Design of a class of reconfigurable hybrid mechanisms for large complex curved surface machining
based on topological graph theory

Litao He¹², Hairong Fang ¹, Dan Zhang²
1. Department of Mechanical Engineering, Beijing Jiaotong University, Beijing 100044, PR China
2. Department of Mechanical Engineering, Lassonde School of Engineering, York University, Toronto M3J 1P3, Canada

Abstract
A design method for hybrid mechanism configuration synthesis based on topological graph theory is proposed to address the challenge of machining large and complex curved surfaces in the aerospace domain. The method leads to the generation of a new class of reconfigurable large extension hybrid mechanisms. Firstly, the spatial mechanism topological graph (SMTG) is obtained by evolving the structure of chemical molecules, which is used to express the spatial mechanism topological relationships. Then, combining graph theory methods with the SMTG, the topological relationships between motion modules of the hybrid mechanism are expressed, and the mechanisms of each motion module are designed. Finally, mechanisms of different functional modules are topologically connected and combined using the proposed topological connection relationship for the motion modules of hybrid mechanisms, thus resulting in a class of reconfigurable hybrid mechanisms (RHMs) with large extensibility. The RHMs exhibit strong structural stability, a large working space, and a small volume of occupation. It can be installed on the ends of industrial robots or in large guide ways to achieve high-precision machining of large complex curved structures.

Keywords: design method; configuration synthesis; topological graph; reconfigurable hybrid mechanism; large extension

1. Introduction
With the development of the aerospace field, the structure of aerospace vehicle is more and more complex and large, and the requirement of manufacturing cycle is required to be shorter. The equal-thickness milling for large complex curved structural parts with surface foam [1,2] (shown in Fig.1) is facing more severe challenges. Traditional manufacturing equipment is single-purpose, with limited working space and lack of flexibility, and some of the work still be carried out manually. However, these methods have been unable to meet the efficiency, accuracy and reliability requirements of equal-thickness milling of large complex curved structural parts, which has restricted the further development of the aerospace industry. As a result, there is a need to explore more flexible automated machining solutions to promote the development of aerospace manufacturing and improve productivity.

![Fig.1. Large complex curved structural parts with foam](image1)

Currently, in aerospace manufacturing, the machining of complex curved structural parts usually use equipment such as three-coordinate CNC machine tools [3,4] and five-axis CNC machine tools [5,6], as shown in Fig.2. Although multi-axis CNC machine tools are the main method of manufacturing complex curved parts and widely used [7,8], they are expensive, have fixed machining methods, lack flexibility and parallel processing capabilities, and are difficult to integrate into machining and measurement systems [9]. In contrast, industrial robots and supporting devices [10-14] have a broad application prospect due to their good flexibility, low cost, and large working space [15]. It is gradually becoming another important way to machine complex curved parts. However, their low machining accuracy and insufficient stability limit their ability to meet the demands of high-precision machining for large components. To address this issue, parallel mechanisms (PMs) have been proposed, which offer higher stiffness and accuracy while maintaining flexibility and precision.

* Corresponding author.
E-mail addresses: hrfang@bjtu.edu.cn (H. Fang), dzhang99@yorku.ca (D. Zhang).

This preprint research paper has not been peer reviewed. Electronic copy available at: https://ssrn.com/abstract=4411884
Mechanisms such as Exechon [16-18], Z3 head [19], and Tricept [20] are successfully used in production machining to prove the feasibility of PMs. However, the PMs currently used in aerospace manufacturing are mainly used as machine spindle heads with small working space and low machining efficiency. They are not applicable to large aerospace components with large variety and small lot sizes [21].

Due to the shortage of existing machining equipment in terms of extension range, efficiency, precision and stability, it cannot be applied to the processing and manufacturing of large complex curved structural parts alone. In order to improve production efficiency and realize automated intelligent processing, it is necessary to adopt flexible manufacturing methods. The flexible manufacturing method can adopt the strategy of "ants gnawing on bones" [22] to process large complex curved workpieces in different areas. By designing multifunctional robotic cells, taking advantage of the series of industrial robots and PMs, high-precision flexible machining can be achieved. Multifunctional robotic cells require PMs with high dexterity and stability, as well as large working spaces and small size occupations. The conventional PM cannot meet these requirements, so it is necessary to redesign the reconfigurable hybrid mechanisms as the machining device at the end of the robot. The kinematic performance of RHM is changed by different operation modes to adapt to different task requirements [23]. Current research on design methods focus on the modular and reconfigurable design of Exechon-like PMs [24], the structural design of reconfigurable PMs based on position qualification [25], the design of reconfigurable parallel robot systems built with various combinations of modules [26], and the reconfiguration of task-based PM topology and geometry combinations [27]. The design of reconfigurable PMs that meet the requirements using efficient methods is a long-term challenge.

To meet the reconfigurable characteristics of large complex surface machining, it is necessary to design RHMs with comprehensive performance such as large working space, large extensibility, and high stability. The construction of topological relationships in mechanism innovation is an important basis for generating new mechanisms [28-30], but there is a lack of effective methods to synthesize RHM configurations. Simple serial combinations cannot ensure the stability and reliability of the spatial structure or exploit the performance of each mechanism. The existing method of graph theory can carry out the topological design of mechanisms [31], but only represents the topological relations of planar linkage mechanisms. There are fewer expressions of topological relations for spatially PMs. Therefore, how to establish the topological and conformational representation of spatial PMs, and then simplifying the design, is still a difficult task for the configuration synthesis of PMs.

In conclusion, there is a lack of effective methods for the configuration synthesis of RHMs, thus it is important to carry out relevant research. In this paper, from the perspective of stability and reliability of the spatial structure, inspired by the topological relationship of the spatial structure of chemical molecules. A design method for RHM configuration synthesis based on topological graph theory is proposed to realize a class of RHM with large extension characteristics.

The structure of this paper is organized as follows: Section 2 analyzes the correlation between chemical molecular structure and configuration design, and evolves to obtain a spatial structure topology graph; Section 3 proposes a design method for configuration synthesis of hybrid mechanism based on topology graph; Section 4 performs configuration synthesis of each mechanism module of RHM; Section 5 carries out mechanism topology connection and combination to obtain a class of reconfigurable large extension hybrid mechanism; Section 6 briefly introduces the application of this mechanism. Conclusions are drawn in Section 7.

2. Correlation between molecular structure topology relations and mechanism design

2.1 Comparison of molecular space structure with PM

The structure of a substance determines its properties [32]. Whether macroscopic or microscopic, structural stability is essential for its long-term existence. The topological laws of chemical molecules determine their spatial configurations [33], and the most representative one is the elements carbon [34]. Elemental carbon has three different isomers: diamond, graphite and C\textsubscript{60} [35-40]. Their spatial structures have different shapes and properties depending on the way the basic units are topologically connected, as shown in Table 1.

<table>
<thead>
<tr>
<th>Chemical substances</th>
<th>Molecular spatial structure</th>
<th>Basic unit</th>
<th>Topological connection form</th>
<th>Spatial structure</th>
<th>Topological structure type</th>
<th>Examples of similar mechanism</th>
</tr>
</thead>
</table>

Table 1 Molecular basic unit topological connection form
All three isomeric spatial structures of carbon elements are composed of several basic units arranged according to certain laws. They are obtained by different forms of topological connections and have stable properties. Each basic unit has its own specific structural characteristics, which also influence the overall structure and properties. By analyzing these structures, the topological connection forms between the basic units of the three molecular structures can be obtained. Namely, chain-to-chain connection forms, layer-to-layer connection forms, and ring-to-ring connection forms. In the study of mechanism, there are also similar structures to these. For example, the mechanisms designed by Huang [41], Xia [42], and Li [43], etc., as shown in Table 1, also have the corresponding structural features.

In summary, the stability of the spatial structure is closely related to the structure of the basic units and the way they are topologically connected to each other. The overall structure can be made to have composite characteristics when the basic units act together through an effective topological connection method. Hence, when the mechanism needs to have a composite function and at the same time the general PM cannot meet the requirements, the configuration can be designed in modules for the required mechanism by drawing on the formation characteristics of molecular spatial structure. By using effective topological connection method to combine between each basic unit, a mechanism configuration with composite function and stable and reliable structure is generated.

2.2 Evolution and expression of topological relationships in molecular spatial structure

There are various forms of spatial PMs. The PM for machining large and complex curved workpieces requires large extension, high stiffness, high dexterity and structural stability. Therefore, it needs to be considered at the level of configuration design. Through the analysis of three different spatial structures of carbon elements, it is found that the structure of the basic unit and the connection method have an important influence on the structural stability. Diamond in nature has a very stable property, and its spatial structure consists of multiple orthotetrahedral molecular units combined by special topological hybrid connection, as shown in Fig.3. Hence, this paper makes reference to the formation and characteristics of the stable spatial structure of diamond. Starting from the basic orthotetrahedral units of diamond structure, the design of mechanism configuration is studied. The aim is to obtain a hybrid mechanism with better performance and to ensure the stability and reliability of the designed mechanism structure.
Chemical molecules are expressed in various ways, among which molecular structural formulae use elemental symbols and short lines to indicate the arrangement and binding of atoms in a molecule [33]. This approach converts the spatial structure relationships into planar connection relationships, thus simplifying the expression of topological relationships of molecular structures. For example, the expression of CH₄ as shown in Fig.4. Hence, the correspondence between the spatial structure of a chemical molecule and the structural formula suggests that the spatial structure can be described using planar topological relations. The association between elements can be expressed in terms of planar topological relations.

The spatial molecular structure can be expressed as the corresponding planar topological connections. However, in mechanism studies, the expression of topological graph form is nearly blank for spatially PMs. And graph theory is often used in mechanism studies to express the topological connection relations of mechanisms. Therefore, in this paper, combining with the graph theory approach, we take methane (CH₄), a representative chemical molecule with orthotetrahedral structure, as an example for the evolution of spatial structure topological relations as well as topological graph expression. Methane (CH₄) molecule has a symmetric orthotetrahedral spatial structure [44], and its structure is shown in Fig.5. The figure preserves the connection relationships between the individual elemental points of the molecular structure and also expresses the spatial characteristics of the external circle of the structure. By simplifying the information about the main geometric features such as the position of the edges and vertices of the structure, it can be evolved into a planar topological graph. The topological evolution of the spatial molecular structure is shown in Fig.6, and the exact process has been discussed and proved in another paper.
After the topological transformation of the spatial molecular structure, the results show that the stable spatial orthotetrahedral structure can be evolved into a more simplified planar topological connection relationship - the topological graph, as shown in Fig.7. That is, the two are interconvertible and there is a certain correspondence between them.

Based on the spatial structure topological graph of the molecular basic unit obtained above, the topological relationship of the overall molecular spatial structure and the topological connection way between each basic unit module are further expressed. Then, a complete topological graph of the molecular spatial structure can be obtained. Hence, on the basis of the way of chain-to-chain connection between the basic units of diamond and their topological relationships, the topological graph of its spatial structure can be obtained, as shown in Fig.8.

Diamond molecular structure is obtained by combining several basic unit modules in a certain spatial topological relationship. Based on the topological relationship between the basic units in the diamond spatial structure, the basic unit topological graphs are evolved and combined. Then the corresponding spatial hybrid structure topological graph can be obtained, as shown in Fig.9, which further verifies the feasibility of the spatial topological graph and provides a new idea for the design of the hybrid mechanism. Therefore, for the mechanism with composite functional requirements in space, the mechanism can be divided into multiple modules. Using the spatial structure topological graph, the basic mechanism unit can be designed first. Then the basic mechanism units can be combined according to the topological connection relationship to get the...
desired hybrid mechanism configuration.

![Diagram of hybrid mechanism configuration]

**Fig.9.** The process for expressing the topological relationship of diamond spatial structure

### 3. Design method for hybrid mechanism configuration synthesis based on topological graph

#### 3.1 Topological graph of spatial hybrid mechanisms proposed

It is considered that the hybrid mechanism is also a combination of different types of mechanisms, but there is a lack of appropriate design methods and processes to achieve the conformational design. Hence, based on the topological diagram of the diamond molecular structure obtained by the evolution of Section 2.2. By referring to the modular topological connections and topological relationships of the basic units of the diamond molecular structure, the configuration design of the hybrid mechanism is carried out to obtain some hybrid mechanisms with excellent performance.

![Topological graph of diamond structure and its basic unit]

**Fig.10.** Topological graph of diamond structure and its basic unit

As shown in Fig. 10, taking the basic unit topological graph of orthotetrahedron in the diamond molecular topological structure, the structure is compared with the structure of the spatial PM as shown in Table 2. Since there are more similarities, the topological relationship of the spatially PM is further expressed by combining
the topological graph of the molecular spatial structure.

<table>
<thead>
<tr>
<th>Type</th>
<th>Expression form</th>
<th>Structural features</th>
<th>Branch characteristics</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orthotetrahedral molecular structure</td>
<td>![Orthotetrahedral molecule]</td>
<td>1. Two vertices 2. Multiple identical branches</td>
<td>Contains multiple chemical elements and chemical bonds</td>
<td>The topological graph expresses the spatial structure and its topological connections.</td>
</tr>
<tr>
<td>Spatially symmetrical PM</td>
<td>![Spatially symmetrical PM]</td>
<td>1. Two platforms 2. Multiple identical branches</td>
<td>Contains multiple components and kinematic pairs</td>
<td>The mechanism diagram illustrates the relative motion relationship between the components of the mechanism.</td>
</tr>
</tbody>
</table>

In this study, the symmetrical PM is studied. Topological analysis and design are performed by describing its topological relations. With reference to the graphical representation of planar mechanisms, the points are used to represent the components and the edges are used to represent the kinematic subsets [45]. And combined with the topological representation of the spatial structure of methane molecules, the topological graph of the spatially symmetric PM is defined as follows:

Assuming that the spatially symmetric PM is Ms. For the mechanism Ms, the number of branch chains of Ms is \( i \), and the number of kinematic pairs of each branch is \( j \), then the number of vertices of the topological graph is \( N_v = i \cdot (j - 1) + 2 \), and the number of edges of the topological graph is \( N_e = i \cdot j \). Similarly, the components are also represented by circles (where the hollow circle represents the movable component, the solid circle represents the fixed component), and the kinematic pairs are represented by edges. At the same time, in order to reflect the characteristic that the used topological graph represents the spatial structure, the topological graph representation continues to use a circular representation. Then, the resulting graph is defined as the topological graph of the spatially symmetric PM. According to the definition, the topological graph expression of the spatially symmetrical PM can be further obtained as shown in Fig.11. In this case, the topological graph represents the uniform form of the spatially symmetric PM with stable structure. That is, the topological graph of the spatially symmetric PM containing \( i \) branch chains and each branch chain contains \( j \) kinematic pair \( K \).

**Fig.11.** The topological graph of spatially symmetrical PM

Further, the modular topology of the basic unit of the hybrid mechanism is utilized. By combining the symmetrical PM topological graphs in a "chain-to-chain" topological connection. Then, the expression of the hybrid topological relationship of the modular mechanism can be obtained. That is, the topological graph of spatial hybrid mechanism, as shown in Fig.12.
The analysis of the topological graph of the spatial hybrid mechanism showed that the fixed and movable components of the two modular mechanisms are overlapped. This means that the fixed components of the second level motion module are connected to the movable components of the first level motion module. That is, a "chain and chain" hybrid topological connection. The mechanism of the second level motion module will further move on the basis of the first level motion module, and the composite function of the hybrid mechanism will be realized by the cooperation of the mechanism of the two modules. Hence, the modular configuration of the hybrid mechanism will be designed based on the topological relationship in the above-mentioned topological graph of the spatial hybrid mechanism.

### 3.2 Design idea of hybrid mechanism

When the functional requirements of the hybrid mechanism are decomposed into multiple modular mechanisms, the design process of the mechanism can be further simplified. To obtain the desired hybrid mechanism easily and quickly, while making the mechanism meet multiple performance requirements, the design process of the hybrid mechanism is described here based on the hybrid mechanism topological graph (HMTG), as shown in Fig.13. By decomposing the complex functional requirements of the mechanism into multiple modules, the modular mechanism is first designed using the SMTG. Then the topological connection forms of the basic topological units in the spatial HMTG are combined to design the hybrid mechanism.
(1) To divide the functional modules according to the overall functional requirements of the mechanism.
(2) To determine the DOF of each modular mechanism according to the multiple functional modules obtained from the division.
(3) Using the topological graph of mechanism, to carry out the configuration synthesis of different functional modules.
(4) Screening the mechanism configurations of different functional modules according to the functional requirements.
(5) Using the screened configurations, and combining the modular topological connection form of the hybrid configuration, to design the mechanism.
(6) To combine and obtain the complete hybrid mechanism.
(7) Verifying the overall functionality and performance of the hybrid mechanism.

According to the modular design method of the hybrid mechanism proposed above, the subsequent configuration design of each functional module will be carried out separately to ensure the performance of each aspect of the mechanism.

3.3 Design method for hybrid mechanism configuration synthesis

According to the functional requirements and the design process, the hybrid mechanism is divided into several kinematic modules to carry out the mechanism configuration synthesis of each module separately. The spatial HMTG is formed by mixing several basic topological graph units. Each of these units can be designed from the SMTG, as shown in Fig.14, so it can be simplified to design each unit. Due to the advantages of PM, using PM for each module can meet the functional requirements. Meanwhile, the spatially symmetric mechanism topological graph based on the evolution of the chemical molecular structure of the orthotetrahedron has strong stability and reliability. Therefore, the spatially symmetric PM topological graph in Section 3.1 is used to synthesize the mechanism configuration of the basic unit of each functional module separately.

A mechanism is a kinematic chain with fixed components, so the type synthesis of a mechanism is essentially the type synthesis of its kinematic chain. Therefore, the type synthesis of the mechanism can be carried out for the kinematic chain to obtain all the mechanism types. Due to the topological graph of the spatially symmetric PM, all the series motion branches are the same. Hence, the configuration synthesis can be performed with one of the series motion branches. Then the mechanism is further arranged symmetrically and configured to meet the requirements. For the modular design method for the hybrid mechanism configuration synthesis proposed in this paper, the branch chains of the modular mechanism are divided into planar motion branches and spatial motion branches. The branch chain configuration synthesis is carried out separately, and the modular configuration synthesis design method is shown in Fig.15.

![Configuration synthesis of the hybrid mechanism](https://ssrn.com/abstract=4411884)

For the planar mechanism, its kinematic chains are generally composed of basic bar groups with relatively simple functional structures. Therefore, combined with the topological graph of the spatially symmetrical PM,
some of the kinematic pairs and kinematic components in each branch chain are replaced by the basic bar group for planar kinematic branch unit configuration synthesis, as shown in Fig.15.

For the spatial mechanism, especially the PM, the motion of the moving platform is the intersection of the motions that can be achieved by all branches. By using the characteristics of the symmetric PM topological graph and combining the displacement subgroups or displacement manifolds of branch motion chains in the Lie group theory [46-49], some motion subsets and motion members in each branch chain are replaced by displacement subgroups or displacement manifolds, as shown in Fig.15. After that, configuration synthesis of the spatial motion branch unit is performed.

Based on the expressions in topological diagrams, a general multi-DOF branch chain (DOF ≥ 6) consisting of only single-DOF kinematic pairs (R and P pairs) in a topological graph branch is required to use at least six kinematic pairs (edges) and seven components (circles). In this study, some basic three dimensional subgroups in space, such as \( \{S(N)\}, \{T\}, \{G(u)\}, \) and so on, are used as overall sub-chain modules to express three motion pairs and four components in the multi-DOF topological graph branch chain, as shown in Fig.16. The topological relationship expression is simplified to carry out the configuration synthesis of the spatial motion branch units.

The subsequent research will be based on the above modular configuration synthesis method, combined with the machining requirements of large complex curved components, using the symmetric topological structure form of the spatially symmetric PM topological graph to carry out the PM configuration synthesis of different functional modules.

This paper aims to design a mechanism with large extensibility, high stability and adaptability to meet the machining demand functions of large complex curved components. To this end, based on the modular design process of hybrid mechanism and the modular configuration synthesis method proposed in Section 3.1, a class of RHM with large extensibility is expected to be obtained. Hence, the mechanism is divided into two
motion modules: the basic extension module and the reconfigurable extension module. The basic extension module requires at least one degree of freedom (DOF) to achieve a wide range of extension in a single direction and to ensure the stiffness. The reconfigurable extension module requires six DOF to achieve attitude and position adjustment with extension. Through the cooperation and collaboration of the two module mechanisms, the design goal of large extensibility and reconfiguration can be finally achieved, as shown in Fig.17.

Based on the above-mentioned configuration synthesis method and design approach, the next step is the configuration synthesis of the different motion modules of the RHM.

4. Configuration synthesis of different motion modules of RHMs
4.1 Configuration synthesis of basic extension modules with single DOF
Since the general kinematic chains are composed of basic bar groups. Therefore, some of the kinematic pairs and kinematic components in the symmetrical PM topological graph branch chain are replaced by basic bar groups, as shown in Fig.18. The graph theory method is used to perform the configuration synthesis of single-DOF extensible branched chain units.

Due to the configuration synthesis of single-DOF extensible branched units has been carried out in the literature [50]. Three planar single-DOF extensible branched chains with large extension capabilities were finally screened out, as shown in Table 3.

Table 3 Topological graph of extendable branch chain configuration and its evolutionary configuration
Using the spatially symmetric PM topological graph, the branch chain can be configured into a PM, then the following three mechanisms of the basic extension motion module can be obtained, as shown in Fig.19.

Fig.19. Three types of extendable PMs

### 4.2 Configuration synthesis of reconfigurable extension modules with 6-DOF

The motion that can be achieved by a PM moving platform is the intersection of all branch motions. Utilizing the symmetric PM topological graph, a 6-DOF reconfigurable extension motion branch unit configuration is synthesized. Based on the expression in the topological graph, some basic three-dimensional subgroups in space, such as \(S(N)\), \(T\), \(G(u)\), and so on, are used as the overall sub-chain modules, as shown in Fig.20, to further simplify the design process.

Fig.20. Design of branch chain for spatially symmetric PM topological graph

As the branch end DOF of each branch chain are determined by the composition of its branch kinematic pairs and the geometric position relationship between the kinematic pairs. Hence, to get the reconfigurable
extensional branches, it is necessary to analyze the equivalent displacement manifold of the motion generated at the end of the kinematic branches first. Based on the DOF that can be generated by the kinematic pairs in space and the correlation that exists between the combination of multiple kinematic pairs, analyzing the independent DOF, then the equivalent displacement manifold of the motion at the end of the mechanism can be obtained. Some of them are listed in this paper, as shown in Table 4. Where \( i \), \( j \) and \( k \) represent the axial directions of the rotation pairs, and \( v \) and \( w \) represent the axial directions of the kinematic pairs, both perpendicular to \( u \), corresponding to the two unit vectors in the motion plane.

\[
\begin{align*}
\text{Number of kinematic pairs} & \quad \text{Equivalent motion displacement manifold at the end of the mechanism} & \quad \text{Axis position relationship of kinematic pair} \\
\text{R-R} & \quad \{ R(A, u) \} \{ R(A, u) \} = \{ R(A, u) \} & \text{Axis parallel} \\
\text{R-R} & \quad \{ R(A, u) \} \{ R(A, u) \} = \{ R(A, u) \} \{ T(i) \} & i \perp u \ , \ T(i) \text{ is the derived DOF of movement} \\
\text{R-R-R} & \quad \{ R(A, v) \} \{ R(A, w) \} \{ R(A, u) \} = \{ S(A) \} & u \cdot v \ , \ w \text{ nonlinear correlation} \\
\text{R-R-R} & \quad \{ R(A, u) \} \{ R(B, u) \} \{ R(C, u) \} = \{ G(u) \} & \text{Axis parallel} \\
\text{R-R-R} & \quad \{ R(A, u) \} \{ R(B, u) \} \{ R(C, u) \} \{ R(D, u) \} = \{ G(u) \} & \text{Axis parallel} \\
\text{R-R-R-R} & \quad \{ R(A, u) \} \{ R(B, u) \} \{ R(C, u) \} = \{ G(u) \} & \text{Axis parallel} \\
\text{R-R-R-R-R-R} & \quad \{ R(N, i) \} \{ R(N, j) \} \{ R(N, k) \} = \{ S(N) \} & \text{Axis intersection, } i \cdot j \cdot k \text{ is the direction of the axis of the rotational pair} \\
\text{R-R-R-R-R-R} & \quad \{ S(N) \} \{ S(N) \} = \{ S(N) \} & \text{Axis intersection} \\
\text{R-R-R-R-R-R} & \quad \{ S(N) \} \{ S(N) \} \{ T \} = \{ S(N) \} \{ T \} & \text{Each group of axes intersects separately} \\
\text{R-R-R-R} & \quad \{ S(N) \} \{ R(B, u) \} \{ T(v) \} = \{ S(N) \} \{ T(v) \} & v \perp u \\
\text{R-R-R-R} & \quad \{ S(N) \} \{ R(B, u) \} \{ R(C, u) \} = \{ S(N) \} \{ T(P_u) \} & w \times w = u \\
\text{R-R-R-R-P} & \quad \{ S(N) \} \{ R(B, u) \} \{ T(u) \} = \{ S(N) \} \{ T(P_u) \} & v \perp u \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>Number of kinematic pairs</th>
<th>Equivalent motion displacement manifold at the end of the mechanism</th>
<th>Axis position relationship of kinematic pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-R</td>
<td>{ R(A, u) } { R(A, u) } = { R(A, u) }</td>
<td>Axis parallel</td>
</tr>
<tr>
<td>R-R</td>
<td>{ R(A, u) } { R(A, u) } = { R(A, u) } { T(i) }</td>
<td>( i \perp u ) \ , \ T(i) \text{ is the derived DOF of movement}</td>
</tr>
<tr>
<td>R-R-R</td>
<td>{ R(A, v) } { R(A, w) } { R(A, u) } = { S(A) }</td>
<td>( u \cdot v ) \ , \ w \text{ nonlinear correlation}</td>
</tr>
<tr>
<td>R-R-R</td>
<td>{ R(A, u) } { R(B, u) } { R(C, u) } = { G(u) }</td>
<td>Axis parallel</td>
</tr>
<tr>
<td>R-R-R</td>
<td>{ R(A, u) } { R(B, u) } { R(C, u) } { R(D, u) } = { G(u) }</td>
<td>Axis parallel</td>
</tr>
<tr>
<td>R-R-R-R</td>
<td>{ R(A, u) } { R(B, u) } { R(C, u) } = { G(u) }</td>
<td>Axis parallel</td>
</tr>
<tr>
<td>R-R-R-R-R-R-R-R</td>
<td>{ R(N, i) } { R(N, j) } { R(N, k) } = { S(N) }</td>
<td>\text{Axis intersection, } i \cdot j \cdot k \text{ is the direction of the axis of the rotational pair}</td>
</tr>
<tr>
<td>R-R-R-R-R-R-R-R</td>
<td>{ S(N) } { S(N) } = { S(N) }</td>
<td>\text{Axis intersection}</td>
</tr>
<tr>
<td>R-R-R-R-R-R-R-R</td>
<td>{ S(N) } { S(N) } { T } = { S(N) } { T }</td>
<td>\text{Each group of axes intersects separately}</td>
</tr>
<tr>
<td>R-R-R-R-R</td>
<td>{ S(N) } { R(B, u) } { T(v) } = { S(N) } { T(v) }</td>
<td>( v \perp u )</td>
</tr>
<tr>
<td>R-R-R-R-R</td>
<td>{ S(N) } { R(B, u) } { R(C, u) } = { S(N) } { T(P_u) }</td>
<td>( w \times w = u )</td>
</tr>
<tr>
<td>R-R-R-R-P</td>
<td>{ S(N) } { R(B, u) } { T(u) } = { S(N) } { T(P_u) }</td>
<td>( v \perp u )</td>
</tr>
</tbody>
</table>

The general 6-DOF branched chain configuration synthesis aims to achieve three rotational DOF and three moving DOF in space, as shown in Fig.21, that is, a 6-DOF kinematic branched chain with 3R3T. Hence, the displacement manifold of the basic 6-DOF branched chain can be obtained as follows:

\[
\{ L \} = \{ T \} \{ S(N) \} = \{ S(N) \} \{ T \}
\]

(1)

![Fig.21. 3R3T motion in six DOF](https://ssrn.com/abstract=4411884)

Based on the basic 6-DOF branched chain displacement manifold, the first level extension mechanism is used to achieve a wide range of extension and to exploit the advantages of reconfiguration. Here, we add a moving pair at the beginning of each second level 6-DOF extensible chain that allows the mechanism to be reconfigured and locked. The extension performance of the mechanism is further increased by using it as an additional driving pair of the 6-DOF branes to obtain a 6-DOF kinematic branched chain with reconfigurable extension. According to the closure and combination law of the multiplication operation in the group, on the basis of the branch displacement manifold with the 6-DOF branched chain, a reconfigurable locked moving pair is added, which does not affect the DOF at the end of the whole branched chain. Hence, the displacement manifold of a 6-DOF kinematic branched chain with reconfigurable extension can be obtained as follows:

\[
\{ L \} = \{ T \} \{ S(N) \} = \{ T(u) \} \{ T \} \{ S(N) \}
\]

(2)

By further equating a multi-DOF kinematic pair or hinge pair to a combination of a rotational or moving
pair, the multiple branch displacement manifold of a 6-DOF branch chain with reconfigurable extension is obtained as follows:

\[ \{L\} = \{T(u)\} \{T\} \{S(N_i)\} = \{T(u)\} \{S(N_i)\} \{T\} \{S(N_i)\} = \{T(u)\} \{S(N_i)\} \{T(v)\} \{T(w)\} \{T(u)\} \{S(N_i)\} \]

where \(v\), \(w\), and \(u\) are three arbitrary linear independent unit vectors and \(N_i\) is the center of rotation of the spherical pair.

Taking the equivalent motion displacement manifold \(\{S(N)\} \{R(B,a)\} \{R(C,u)\} = \{S(N)\} \{T(P_a)\}\), \(\{S(N)\} \{R(B,u)\} = \{S(N)\} \{T(v)\}\) in Table 4, and bringing them into equation (3) in turn, we have

\[ \{L\} = \{T(u)\} \{S(N_i)\} \{T(v)\} \{T(w)\} \{T(u)\} \{S(N_i)\} = \{T(u)\} \{S(N_i)\} \{T(P_a)\} \{T(u)\} \{S(N_i)\} = \{T(u)\} \{S(N_i)\} \{R(B,u)\} \{R(C,u)\} \{T(u)\} \{S(N_i)\} = \{T(u)\} \{S(N_i)\} \{T(u)\} \{S(N_i)\} = \{T(u)\} \{S(N_i)\} \{R(B,v)\} \{S(N_i)\} \]

On the basis of the branch displacement manifold obtained from the above analysis, the closure and combination law of the multiplicative operation of the group are used. Initially, the 6-DOF reconfigurable extensional kinematic branch chains containing multi-DOF kinematic pairs can be obtained, as shown in Table 5.

| Table 5 Branching displacement manifolds composed of multi-DOF kinematic subgroups |
|---------------------------------|---------------------------------|
| Displacement manifold of the kinematic branch chain \(\{L\}\) | Corresponding kinematic chain |
| \(\{T(u)\} \{S(N_i)\} \{T(u)\} \{S(N_i)\}\) | \(^a\)P-S\(_{N_i}\), \(^a\)PS\(_{N_i}\) |
| \(\{T(u)\} \{T(u)\} \{S(N_i)\} \{S(N_i)\}\) | \(^a\)P-\(^a\)P\(_{S_{N_i}}\)S\(_{N_i}\) |
| \(\{T(u)\} \{S(N_i)\} \{R(B,v)\} \{S(N_i)\}\) | \(^a\)P-\(^a\)RS\(_{N_i}\) |
| \(\{T(u)\} \{R(B,v)\} \{S(N_i)\} \{S(N_i)\}\) | \(^a\)P-\(^a\)RS\(_{S_{N_i}}\) |

Furthermore, due to the existence of a partial rotation DOF in \(\{S(N_i)\} \{T(u)\} \{S(N_i)\}\), \(\{S(N_i)\}\) can be simplified as \(\{S_i(N_i)\} = \{R(N,i)\} \{R(N,j)\}\). Meanwhile, \(\{S(N_i)\}\) can be equivalent to a 3R spherical subchain composed of three rotational pairs whose axes intersect at one point, that is, \(\{S(N_i)\} = \{R(N,i)\} \{R(N,j)\} \{R(N,k)\}\). Where \(i, j, k\) denote the direction of the axes of the three rotational pairs, and \(N_i\) denotes the center of the sphere where the axes of the three rotational pairs intersect, and when taken into equation (4), we have

\[ \{L\} = \{T(u)\} \{T\} \{S(N_i)\} = \{T(u)\} \{S(N_i)\} \{T(u)\} \{S(N_i)\} = \{T(u)\} \{S_i(N_i)\} \{T(u)\} \{S(N_i)\} = \{T(u)\} \{R(N,i)\} \{R(N,j)\} \{T(u)\} \{R(N,i)\} \{R(N,j)\} \{R(N,k)\} \]

Through simplification and combination of kinematic pairs, the branch displacement manifolds and corresponding branch kinematic chains of this type of mechanism which contains U pair and S pair displacement subgroups in general can be obtained, as shown in Table 6.

| Table 6 Branch displacement manifold with U-pair and S pair displacement subgroups |
|---------------------------------|---------------------------------|
| Displacement manifold of the kinematic branch chain \(\{L\}\) | Corresponding kinematic chain |
| \(\{T(u)\} \{S_i(N_i)\} \{T(u)\} \{S(N_i)\}\) | \(^u\)P\(_{R_i\text{R}}\)\(_{N_i}\), \(^u\)PS\(_{S_{N_i}}\) |
| \(\{T(u)\} \{S(N_i)\} \{T(u)\} \{S_i(N_i)\}\) | \(^u\)P-\(^u\)P\(_{S_{N_i}}\)\(_{N_i}\) |
| \(\{T(u)\} \{T(u)\} \{S_i(N_i)\} \{S_i(N_i)\}\) | \(^u\)P-\(^u\)PS\(_{N_i}\)\(_{S_{N_i}}\) |
| \(\{T(u)\} \{T(u)\} \{S_i(N_i)\} \{S(N_i)\}\) | \(^u\)P-\(^u\)P\(_{R_i\text{R}}\)\(_{N_i}\) |
| \(\{T(u)\} \{U(N,i,j)\} \{T(u)\} \{S(N_i)\}\) | \(^u\)P-\(^u\)U\(_{N_i}\)\(_{S_{N_i}}\) |

This preprint research paper has not been peer reviewed. Electronic copy available at: https://ssrn.com/abstract=4411884
Based on the design requirements in terms of topology, functionality and engineering constraints, the branch chain configurations were screened. After sorting, the 6-DOF reconfigurable extension branches, equivalent branches and the corresponding structural diagrams were obtained as shown in Table 7.

Table 7 6-DOF reconfigurable extension branched configurations

<table>
<thead>
<tr>
<th>No.</th>
<th>6-DOF reconfigurable kinematic branch chain</th>
<th>Diagram of the kinematic chain</th>
<th>Equivalent kinematic chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \text{P-}[R^1 R^3] R \text{N}_i ) ( R \text{A}[R^1 R^3] R \text{N}_i )</td>
<td>![Diagram 1]</td>
<td>( \text{P-}[S \text{N}_i \text{R} \text{N}_i )</td>
</tr>
<tr>
<td>2</td>
<td>( \text{P-}[R^1 R^3] R \text{N}_i ) ( [R^1 R^3] R \text{N}_i )</td>
<td>![Diagram 2]</td>
<td>( \text{P-}[R \text{S} \text{N}_i \text{R} \text{N}_i )</td>
</tr>
<tr>
<td>3</td>
<td>( \text{P-}[R^1 R^3] R \text{N}_i ) ( [R^1 R^3] R \text{N}_i )</td>
<td>![Diagram 3]</td>
<td>( \text{P-}[S \text{N}_i \text{R} \text{N}_i )</td>
</tr>
</tbody>
</table>
The symmetrical PM with six DOF reconfigurability can be constructed by combining a variety of
different kinematic branes obtained by the above configurations. To get the better extension performance, P pair needs to be introduced in the branched chain kinematic pair configuration with no more than two. Meanwhile, to meet the requirements of engineering applications, achieve large extension and reduce space occupation, it is necessary to use P pair as the driving pair and adopt multi-DOF kinematic pair or hinge. Considering the machining accuracy, control and assembly problems of S pair in actual application, the combination of (RU) is used to replace S pair equivalently, then seven feasible configurations of 6-DOF reconfigurable extension kinematic branch chains are screened and obtained, as shown in Table 8.

Utilizing the spatially symmetric PM topological graph, the branch chains are arranged into 6-DOF PMs. Because of the limited length of this paper, only the three 6-DOF reconfigurable large extension branches \( ^{\text{a}} \text{P-S}_N, ^{\text{b}} \text{P}(^{\text{c}} \text{U}_{N_i}^{\text{j}} \text{R}_{N_i}) \), \( ^{\text{a}} \text{P-}(^{\text{b}} \text{R}_{N_i}^{\text{j}} \text{U}_{N_i}^{\text{b}}) \text{P}(^{\text{c}} \text{U}_{N_i}^{\text{j}} \text{R}_{N_i}) \) and \( ^{\text{a}} \text{P-}(^{\text{b}} \text{R}_{N_i}^{\text{j}} \text{U}_{N_i}^{\text{b}}) \text{P}^{\text{c}} \text{U}_{N_i}^{\text{j}} \) are arranged into 6-DOF reconfigurable extension modules of the hybrid mechanism, as shown in Fig.22.

### Table 8 6-DOF reconfigurable extension kinematic branch chains

<table>
<thead>
<tr>
<th>No.</th>
<th>Type of reconfigurable branched chain</th>
<th>Equivalent branched chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(^{\text{a}} \text{P-S}_N ) (^{\text{b}} \text{PS}_N)</td>
<td>(^{\text{a}} \text{P-S}<em>N, ^{\text{b}} \text{P}(^{\text{c}} \text{U}</em>{N_i}^{\text{j}} \text{R}_{N_i}) )</td>
</tr>
<tr>
<td>2</td>
<td>(^{\text{a}} \text{P}^{\text{c}} \text{U}_{N_i}^{\text{j}} ) (^{\text{b}} \text{PS}_N)</td>
<td>(^{\text{a}} \text{P-}(^{\text{b}} \text{R}<em>{N_i}^{\text{j}} \text{U}</em>{N_i}^{\text{b}}) \text{P}(^{\text{c}} \text{U}<em>{N_i}^{\text{j}} \text{R}</em>{N_i}) )</td>
</tr>
<tr>
<td>3</td>
<td>(^{\text{a}} \text{P-S}<em>N ) (^{\text{b}} \text{P}^{\text{c}} \text{U}</em>{N_i}^{\text{j}})</td>
<td>(^{\text{a}} \text{P-}(^{\text{b}} \text{R}<em>{N_i}^{\text{j}} \text{U}</em>{N_i}^{\text{b}}) \text{P}^{\text{c}} \text{U}_{N_i}^{\text{j}} )</td>
</tr>
</tbody>
</table>

(a) 6-\_SPUR  (b) 6-\_P-(RU)PUR  (c) 6-\_P-(RU)PU

**Fig.22. 6-DOF reconfigurable large extension PM**

### 5. Topological connection of different modules for reconfigurable large extension hybrid mechanism

#### 5.1 Determination of the basic extension module

Three branched chain configurations with large extension capabilities were obtained based on the synthesis of single-DOF extensible kinematic branched unit configurations in Section 4.1. By analyzing the extensibility, stiffness and load carrying capacity of the extensible branched unit in the literature [50], the properties of the three configurations were obtained as shown in Table 9.

<table>
<thead>
<tr>
<th>Basic branch chain unit</th>
<th>Dimension parameters of branched chain</th>
<th>Extensibility ( \Theta )</th>
<th>Maximum stress</th>
<th>Percentage of ( [\sigma] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRRRR</td>
<td>( \Theta_1 = 1.215 )</td>
<td>22.84 MPa</td>
<td>16.1%</td>
<td></td>
</tr>
</tbody>
</table>
From the analysis of the above table, in the same conditions, there are differences in the performance of the three mechanisms of the base extension module obtained by configuration synthesis. Among them, the PM configured with PRRR-3R branch chain has the best extension performance, but its stiffness index is poor. In contrast, the extension performance of the PRRR branch chain is average, but its stiffness index is better than the other two extendable branch chain units, and its overall performance is better.

To provide a reliable base for extension, taking all aspects into consideration, the PM branch chain in the base extension module with PRRR branch chain unit can meet the requirements. Hence, it is configured as a 6-PRRRR extendable PM for subsequent structural topological connection.

5.2 Topological hybrid connection of different modules

Based on the proposed modular design method of the hybrid mechanism, different motion module mechanisms of the RHM are obtained through configuration synthesis. To make full use of the performance of each module mechanism, while reducing the space occupied by the mechanism. According to the topological connection relationship of the motion modules in the topological graph of the spatial hybrid mechanism in Section 3.1, the mechanisms of the two motion modules in Section 4.1 and Section 4.2 are topologically connected and combined.

On the basis of the topological connection relationship in the topological graph of the spatial hybrid mechanism, the topological hybrid connection of different modular mechanisms is carried out, as shown in Fig.23. The hollow circles in the topological graph represent the movable components and the solid circles represent the fixed components. Analyzing the topological graph of the spatially hybrid mechanism, it can be shown that the fixed components of the two modular mechanisms overlap with the movable components. It indicates that the fixed components of the second level motion module are connected to the movable components of the first level motion module, and the topological connection between the two modules is a "chain and chain" connection. That is, the branch chain of the first level motion module is connected to the branch chain of the second level motion module. The branch chain of the first level motion module can be used as a fixed platform for the second level motion module, providing a "fixed base" for the second level motion module. When the first level motion module does not generate motion, the whole is equivalent to the fixed platform of the second level motion module. Hence, a new type of hybrid mechanism can be obtained by this combination of topological connections.
Based on the topological connection relationship between different modules in the topological graph of the hybrid mechanism, variable scale and reconfiguration are realized by interconnecting the branch chains of the modular mechanism. In order to expand the mechanism motion space, realize a wide range of motion at the end of the mechanism and multiple working modes. Therefore, combining the 6-PRRRR parallel extensible mechanism in the basic extension motion module with the three 6-DOF reconfigurable large extension PMs in Section 4.2, a class of RHM with large extension characteristics can be obtained, and the configurations are shown in Fig.24, respectively.

![Figure 23: Topological connection and combinations of modular mechanisms](image)

![Figure 24: Reconfigurable large extension hybrid mechanism](image)

In this paper, a class of RHMs with large extension characteristics is obtained by using the modular hybrid mechanism design method, and the overall performance will be analyzed subsequently. For the other mechanisms not mentioned above, they can be selected from the first level basic extension module and the second level 6-DOF reconfigurable extension module, respectively, and combined according to the modular topological connection of the hybrid mechanism.

### 6. Application of reconfigurable large extension hybrid mechanism

The RHM with large extensibility is obtained based on the configuration synthesis described above. The overall mechanism achieves large extension in space mainly through multi-level extension superposition. Meanwhile, the movable platform at the end of the mechanism has six DOF, which is highly flexible and can realize different position and attitude adjustment. The symmetrical structure with evenly distributed six branch chains ensures that the mechanism has large stiffness and provides stable support when extending along a certain direction.

The reconfiguration of the RHM with large extensibility is realized by locking the different driving pairs P to switch the extension motion of the mechanism with different extension modes. Three times of extension are performed during the movement of the mechanism, and the movement of large extension in space is achieved by locking different driving pairs (P, P1, P2) in different stages, as shown in Fig.25.
For large complex curved structural parts, the huge size of the structure makes it possible to divide them into multiple machining areas. The flexible manufacturing method can be used for simultaneous machining by arranging multiple multifunctional robot cells in the form of "ants gnawing on bones". In this paper, a RHM configuration with large extension characteristics is designed to be used at the end of the industrial robot, as shown in Fig.25. With the advantage of serial industrial robots and RHM, the robot provides a wide range adjustment of spatial posture and position, and the RHM performs partial extension and posture adjustment.

By combining PMs with automated intelligent equipment, the advantages of each can make up for the shortcomings of single equipment in terms of extension range, efficiency, precision and stability, to improve the production efficiency and realize the machining of large and complex curved structures in space.

7. Conclusion

This paper presents a novel design method for the configuration synthesis of hybrid mechanisms based on spatial mechanism topological graph (SMTG) theory, resulting in a new class of reconfigurable hybrid mechanisms (RHMs) with large extensibility. The proposed method overcomes the limitations of previous design methods for planar mechanisms by utilizing the spatial structure evolution of chemical molecules to obtain the SMTG, which accurately expresses the topological connection relationship between the modules of the hybrid mechanism. By combining the topological relationship expression and configuration synthesis of each module with the graph theory method, two types of spatially extended mechanisms are obtained, leading to a class of RHM configurations with large extensibility. This innovation provides a solution to the challenging problem of machining large complex curved surfaces in the aerospace industry, as this type of mechanism can be installed on the end of industrial robots or in large guide ways to realize the machining of large complex curved structures. Overall, this study provides a novel method for the field of hybrid mechanism design and aerospace manufacturing. Conclusions are drawn as follows:

(1) A spatial mechanism topological graph (SMTG) is obtained through the evolution of molecular spatial structure. This enables a design of spatial mechanisms using the graph theory method, which overcomes the limitation that the graph theory method can only design planar mechanisms.

(2) A design method for hybrid mechanism configuration synthesis based on topological graph theory is proposed. This method simplifies the design process and improves the performance of general mechanisms by considering the topological connection relationship in the hybrid mechanism topological graph.

(3) A new class of reconfigurable hybrid mechanism with large extensibility is obtained based on the proposed design method for hybrid mechanism. This mechanism has strong structural stability, a large working space, a small volume of occupation, and is suitable for machining large complex curved surfaces.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Acknowledgments
The authors gratefully acknowledge the financial support of the Fundamental Research Funds for the Central Universities under grant no. 2018JBZ007 and the National Natural Science Foundation of China under grant no. 51975039. The third author would like to thank the financial support from the Natural Sciences and Engineering Research Council of Canada (RGPIN-2022-04624) and gratefully acknowledge the financial support from the York Research Chairs (YRC) program. In addition, the first author would like to acknowledge the China Scholarship Council (CSC) (No. 202207090069) for financial support and the use of the research facilities at Lassonde School of Engineering at York University.

References