$

where I have plugged in for $v_{BO}(b, 0, n)$ via (156).

CASE C2 If $v_{BO}(b, 0, n) < 0$, the bailin is likewise feasible via (158), and since $-v_{BO}(b, 0, n) > 0$.

CASE D Consider state realizations in $[\theta_p(\hat{b}), \theta_p(b)]$. Via equation (42) and (36) such state realizations correspond to withdrawals in the range $[n_p(b), n_p(\hat{b})]$. Fix any withdrawal level in $[n_p(b), n_p(\hat{b})]$. Note, this withdrawal range is non-empty whenever $\hat{b} < b$, because the critical state $\theta_p(b)$ varies in the deposit guarantee. The payoff difference following the original bailout equals

$$v_{BO}(b, 0) = u(b - \tau(b, \varepsilon)) - u(r_1 - \tau(b, \varepsilon)), \quad (159)$$

whereas the payoff difference following the less costly alternative equals

$$v_{BI}(b - z, \kappa(z)) = u \left( \frac{R(\theta(n, \theta^*))(1 - nr_1)}{1 - n} - \tau(b - z) \right) - u(r_1 - \tau(b - z) - \kappa(b, n)) \quad (160)$$

Solving (160) for $\kappa$ while imposing $v_{BI}(b - z, \kappa(z)) = v_{BO}(b, 0, n)$, and assume $v_{BO}(b, 0) > 0$. Then, by the same argument as above,

$$u(r_1 - \tau(b - z) - \kappa(b, n)) = u \left( \frac{R(\theta(n, \theta^*))(1 - nr_1)}{1 - n} - \tau(b - z) \right) - v_{BO}(b, 0) > 0 \quad (161)$$
and the zero bailout, jointly with a zero tax and bailin \( \kappa \) is feasible and given by

\[
\kappa(b,n) = r_1 - u^{-1} \left( u \left( \frac{R(\theta(n,\theta^*))}{1-n}(1-nr_1) \right) - v_{BO}(b,0) \right) \tag{162}
\]

\[
= r_1 - u^{-1} \left( u \left( \frac{R(\theta(n,\theta^*))}{1-n}(1-nr_1) \right) - \left( u(b-\tau(b,\varepsilon)) - u(r_1-\tau(b,\varepsilon)) \right) \right) > 0
\]

where I have plugged in for \( v_{BO}(b,0) \) via (159). For \( v_{BO}(b,0) \leq 0 \) the zero bailout with bailin given in (162) is likewise feasible.

**CASE E** Consider states in the upper dominance region \( \theta \in [\theta, 1] \). For such states all patient agents roll over, \( n = \lambda \). Further, the asset already pays in \( t = 1 \) and the payoff difference following the original bailout becomes

\[
v_{BO}(b,0,n) = u \left( \frac{R(\theta(n,\theta^*)) - nr_1}{1-n} - \tau(b,\varepsilon) \right) - u(r_1-\tau(b,\varepsilon)) \tag{163}
\]

because liquidation is not necessary to finance withdrawals. For such low withdrawals, also the less costly alternative bailout is not provided and the payoff difference (plugging in a zero bailout) to the less costly policy equals

\[
v_{BI}(b-z,\kappa(z)) = u \left( \frac{R(\theta(n,\theta^*)) - nr_1}{1-n} \right) - u(r_1-\kappa(b,n)) \tag{164}
\]

Following the same steps as in case C, the zero bailin is feasible and given by

\[
\kappa(n) = r_1 - u^{-1} \left( u \left( \frac{R(\theta(n,\theta^*))}{1-n}(1-nr_1) \right) - v_{BO}(b,0,n) \right)
\]

\[
= r_1 - u^{-1} \left( u \left( \frac{R(\theta(n,\theta^*))}{1-n}(1-nr_1) \right) - \left( u \left( \frac{R(\theta(n,\theta^*))}{1-n}(1-nr_1) - \tau(b) \right) - u(r_1-\tau(b)) \right) \right) > 0
\]

where I have plugged in for \( v_{BO}(b,0,n) \) via (163).

This completes the derivation of the stability-equivalent bailin.