Application of Peter Chew Rule

In Pool Game

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Abstract.
The purpose of Peter Chew's Rule for solution of triangle is to provide a simple method compare current methods to aid in mathematics teaching and learning, especially if similar COVID-19 problems arise in the future. Therefore, applying the Peter Chew's Rule for solution of triangle in Pool Game problem can make calculation of Pool Game problem simple and easier. The purpose of Peter Chew's Rule for solution of triangle is the same as Albert Einstein's famous quote Everything should be made as simple as possible, but not simpler.

Keywords: Pool Game, Peter Chew Rule, solution of triangle.
1. Introduction

1.1 The Hidden Mathematics Behind a Pool Shot

Exploring the Trigonometry, Vectors, Physics, and Probability Involved in a Game of Billiards.

Trigonometry

Trigonometry, the study of triangles and their properties, plays a crucial role in understanding the geometry of a pool shot. According to the American Mathematical Society, “trigonometry allows us to calculate the angles and distances involved in a pool shot.” For example, the angle at which the cue ball strikes the target ball and the angle of reflection can both be calculated using trigonometry.
1.3 USE MATH TO IMPROVE POOL AND POOL TO IMPROVE MATH

If your child is taking, or will soon take, geometry you could inspire their interest in angles and physics by inviting them to play pool. Cue sports, which includes pool and billiards, offer tons of opportunity to add math to the conversation.

Use math to predict where the ball will go if you hit from various angles and distances, so you can make the best shot. It will help you and your kids shoot more accurately, and win more often. The most popular game on the pool table is “8 ball” but there are other games, too. No matter which game you play, understanding the underlying rules of geometry, trigonometry, and physics, will help you sink a ball into a pocket.

**Straight Shots**

A “straight shot” is when a player hits the cue ball into the object ball directly into the pocket, without bouncing off the rail. In other words, the cue ball, object ball, and pocket are in a straight line. Straight shots are usually the easiest shot to make, but the degree of difficulty depends on the distance between the cue ball, object ball, and the pocket.

Errors like spin or “English” get amplified the greater the distance is between objects. Proper body position, including the angle of the elbow, and the distance of the hand from the end of the cue stick improves accuracy. Use math to teach your kids how to improve their straight shot!

Mathematician Rick Mabry used trigonometry to find the most difficult straight shot, which is when the distance from the pocket to the cue ball is 1.618 times the distance from the pocket to the object ball. You may recognize this famous number as Phi, or the golden ratio.
**Angle Shots**

Players make an “angle shot,” “cut shot”, or “slice shot” when a straight shot is not available. Most pool shots will be an angle shot. The degree of difficulty depends on the angles involved. The larger the angle the harder it will be to make the shot.

Intermediate and advanced pool players use “The Law of Reflection.” The law says a ball will bounce off the side of the table (the rail) at the same angle at which it hits the rail. For example, if a ball hits the rail at an angle of 40 degrees, it will bounce off the rail at an opposite angle of 40 degrees.

Using this law, you can predict where the ball will go based on the angle it approaches the rail. Good pool players are able to judge these angles by sight. Learn more about angle shots by watching [this video](https://www.pbs.org) from PBS.

If your children want to impress their friends with their pool skills, show them how to use math to improve their game. Next time you go plan to go a restaurant with a pool table, or if you have one in your basement, look at these resources first.
2. Current Method and Peter Chew Rule Method.

Example: Given \( \angle B = 35^\circ \), \( AB = 6 \) cm and \( BC = 3 \) cm. Find \( \angle C \).

Solution:

**Current Method 1:**

Cosine Rule plus Sine rule,

**Cosine Rule,**

\[
b^2 = a^2 + c^2 - 2ac \cos B
\]

\[
b^2 = 3^2 + 6^2 - 2(3)(6) \cos 35^\circ
\]

\[= 15.51\]

\[b = 3.938\]

**Sine Rule,**

\[
\frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}
\]

\[
\frac{3.938}{\sin 35^\circ} = \frac{6}{\sin \angle C}
\]

\[
\sin \angle C = \frac{6 \sin 35^\circ}{3.938}
\]

\[= 0.8739\]

\[\angle C = 119.0849^\circ, 60.9151^\circ \text{ (Reject)}\]
Current Method 2:

Cosine Rule plus Cosine rule,

\[ b^2 = a^2 + c^2 - 2ac \cos \angle B \]

\[ b^2 = 3^2 + 6^2 - 2(3)(6) \cos 35^\circ \]

\[ = 15.51 \]

\[ b = 3.938 \]

\[ c^2 = a^2 + b^2 - 2ab \cos \angle C \]

\[ 6^2 = 3^2 + 3.938^2 - 2(3)(3.938) \cos \angle C \]

\[ 23.628 \cos \angle C = -11.49 \]

\[ \cos \angle C = -0.4863 \]

\[ \angle C = 119.0977^\circ \]

Peter Chew Rules.

\[ \tan \angle C = \frac{c \sin \angle B}{a - c \cos \angle B} \]

\[ \tan \angle C = \frac{6 \sin 35^\circ}{3 - 6 \cos 35^\circ} \]

\[ = -1.797 \]

\[ \angle C = 119.952^\circ \]

Note: The actual answer is 119.0926395°
3. Application of Peter Chew Rule for Solution Of Triangle in Pool Game.

Example 1: In a game of pool, a player must put the eight ball into the bottom left pocket of the table. Currently, the eight ball is 6.8 feet away from the bottom left pocket. However, due to the position of the cue ball, she must bank the shot off of the right side bumper. If the eight ball is 2.1 feet away from the spot on the bumper she needs to hit and forms a $168^\circ$ angle with the pocket and the spot on the bumper, at what angle does the ball need to leave the bumper?

Note: This is actually a trick shot performed by spinning the eight ball, and the eight ball will not actually travel in straight-line trajectories. However, to simplify the problem, assume that it travels in straight lines.
Solution: In the scenario above, we have the SAS case, which means that we need to use the Law of Cosines to begin solving this problem. The Law of Cosines will allow us to find the distance from the spot on the bumper to the pocket (y). Once we know y, we can use the Law of Sines to find the angle (X).

\[ y^2 = 6.8^2 + 2.1^2 - 2(6.8)(2.1)\cos 168^\circ \]

\[ = 78.59 \]

\[ y = 8.86 \text{ feet} \]

**Note:** A more accurate calculation should be \[ \sqrt{78.59} = 8.8651 \]

2. The distance from the spot on the bumper to the pocket is 8.86 feet. We can now use this distance and the Law of Sines to find angle X. Since we are finding an angle, we are faced with the SSA case, which means we could have no solution, one solution, or two solutions. However, since we know all three sides this problem will yield only one solution.

\[ \frac{\sin 168^\circ}{8.86} = \frac{\sin X^\circ}{6.8} \]

\[ \frac{6.8 \sin 168^\circ}{8.86} = \sin X^\circ \]

\[ 0.1596 = \sin X^\circ \]

\[ X^\circ = 8.77^\circ \]

**Note:** A more accurate calculation should be \[ \sin X^\circ = \frac{6.8 \sin 168^\circ}{8.8651} = 0.15948 \]

\[ X^\circ = 9.177^\circ \]

Peter Chew Rules. \[ \tan x = \frac{6.8 \sin 168^\circ}{2.1 \cdot 6.8 \cos 168^\circ} \]

\[ = 0.1616 \]

\[ x = 9.1796^\circ \]
Example 2: In a game of pool, a player must put the eight ball into the bottom left pocket of the table. Currently, the eight ball is 5.3 feet away from the bottom left pocket. However, due to the position of the cue ball, she must bank the shot off of the right side bumper. If the eight ball is 3.2 feet away from the spot on the bumper she needs to hit and forms a $150^\circ$ angle with the pocket and the spot on the bumper, at what angle does the ball need to leave the bumper?

![Diagram of pool table with eight ball and angles labeled]

Note: This is actually a trick shot performed by spinning the eight ball, and the eight ball will not actually travel in straight-line trajectories. However, to simplify the problem, assume that it travels in straight lines.
Solution: In the scenario above, we have the SAS case, which means that we need to use the Law of Cosines to begin solving this problem. The Law of Cosines will allow us to find the distance from the spot on the bumper to the pocket (y). Once we know y, we can use the Law of Sines to find the angle (X).

\[ y^2 = 5.3^2 + 3.2^2 - 2(5.3)(3.2)\cos 150^\circ \]

\[ = 67.71 \]

\[ y = 8.2286 \text{ feet} \]

2. The distance from the spot on the bumper to the pocket is 8.86 feet. We can now use this distance and the Law of Sines to find angle X. Since we are finding an angle, we are faced with the SSA case, which means we could have no solution, one solution, or two solutions. However, since we know all three sides this problem will yield only one solution.

\[ \frac{\sin 150^\circ}{8.2286} = \frac{\sin X^\circ}{5.3} \]

\[ \frac{5.3 \sin 150^\circ}{8.2286} = \sin X^\circ \]

\[ 0.3220 = \sin X^\circ \]

\[ X^\circ = 18.78^\circ \]

Peter Chew Rules

\[ \tan x = \frac{5.3 \sin 150^\circ}{3.2 - 5.3 \cos 150^\circ} \]

\[ = 0.3402 \]

\[ x = 18.79^\circ \]
Example 3: In a game of pool, a player must put the eight ball into the bottom left pocket of the table. Currently, the eight ball is 6.3 feet away from the bottom left pocket. However, due to the position of the cue ball, she must bank the shot off of the right side bumper. If the eight ball is 1 foot away from the spot on the bumper she needs to hit and forms a $150^\circ$ angle with the pocket and the spot on the bumper, at what angle does the ball need to leave the bumper?

Note: This is actually a trick shot performed by spinning the eight ball, and the eight ball will not actually travel in straight-line trajectories. However, to simplify the problem, assume that it travels in straight lines.
Solution: In the scenario above, we have the SAS case, which means that we need to use the Law of Cosines to begin solving this problem. The Law of Cosines will allow us to find the distance from the spot on the bumper to the pocket (y). Once we know y, we can use the Law of Sines to find the angle (X).

\[ y^2 = 6.3^2 + 1^2 - 2(6.3)(1)\cos 80^\circ \]
\[ = 38.50 \]
\[ y = 6.205 \text{ feet} \]

2. The distance from the spot on the bumper to the pocket is 8.86 feet. We can now use this distance and the Law of Sines to find angle X. Since we are finding an angle, we are faced with the SSA case, which means we could have no solution, one solution, or two solutions. However, since we know all three sides this problem will yield only one solution.

\[ \frac{\sin 80^\circ}{6.205} = \frac{\sin X^\circ}{6.3} \]
\[ \frac{6.3 \sin 80^\circ}{6.205} = \sin X^\circ \]
\[ 0.9999 = \sin X^\circ \]
\[ X^\circ = 90.81^\circ, \quad 89.19^\circ (Reject) \]
Peter Chew Rules

\[ \tan x = \frac{6.3 \sin 80^\circ}{1 - 6.3 \cos 80^\circ} \]

= -66.01

\[ x = 90.84^\circ \]

Note: For this problem, the general calculation error is to assume that \( x = 89.19^\circ \).

We can double check the answer using current method 2 (cosine rule plus cosine rule).

Current Method 2:

Cosine rule

\[ y^2 = 6.3^2 + 1^2 - 2(6.3)(1)\cos 80^\circ \]

= 38.50

\[ y = 6.205 \text{ feet} \]

Cosine rule

\[ 6.3^2 = 6.205^2 + 1^2 - 2(6.205)(1)\cos x^\circ \]

12.41 \( \cos x^\circ \) = -0.187975

\[ \cos x^\circ = -0.01515 \]

\[ x = 90.87^\circ \]

Since both tangent and cosine of obtuse angles are negative, we can easily tell whether the angle is obtuse or acute. As we have seen, the Peter Chew rule is a simple method because it only needs to be used once to get the answer.
Example 4: In a game of pool, a player must put the eight ball into the bottom left pocket of the table. Currently, the eight ball is 7.2 feet away from the bottom left pocket. However, due to the position of the cue ball, she must bank the shot off of the right side bumper. If the eight ball is 1.5 feet away from the spot on the bumper she needs to hit and forms a $70^\circ$ angle with the pocket and the spot on the bumper, at what angle does the ball need to leave the bumper?

Note: This is actually a trick shot performed by spinning the eight ball, and the eight ball will not actually travel in straight-line trajectories. However, to simplify the problem, assume that it travels in straight lines.
Solution: In the scenario above, we have the **SAS** case, which means that we need to use the Law of Cosines to begin solving this problem. The Law of Cosines will allow us to find the distance from the spot on the bumper to the pocket \( (y) \). Once we know \( y \), we can use the **Law of Sines** to find the angle \( (X) \).

\[
y^2 = 7.2^2 + 1.5^2 - 2(7.2)(1.5)\cos70^\circ
\]

\[
= 46.70
\]

\[
y = 6.834 \text{ feet}
\]

2. The distance from the spot on the bumper to the pocket is 8.86 feet. We can now use this distance and the **Law of Sines** to find angle \( X \). Since we are finding an angle, we are faced with the SSA case, which means we could have no solution, one solution, or two solutions. However, since we know all three sides this problem will yield only one solution.

\[
\frac{\sin 70^\circ}{6.834} = \frac{\sin X^\circ}{7.2}
\]

\[
7.2 \sin 70^\circ = 6.834 \sin X^\circ
\]

\[
0.9900 = \sin X^\circ
\]

\[
X^\circ = 98.11^\circ, \quad 81.89^\circ (\text{Reject})
\]
Peter Chew Rules

\[ \tan x = \frac{7.2 \sin 70^\circ}{1.5 - 7.2 \cos 70^\circ} \]

\[ = -7.029 \]

\[ x = 98.10^\circ \]

Note: For this problem, the general calculation error is to assume that \( x = 81.89^\circ \).

We can double check the answer using current method 2 (cosine rule plus cosine rule).

Current Method 2:

Cosine rule

\[ y^2 = 7.2^2 + 1.5^2 - 2(7.2)(1.5)\cos 70^\circ \]

\[ = 46.70 \]

\[ y = 6.834 \text{ feet} \]

Cosine rule

\[ 7.2^2 = 6.834^2 + 1.5^2 - 2(6.834)(1.5)\cos x^\circ \]

\[ 20.502 \cos x^\circ = -2.8864 \]

\[ \cos x^\circ = -0.1408 \]

\[ x = 98.09^\circ \]

Since both tangent and cosine of obtuse angles are negative, we can easily tell whether the angle is obtuse or acute. As we have seen, the Peter Chew rule is a simple method because it only needs to be used once to get the answer.
4. Conclusion

The purpose of Peter Chew's Rule for solution of triangle is to provide a simple method to compare current methods to aid in mathematics teaching and learning, especially if similar COVID-19 problems arise in the future. Therefore, applying the Peter Chew's Rule for solution of triangle in Pool Game problem can make calculation of Pool Game problem simple and easier.

The purpose of Peter Chew's Rule for solution of triangle is the same as Albert Einstein's famous quote: Everything should be made as simple as possible, but not simpler.

In addition, Albert Einstein's also quote:

i) We cannot solve our problems with the same thinking we used when we created them.

ii) If you can't explain it simply you don't understand it well enough.

iii)“Genius is making complex ideas simple, not making simple ideas complex.”.

iv) “Any intelligent fool can make things bigger and more complex. It takes a touch of genius - and a lot of courage - to move in the opposite direction.”

v) God always takes the simplest way.

vi) When the solution is simple, God is answering.

Isaac Newton quote Nature is pleased with simplicity. And nature is no dummy.

From the Albert Einstein's and Isaac Newton quote above, it can be seen that simplifying knowledge is very important.
5. Reference


