Dynamic test of the continuously variable weak force by a torsion pendulum with pre-applied stress

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1. Introduction

Gravitational waves (GWs) are perturbations of the space–time curvature caused by the acceleration of mass and travel at the speed of light [1]. In September 2015, the GW signal GW150914 was first detected by LIGO, which opened a new era in GW astronomy [2]. There are abundant sources of GW in the $10^{-4}$–1 Hz frequency band, which can only be obtained by space GW detection [3, 4, 5]. The laser interferometer space antenna (LISA) is a space GW detection project led by the European Space Agency (ESA) in collaboration with the National Aeronautics and Space Administration (NASA); it is designed to form a regular triangular constellation operating in a solar orbit [6, 7]. The TianQin project orbiting the Earth [8, 9, 10] and the Taiji project orbiting the Sun [11, 12] proposed by Chinese researchers are also space GW detection missions, and the observatories are expected to be launched in the next 10–15 years.

In space GW detection, to offset the non-conservative weak force from sunlight pressure and obtain an ultra-stable satellite platform, micro-thrusters must generate a continuously variable weak thrust for precise drag-free control [13, 14]. This weak thrust must have a range of 0.1–100 μN and a noise below 0.1 μN · Hz$^{-1/2}$ from $10^{-4}$ to 1 Hz to achieve the desired goal [8]. As preliminary verification satellites of space GW detection missions, the LISA pathfinder, Taiji-1, and TianQin-1 have all conducted in-orbit tests and obtained the expected results, gradually approaching the final goal [15, 16, 17]. Before launching precision space science missions, high-precision ground tests are required to guarantee success. Therefore, the continuously variable weak forces must be tested.

Currently, several scientific research institutions, such as the Office National d’Études et Recherches Aérospatiales (ONERA) [18], Thales Alenia Space [19], ESA [20], NASA [21], Busek Company [22], Airbus [23], Politecnico di Torino (Polito) [24], George Washington University (GWU) [25], Huazhong University of Science and Technology (HUST) [26], Space Engineering University [27], and University of Chinese Academy of Sciences (UCAS) [28], have conducted tests and evaluations for micro-thrust on the ground. According to the requirements of space GW detection, the test frequency must be as low as $10^{-4}$ Hz, for which the torsion pendulum is the most appropriate because of its ultra-low natural frequency, which results in high sensitivity. Therefore, Yang et al. developed a ground test system at HUST that demonstrated a sensitivity of 0.1 μN · Hz$^{-1/2}$ within a range of $10^{-4}$ to 0.1 Hz. This system was built using the conventional torsion pendulum method and was utilized to test the impulse of a pulsed plasma thruster (PPT) [26]. Yang et al. developed a ground test system at UCAS that demonstrated a sensitivity of 0.1 μN · Hz$^{-1/2}$ ranging from $10^{-3}$ to 1 Hz using a pivot torsion pendulum [28]. However, the thrusters in space GW detection missions produce continuously variable micro-thrusters, and the existing test systems on ground have not gathered in-depth research on dynamic micro-thrust. Given the prevailing testing procedures, the dynamic weak force considerably influences the system stability. This instability makes assessing the continuously variable thrust challenging, particularly in high-sensitivity structures such as the torsion pendulum, where the coupling of non-sensitive modes becomes highly noticeable. Furthermore, there has been limited research on the calibration of the dynamic transfer function of a test system, which only remains in static calibration, while the micro-thrust is continuously variable. In space GW detection missions, studies on non-sensitive modes, dynamic transfer function calibration, and continuous variable weak force tests are crucial and practical.

In this study, a ground test system was developed for continuously variable weak forces in space GW detection.
missions. Pre-applied stress was used to improve the non-sensitive modes. The modeling, analysis, and verification are presented in Sections 2 and 3. A high-precision capacitance displacement reading system [29, 30], a PID feedback, and electrostatic force are designed and implemented to perform the dynamic calibration tests, as presented in Sections 4 and 5. The conclusions are presented in Section 6.

2. Measurement principle and modeling

A classic torsion pendulum was suspended using a wire, as shown in Figure 1(a). Point A is the fixed point of the wire, point O is the connection point between the wire and pendulum, and B and B’ are the balance masses of the torsion pendulum. The deflection angle of the torsion pendulum or the displacement of the balance masses can reflect the small motion applied to the torsion pendulum. The equation of motion can be expressed as follows:

\[
J \ddot{\theta} (t) + \lambda \dot{\theta} (t) + k \theta (t) = F (t) \cdot R.
\]

(1)

Where \( J \) is the moment of inertia of the torsion pendulum, \( \lambda \) is the damping factor, \( k \) is the torsional stiffness of the wire, \( \theta \) is the angular displacement of the torsion pendulum, \( s \) is the Laplace operator, and \( F \) and \( R \) are the external forces applied to the torsion pendulum and moment arm length, respectively. Subsequently, the transfer function of the torsional pendulum system can be expressed as follows:

\[
H_p = \frac{\Delta \theta (s)}{F (s)} = \frac{R}{J s^2 + \lambda s + k}.
\]

(2)

\[F \]

Figure 1: Schematic of the torsion pendulum structure. (a). Simplified diagram of the classic torsion pendulum in the free state. (b). Simplified diagram of the motion of the classic torsion pendulum when an external force \( \vec{F} (F_x, F_y, F_z) \) is applied. (c). Simplified diagram of a double-wire torsion pendulum in the free state; point \( A' \) is the fixation point of one end of the lower wire.

According to Quinn et al.[31], the equation for the torsional stiffness of a wire \( k \) is as follows:

\[
k = \frac{\pi D^4}{32I} \left( G + \frac{4mg}{\pi D^2} \right).
\]

(3)

Where \( l \) denotes the length of the suspended wire, \( D \) is the diameter of the wire, \( m \) is the mass of the torsion pendulum, and \( G \) is the shear modulus of the wire. \( k \) can be represented in the equation (Equation 1) of motion as follows:

\[
k = J \omega_0^2,
\]

(4)

where \( \omega_0 \) denotes the intrinsic frequency of the torsion pendulum.

According to Equations (3) and (4), the torsional stiffness \( k \) and intrinsic frequency \( \omega_0 \) are obtained by designing the wire length \( l \), wire diameter \( D \), and moment of inertia \( J \) of the torsion pendulum.

However, in practice, the external force also influences the motion of the non-sensitive modes of the torsion pendulum, as shown in Figure 1(b). When an external force \( \vec{F} \) is applied, the influence of the non-sensitive modes is apparent. Therefore, we must analyze the torsional pendulum modes and suppress the coupling motion of the non-sensitive modes to test the continuous variable weak force.

To facilitate the analysis, we preset the parameters for the classical torsion pendulum structure shown in Figure 1(a) based on experience, as shown in Table 1. Copper blocks of mass 2 kg were suspended at both ends of the torsion pendulum (B and B’) as a weak force generator and counterweight, respectively. A tungsten wire was selected owing to its high carrying capacity.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire length</td>
<td>0.03</td>
<td>m</td>
</tr>
<tr>
<td>Wire diameter</td>
<td>300</td>
<td>( \mu m )</td>
</tr>
<tr>
<td>Pendulum mass</td>
<td>2.56</td>
<td>kg</td>
</tr>
</tbody>
</table>

According to the parameters in Table 1, the moment of inertia of the torsional pendulum in the sensitive mode is \( J = 0.05 \text{ kg} \cdot \text{m}^2 \), and the intrinsic frequency of the torsion pendulum can be obtained as \( f = 3.28 \times 10^{-2} \text{ Hz} \) using Equations (3) and (4). The finite element simulation software ANSYS was used for synchronous verification, and the results are presented in Table 2, where mode 1 is the sensitive mode and the simulation value of the intrinsic frequency is \( 2.96 \times 10^{-2} \text{ Hz} \), which is consistent with the theoretically calculated value.

<table>
<thead>
<tr>
<th>Mode orders</th>
<th>Mode of motion</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Torsion around the Z-axis</td>
<td>2.96x10^{-2}</td>
</tr>
<tr>
<td>2</td>
<td>Torsion around the Y-axis</td>
<td>3.20x10^{-2}</td>
</tr>
<tr>
<td>3</td>
<td>Translation along the Y-axis</td>
<td>9.84x10^{-2}</td>
</tr>
<tr>
<td>4</td>
<td>Translation along the X-axis</td>
<td>0.48</td>
</tr>
<tr>
<td>5</td>
<td>Torsion around the X-axis</td>
<td>1.01</td>
</tr>
<tr>
<td>6</td>
<td>Translation along the Z-axis</td>
<td>52.29</td>
</tr>
</tbody>
</table>
The first six modes in Table 2 include torsion around the $X$, $Y$, and $Z$-axes and translation along the $X$, $Y$, and $Z$-axes. The frequencies of the non-sensitive modes (modes 2 and 3) and sensitive mode (mode 1) are close, resulting in the motion of these modes having almost the same sensitivity. Subsequently, when the torsion pendulum is subjected to an external force $\vec{F}(F_x, F_y, F_z)$, the coupled motion shown in Figure 1(b) occurs, which affects the test for the continuous variable weak force. Therefore, we adopted an improved structure for the double-wire suspension, as shown in Figure 1(c). The point $A'$ is the fixation point on the lower wire.

### Table 3
Parameters of double-wires torsion pendulum

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper wire length</td>
<td>0.06</td>
<td>m</td>
</tr>
<tr>
<td>Lower wire length</td>
<td>0.06</td>
<td>m</td>
</tr>
<tr>
<td>Wire diameter</td>
<td>300</td>
<td>μm</td>
</tr>
<tr>
<td>Pendulum mass</td>
<td>2.56</td>
<td>kg</td>
</tr>
</tbody>
</table>

The preset parameters of the double-wire torsion pendulum are listed in Table 3. The first six modes obtained from the ANSYS simulations are listed in Table 4. The intrinsic frequency (mode 1) is $2.99 \times 10^{-2}$ Hz, which is consistent with the simulation value of the classic torsion pendulum and the theoretically calculated value. Table 4 shows that the frequencies of modes 2 and 3, which have a significant coupling effect on the sensitive mode 1 in the classic structure, increase by approximately 3.1 times and 2.3 times, respectively, through the double-wire structure, implying that the double-wire structure can improve the effect of the non-sensitive modes of the torsion pendulum.

### Table 4
First six modes of double-wires torsion pendulum

<table>
<thead>
<tr>
<th>Mode orders</th>
<th>Mode of motion</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Torsion around the $Z$-axis</td>
<td>$2.99 \times 10^{-2}$</td>
</tr>
<tr>
<td>2</td>
<td>Torsion around the $Y$-axis</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>Translation along the $X$-axis</td>
<td>0.23</td>
</tr>
<tr>
<td>4</td>
<td>Translation along the $Y$-axis</td>
<td>0.23</td>
</tr>
<tr>
<td>5</td>
<td>Torsion around the $X$-axis</td>
<td>0.66</td>
</tr>
<tr>
<td>6</td>
<td>Translation along the $Z$-axis</td>
<td>52.37</td>
</tr>
</tbody>
</table>

### 3. Model analysis with pre-applied stress

Based on the improved structure of the double wires in Section 2, an optimized double-wire torsion pendulum is designed, and a simplified diagram of the equilibrium is shown in Figure 2. $a$ is the length of the half-horizontal arm of the torsion pendulum, and $b$ represents the half-length of the vertical beam. $B_1$ and $B_1'$ are the horizontally separated ends of the torsion pendulum, and $C_1C_1'$ represents a rigid vertical beam, which is important for suppressing the non-sensitive modes, as listed in Table 4. $T$ is the pre-applied stress acting on the lower wire. The first three modes shown in Table 4, namely, the sensitive mode of torsion around the $Z$-axis, the non-sensitive mode of torsion around the $Y$-axis, and the non-sensitive mode of translation along the $X$-axis, are mainly discussed in this section because of their close frequencies.

Considering the rigorous analysis and experiment, the parameters of the torsion pendulum are obtained from the experimental measurements, as shown in Table 5. The lengths of the upper and lower tungsten wires are $l_{01}$ and $l_{02}$, respectively, when not suspended. In free equilibrium after suspension, as shown in Figure 2(a), the stresses generated by the upper and lower wires are $F_{11}$ and $T$, respectively. The lengths of the upper and lower wires after stretching are $l_{11}$ and $l_{12}$. When an external force $F_Y$ is applied along the $Y$-axis (as shown in Figure 2(b)) to one end of the torsion pendulum, the torsion pendulum moves around the sensitive axis-$Z$ axis. According to Equation (3), the stiffness $k_z$ of the torsion pendulum in the sensitive mode (torsion around the $Z$-axis) can be expressed as follows:

$$k_z = \frac{\pi D^4}{32 l_{01}} \left( G + \frac{4 (mg + T)}{\pi D^2} \right) + \frac{4 D^4}{32 l_{02}} \left( \frac{l_{11}^2 + b}{l_{11}} + \frac{l_{12}^2 + b}{l_{12}} \right) T.$$

When an external force $F_z$ is applied along the $Z$-axis (as shown in Figure 2(c)) or an external force $F_x$ is applied along the $X$-axis (as shown in Figure 2(d)) to one end of the torsion pendulum, it moves to a new equilibrium, and the stress of the upper wire is expressed by $F_{21}$ and $F_{31}$, respectively. According to the equation of force equilibrium and the tension equation of the tungsten wire, the torsion stiffness $k_y$ around the $Y$-axis and translation stiffness $k_x$ along the $X$-axis under Taylor approximation can be obtained as follows:

$$k_y = \frac{F_{11} b (l_{11} + b)}{al_{11}} + \frac{b (l_{12} + b)}{al_{12}} T,$$

![Figure 2](https://ssrn.com/abstract=4549401)
Different pre-applied stresses to torsion around the Z-axis with different pre-applied stresses $T$. (c) Sensitivity of the mode corresponding to torsion around the Y-axis with different pre-applied stresses $T$.

![Image](image.jpg)

**Figure 3:** Sensitivities ($S_x$, $S_y$, $S_z$) of the corresponding three modes (torsion around the Z-axis, Y-axis, translation along the X-axis) with the different pre-applied stresses $T$ ($T = 0, 10, 20$ N). (a) Sensitivity of the mode corresponding to torsion around the Z-axis with different pre-applied stresses $T$. (b) Sensitivity of the mode corresponding to torsion around the Y-axis with different pre-applied stresses $T$. (c) Sensitivity of the mode corresponding to translation along the X-axis.

Where $J_z$ and $J_y$ are the moments of inertia of torsion around the Z- and Y-axis directions, respectively, which can be calculated as $J_z = 0.089 \text{ kg} \cdot \text{m}^2$ and $J_y = 0.090 \text{ kg} \cdot \text{m}^2$. Combined with the experimentally measured parameters of the double-wire torsion pendulum in Table 5, the influence of the pre-applied stress $T$ on the first three modes is examined in detail using MATLAB, as shown in Figure 3. The theoretical analysis results indicate that with an increase in the pre-applied stress $T$, the intrinsic frequencies of the torsion around the Y-axis (Figure 3(b)) and translation along the X-axis (Figure 3(c)) both increase and the response sensitivities of the two modes decrease. However, the sensitivity of the sensitive mode (torsion around the Z-axis mode) is almost constant, as shown in Figure 3(a). Therefore, pre-applied stress can theoretically suppress the response of the non-sensitive modes.

**Table 5**

Parameters of double-wires torsion pendulum measured by experiment

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Identifier</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper wire length</td>
<td>$l_{01}$</td>
<td>0.064</td>
<td>m</td>
</tr>
<tr>
<td>Lower wire length</td>
<td>$l_{02}$</td>
<td>0.050</td>
<td>m</td>
</tr>
<tr>
<td>Wire diameter</td>
<td>$D$</td>
<td>300</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>Horizontal arm length</td>
<td>$2a$</td>
<td>0.43</td>
<td>m</td>
</tr>
<tr>
<td>Vertical beam length</td>
<td>$2b$</td>
<td>0.12</td>
<td>m</td>
</tr>
<tr>
<td>Pendulum mass</td>
<td>$m$</td>
<td>2.56</td>
<td>kg</td>
</tr>
</tbody>
</table>

Furthermore, the above results were verified experimentally, as shown in Figure 4. A force sensor was connected to...
the fixation point $A'$ to measure the pre-applied stress $T$ on the lower wire, as shown in Figure 4(a). Stresses of $T=0$ N, 10 N, and 20 N were applied. External forces in different directions ($F_y$, $F_z$, $F_x$) were applied to one end of the torsion pendulum along the X-, Y-, and Z-axes as an incentive to study the corresponding three modes (torsion around the Z-axis, torsion around the Y-axis, and translation along the X-axis). Subsequently, the intrinsic frequency of each measurement was read using a capacitance displacement sensing circuit. Figure 4(b) shows the experimental setup.

The intrinsic frequencies of the different modes in Figure 3 were extracted and compared with the experimental results, as shown in Figure 5. Within the error bar range, the theoretical analysis and experimental results are in agreement. The intrinsic frequency of the sensitive mode is unaffected by changes in the pre-applied stress. However, the frequencies of the non-sensitive modes increase with the pre-applied stress. The error bar was derived primarily from the force sensor with an accuracy of 0.5 N. Therefore, the pre-applied stress can effectively suppress the influence of the non-sensitive modes. In the final experiments, the appropriate pre-applied stress must be selected according to the demand for non-sensitive mode suppression.

4. Experiment design and construction

The measurement principle of the torsion pendulum is shown in Figure 6. When an external force $F_{in}$ is applied to the torsion pendulum, there is a displacement $\theta$. $\theta_1$ and $\theta_2$ are the angular displacement output signals at the ends of the torsion pendulum. Capacitance displacement sensing is used to read out the symmetric ends[32]. The PID controller and electromagnetic actuator were used to control the torsion pendulum to the zero position in real-time to improve the stability and linearity of the system. The input force $F_{in}$ can then be obtained from the output of the PID $V_{pid}$. $H_{tp}$ is the transfer function of the torsion pendulum, $H_c$ is the transfer function of the capacitance sensing circuit, $H_{pid}$ is the transfer function of the PID controller, and $H_f$ is the transfer function of the electromagnetic actuator.

To evaluate the dynamic performance of the test system, it was necessary to perform calibration using a standard force. In this experiment, the electrostatic force between the parallel plates was used as a weak force generator. The electrostatic force equation is as follows:

$$F = \frac{\varepsilon S U^2}{2d_0^2} = \frac{C_0 U^2}{2d_0^2}.$$ (9)

Where $C_0$ is the capacitance between the plates, which was measured using an E4980A LCR meter of KEYSIGHT. $S$ is the opposite area between the plates, $\varepsilon$ is the vacuum dielectric constant, $d_0$ is the distance between the parallel plates, and $U$ is the voltage applied between the plates. Weak force calibration includes static and dynamic calibrations. Static calibration generates different electrostatic forces by applying different direct current (DC) voltages. Dynamic calibration is used to input alternating current (AC) voltages at different frequencies. Finally, the sensitivity was obtained by fitting the data.

### Table 6

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
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<tbody>
<tr>
<td>Upper wire length</td>
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<td>m</td>
</tr>
<tr>
<td>Lower wire length</td>
<td>0.04</td>
<td>m</td>
</tr>
<tr>
<td>Wire diameter</td>
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<td>$\mu$m</td>
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<td>Horizontal arm length</td>
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<td>Vertical beam length</td>
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<td>m</td>
</tr>
<tr>
<td>Pendulum mass</td>
<td>2.88</td>
<td>kg</td>
</tr>
</tbody>
</table>
The weak force calibration and test system used in this experiment are shown in Figure 7. The upper wire was fixed to a micro-rotating table, which was convenient for accurately adjusting its position. The lower wire was fixed to the force sensor and another micro-rotating table. The capacitance structure was adjusted using a triaxial adjustment mechanism. A level was used to adjust the torsion pendulum to a balanced position. The electromagnetic actuator consisting of a coil, four magnets, and a planar coil was fixed at both ends of the torsion pendulum. The weak force generator was adjusted using a precision lifting platform and adjusting mechanism, and a parallel plate capacitance was formed with the plate on the torsion swing arm. One end of the torsion pendulum was equipped with a thruster-mounting port.

We considered the actual continuously variable weak force conditions in the experiment; therefore, the final double-wire torsional pendulum structure was further optimized, and the parameters are listed in Table 6. For this set of parameters, the intrinsic frequency of the sensitive mode of the torsional pendulum structure is 0.07 Hz. A stress of 20 N was pre-applied to the lower wire of the structure, which caused the lowest frequency of the non-sensitive modes to be 3.62 Hz. According to the estimation of Equation (4), the highest sensitivity of the non-sensitive modes of the torsion pendulum in the final experiment was 2733 times lower than that of the sensitive mode. Therefore, in the weak force measurement with a maximum of 100 μN, the influence of the non-sensitive modes is below 0.1 μN.

The plates of the standard force generator used in the calibration are of size 40×40 mm². Through fine adjustment, the distance between the parallel plates d₀ was set as 272.8 μm, and the measured capacitance C₀ was 51.90(±0.01) pF. Upon applying an adjustable voltage of 1—33 V using a programmable voltage source, a test range of 0.1 μN to 100 μN can be covered by KEYSIGHT B2961B. In the experiment, static calibration was performed by changing the DC voltage, and dynamic calibration was performed using the AC voltage with a sweeping frequency.

5. Analysis of the experimental results

The complete evaluation of the developed weak force test system included a static calibration, resolution test, response time test, dynamic calibration, and weak force noise test.

In static calibration, the output voltages of the test system with a set of step signals are obtained, as shown in Figure 8(a). The output voltage was fitted to the force calculated using Equation (9), as shown in Figure 8(b). The fitting static sensitivity was 1.70(±0.05) × 10⁴ V N⁻¹. The 0.1 μN-reciprocating steps are shown in Figure 8(c). The time-domain resolution exceeded 0.1 μN. The response time was tested with a step of approximately 6 μN, and the result was 1.6 s, as shown in Figure 8(d).

The dynamic calibration uses an AC voltage of 10 V to generate a weak dynamic force of sweeping frequencies from 10⁻⁴ to 2 Hz. The results are shown in Figure 9. The error bars are mainly derived from the capacitance measurement accuracy. The theoretical dynamic sensitivity (or transfer function response) is in good agreement with the experimental results. The -3 dB bandwidth of the weak force test system (0.63 Hz) is consistent with the reciprocal of the response time. The transfer function response of the system is gentle, indicating that the system is stable when a dynamic weak force test is performed.

The system was tested for an extended period in a stable external environment during the noise test of the weak force test system. Figure 10 shows the weak force noise power spectral density at 16 h. The results show that the weak force noise is better than 0.1 μN · Hz⁻¹⁄₂ in the frequency band from 0.9×10⁻⁴ Hz to 1.4 Hz, and the noise at low frequencies increases gradually owing to the influence of the ambient temperature.

6. Conclusion

In this study, the pre-stress applied to the double-wire torsion pendulum structure was proved theoretically and experimentally to be effective in suppressing the non-sensitive modes, which has particular significance in regards to testing continuously variable weak forces. Based on this, we developed a weak force test system combining a capacitance displacement sensor and precision PID feedback control and conducted a complete evaluation test. The system resolution was better than 0.1 μN, the response time was approximately 1.6 s, the dynamic performance of the system was stable, and the weak force test noise reached 0.1 μN · Hz⁻¹⁄₂ within the frequency band from 0.9 × 10⁻⁴ Hz to 1.4 Hz. The results meet the requirements of micro-thrust in space GW missions, such as the TianQin project.

Acknowledgments

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Figure 7: Calibration and test system built in the experiment.

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Figure 8: Results of the step experiments using the DC voltage source. (a) Set of 0.1 ~ 105 μN variable steps calibration. (b) Static sensitivity fitting curve. (c) Resolution test of 0.1 μN reciprocating steps. (d) Response time test.

Figure 9: Dynamic calibration of the system transfer function. The black curve is the ideal curve, the blue circle represents the experimental data under different frequencies, and the red curve is the dynamic sensitivity fitted with experimental data.

Figure 10: Noise power spectrum density of the weak force test system. The dashed red line is the noise of 0.1 μN⋅Hz^{−1/2}. The solid blue line is the force noise tested in the experiment.

References
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