Output-Oriented Agricultural Subsidy Design

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Forthcoming in Management Science
March 18, 2023

Abstract: Many governments subsidize the agricultural industry, trying to raise the market outputs either for domestic needs or for export. In many countries, particularly developing countries, the producers’ market may be fragmented, involving a large number of farmers with variable productivity levels. The format of subsidies can have significantly different implications for farmers in different market segments. In this study, we examine four types of subsidies. A planting subsidy is paid to a farmer based on the amount of input, and a harvesting subsidy compensates a farmer for the cost incurred during the process of output collection and distribution. The government may also offer a combined subsidy under which a farmer gets paid for both plantation and harvesting, or offer a selective subsidy under which a farmer can choose to be subsidized on either plantation or harvesting but not both. In addition to examining the efficiency of budget spending and social welfare, two common performance measures studied in various contexts, we thoroughly analyze the implications of subsidies on the output and wealth distributions among the farmers. In general, subsidizing on harvesting or overly compensating on plantation can increase the disparity among the farmers, while an appropriate level of planting subsidy helps to balance the distributions of the farmers’ output and profit. A comprehensive evaluation of the government’s policies reveals that the harvesting subsidy, while inducing the most dispersed output and profit distributions, leads to the most efficient use of input resources and the highest social welfare. The planting subsidy, though being the most effective in balancing the farmer income for a moderate output increase, performs poorly in budget spending, resource usage, and welfare generation when the government sets an aggressive target for output increase. In such a situation, the combined subsidy can offer the most evenly distributed farmer income, with the least amount of budget needed to achieve the output target.

Key words: subsidy and tax; agriculture products; output and wealth distribution; majorization order

Electronic copy available at: https://ssrn.com/abstract=4378693
1 Introduction

Almost every country subsidizes agriculture. Most agricultural subsidy programs aim toward increasing the output in order to meet the need for domestic consumption or to increase export. For example, the Haiti government, together with the Food and Agriculture Organization of the United Nations, incepted a $10.2 million subsidy to raise the outputs of small farmers.\(^1\) The Chinese government issued the Number One Document in 2014 to emphasize the spending on boosting production to ensure self-sufficiency for the domestic market and reduce the reliance on food imports (Gale et al. 2015). In India, two state governments issued a subsidy of $700 per ton of dairy export, and the federal government approved an extra 10% match-up in 2018, aiming toward alleviating the oversupply of dairy products in the domestic market.\(^2\) During the COVID-19 pandemic, the World Bank worked with many countries to alleviate agricultural supply chain disruption and to support their farmers to continue plantation during the pandemic.\(^3\)

The most commonly seen programs subsidize the producers on either planting or harvesting. A *planting subsidy* provides the farmers access to quality inputs for production, including seeds, fertilizers, kerosene, and machinery. For example, the Indian government spent over $22 million to help farmers with irrigation, fertilizers, and electricity in 2017.\(^4\) The Chinese government has instituted various programs to aid the acquisition of machinery since 2004 (Gale 2013). A *harvesting subsidy* allows the farmers to reduce the cost in the process of output collection, storage, and distribution. Subsidies aiming toward encouraging exports belong to the category of harvesting subsidies. Other examples include the transportation subsidy issued by the Indian government in 2019\(^5\) and the price subsidies offered by the Thailand government in 2019.\(^6\)

Though increasing attention has been paid to subsidy programs in the Operations Management community, the focus has been given to understanding the implications on the producer’s incentives


\(^2\) See https://www.farmpolicyfacts.org/2018/10/subsidy-spotlight-india-2/ (last accessed September 21, 2022)


in monopoly or duopoly markets. The agriculture industry, however, often consists of a highly fragmented producer base, particularly in developing countries. The design of the subsidy programs must consider not only the efficient spending of the government budget to raise outputs, but also the fairness in farmers’ output and wealth distributions, the utilization of resources, and the welfare of society. The objective of this study is to understand the responses of the farmers’ market to different subsidy programs, and to evaluate the implications on the key performance indicators to guide the policy design.

Specifically, we consider an industry consisting of multiple farmers with variable productivity levels reflected by their yields or, equivalently, input-to-output ratios. The variability in farmers’ production efficiency is often a result of different experiences, capital, technologies, soil, irrigation, and weather conditions. In an attempt to raise the market output level of a certain product, the government offers a subsidy program. There can be different formats of subsidies. A planting subsidy compensates the farmers based on the input they invest in production, a harvesting subsidy reduces the farmers’ cost of bringing the harvested crop to the market, a combined subsidy pays the farmers on both planting and harvesting, and a selective subsidy offers the farmers the choice of getting compensations on either planting or harvesting, but not both.

When no subsidy is offered, a farmer with high productivity should produce more than one with low productivity. Government subsidy may change the output distribution among the farmers. Specifically, overly subsidizing on plantation can induce the low-yield farmers to produce aggressively, forcing the high-yield farmers to reduce their outputs. In contrast, subsidizing on harvesting enhances the competitive advantages of the high-yield farmers, resulting in an increased discrepancy among the farmers. Because of these effects, when the farmers are given the choice between the planting subsidy and harvesting subsidy, the low-yield farmers would pick the planting subsidy and the high-yield farmers would choose the harvesting subsidy.

In addition to achieving the target output level, the government may also evaluate the overall industry resource use and the market output distribution in deciding the budget spending on subsidies. Our analysis suggests that a smaller per-unit subsidy should be offered when the number of farmers in the industry is larger or when the consumers in the market are less price sensitive. Interestingly, no matter which subsidy format is chosen, the average subsidy per unit output (converted based on the industry-wide average productivity) stays the same across all programs. The overall spending on the subsidies, however, varies from program to program because the compensation to individual farmers depends on the industry productivity distribution.
The effectiveness of the subsidy programs depends critically on the target market output level that the government attempts to achieve. When the target is not too high (compared with the market output level without subsidy), the combined subsidy calls for taxing on harvesting, while subsidizing on plantation. Such a scheme leads to the most efficient government spending, making the combined subsidy the top performer among the four programs in budget planning. Moreover, the combined subsidy induces the least dispersion in farmers’ outputs and profits when the target level is set high or low. Governments, who are concerned about farmer poverty and job security, would potentially favor such an outcome. However, the combined subsidy can induce an excessive use of input resources and low social welfare.

The planting subsidy can effectively reduce the disparity among the farmers in their outputs and incomes when the target output level is in an intermediate range. However, this subsidy program requires the highest budget spending and resource consumption, while generating the lowest social welfare, when the target output level is set high. In contrast, though the harvesting subsidy results in the highest dispersion in the farmers’ output and profit distributions, it achieves the most efficient use of input resources and the highest social welfare. The performance of a selective subsidy, in general, sits in the middle among the four programs in terms of budget spending, resource usage, wealth distribution, and social welfare. These findings underscore the importance of comprehensive evaluations of the subsidy programs in the government’s policy design.

The remainder of the paper is organized as follows. In Section 2, we review the related literature and articulate our contributions. Section 3 presents the problem formulation, and discusses a benchmark model without subsidy. We analyze individual farmers’ responses to the subsidy programs, as well as the overall industry outcomes, in Section 4. In Section 5, we study the government’s subsidy design problem, and evaluate key performance indicators for the subsidy programs. We conclude our study in Section 6. Proofs of all formal results and model extensions (to random yields and nonlinear costs) are relegated to the online appendix.

2 Literature Review

Government intervention, in particular, subsidy and tax policies, to the agriculture industry is a central topic in agricultural economics. The major focus in this stream of literature is on the macroeconomic implications of subsidy and tax policies (e.g., Holloway 2002, Wise 2005, Schmitz et al. 2006, Yu 2013). The evaluation of the intervention policies is usually along the dimensions of income distribution (e.g., Bekkerman et al. 2019), productivity level (e.g., Frick and Sauer 2018),
and welfare effect (e.g., Cui et al. 2011). Majority of these studies conduct analysis on country-wide or industry-wide data to empirically analyze a specific policy. Output-oriented planting and harvesting subsidies have been examined in various contexts (e.g., Lunduka et al. 2013, Arndt et al. 2016, Wang and Wei 2021). In contrast to these studies, we formulate an analytical model to describe individual farmers’ responses to the subsidy programs, and characterize the properties of the industry equilibrium. Complementing the aforementioned empirical studies, we provide a comprehensive evaluation of four different subsidy programs by analyzing their performance on budget planning, output and profit distributions, resource usage, and welfare generation.

There is a growing body of literature on agricultural supply chains (see the surveys by Lowe and Preckel 2004, Sodhi and Tang 2014). Researchers have examined various aspects including resource planning (e.g., Kazaz and Webster 2011, Huh and Lall 2013, Zhang and Swaminathan 2020), information disclosure (e.g., Chen et al. 2013a,b, Devalkar et al. 2018), and farmer incentives (e.g., Tang et al. 2015, Levi et al. 2020). In particular, Alizamir et al. (2019) compare the Price Loss Coverage and Agriculture Risk Coverage programs in the U.S. Guda et al. (2021) analyze the price support policy aimed toward maximizing the social welfare. The price support scheme in their setting can be equivalently converted to a harvesting subsidy in our context (i.e., the government chooses the amount of price reduction instead of the selling price). In contrast to these studies, we focus on comparing the different subsidy formats along the key performance indicators to guide the policy design. In particular, we evaluate the efficiency of resource usage and the distribution among the farmers, two key dimensions that are not well-understood in the supply chain literature.

Our paper is most closely related to Tang et al. (2019), who study subsidies in a two-farmer market. Our study differs from Tang et al. (2019) along three important dimensions. First, they use the farmers’ profit difference to measure fairness, a common approach adopted in the operations literature (e.g., Cui et al. 2007, Ho et al. 2014, Katok et al. 2014). Insights generated from this measure may not remain when an alternative measure (e.g., the squared difference of profits) is applied or when there are more than two farmers involved. The notion of majorization order adopted in our analysis, in contrast, is flexible enough to cover most difference measures and is applicable to an arbitrary number of farmers. Second, our model generates a rich set of outcomes that the government may choose to overly subsidize planting or tax on harvesting (i.e., impose a negative subsidy). Third, Tang et al. (2019) focus on welfare maximization by the government. We, instead, focus on output-oriented subsidy design, while thoroughly evaluating different performance measures including budget, disparity, resource, and welfare.
Subsidy programs have been studied in other contexts including medicine distribution (Arifo˘ glu et al. 2012, Mamani et al. 2012, Adida et al. 2013, Taylor and Xiao 2014, 2019, Kazaz et al. 2016), green technology adoption (Lobel and Perakis 2011, Alizamir et al. 2016, Cohen et al. 2016), bilateral coordination (Raz and Ovchinnikov 2015, Yu et al. 2020), and not-for-profit operations (Berenguer et al. 2017, Feng and Shanthikumar 2018b). These studies focus on a vertical relationship that involves a single producer. The agriculture industry often features a large number of producers, requiring a very different analysis.

3 The Problem

Consider the farming industry for a certain agricultural product, say a crop. The farmers’ market is fragmented, and the farmers vary from one another in their productivity levels, reflected by their input-to-output ratios in farming. The government, aiming toward increasing the overall market output to alleviate the crop shortage in the consumer market, initiates a farmer subsidy program.

The Farmers’ Market. There are \( \tilde{n} \) farmers in the market, indexed by \( j \in \tilde{N} = \{1, 2, \ldots, \tilde{n}\} \), who may grow the crop. The farmers plant at the beginning of the season and harvest at the end of the season. The cost for plantation (including, e.g., land, labor, tools, seeds, and fertilizers) is \( c_P \) per unit of input and the cost for harvesting (including, e.g., labor, tools, packaging, storage, and distribution) is \( c_H \) per unit of output. The farmers vary from one another in the soil condition, capability and productivity, and technology used, which results in different yield rates at harvesting. For farmer \( j \) to harvest \( q_j \) units, an input of \( x_j = z_j q_j \) units needs to be planted. In other words, the input-to-output ratio is \( z_j \), or the yield rate is \( 1/z_j \). Without loss of generality, we assume

\[
    z_1 \leq z_2 \leq \cdots \leq z_{\tilde{n}}.
\]

That is, farmer 1 is the most productive, while farmer \( \tilde{n} \) is the least productive.

Depending on the farmers’ cost structure and the market price, not all farmers may actively produce the crop. Given the farmers’ output vector \( \mathbf{q} = (q_1, q_2, \ldots, q_{\tilde{n}}) \), we define \( N = \{ j \in \tilde{N} : q_j > 0 \} \) as the set of active farmers with \( n = |N| \). Denote

\[
    \bar{z} = \frac{\sum_{i \in N} z_i}{n} \quad \text{and} \quad \nu = \frac{\sum_{i \in N} z_i^2}{n} - \bar{z}^2
\]

as the average and the variability of the input-to-output ratio, respectively.

Upon harvesting, the farmers bring their outputs to the market for sale. The market price of the crop is determined by the overall output. Specifically, the market-clearing price for the output
vector $\mathbf{q} = (q_1, q_2, \ldots, q_n) \geq 0$ is

$$p = \alpha - \beta \sum_{i \in \mathcal{N}} q_i, \quad \alpha > 0, \beta > 0.$$ 

This demand model has been widely applied to study agricultural markets (e.g., Hueth and Marcoul 2006, Shi et al. 2010, Agbo et al. 2015) and competing supply chains (e.g., An et al. 2015, Chen and Tang 2015, Tang et al. 2015, Alizamir et al. 2019). As the farmers are heterogeneous, the equilibrium may cover a wide range of market outcomes. Depending on the distribution of the farmers’ productivity, the market may be highly concentrated with a few dominant farmers, or the market may be highly fragmented consisting of a large number of farmers each with a small output.

**The Government’s Subsidy Program.** The subsidy program is output-oriented, as we discuss in Section 1. The government may set a target for the overall market output level and attempt to efficiently implement the program with a minimal budget. Alternatively, the government may try to maximize the overall output while refining the subsidy spending within an allocated budget. Mathematically, the two scenarios are dual of each other.

We model the two widely used subsidies, a payment $s_P$ for each unit planted and a payment $s_H$ for each unit harvested. The government may choose to offer a single subsidy in the form of either the planting subsidy $s_P$ or the harvesting subsidy $s_H$. The government may also offer a combined subsidy of $(s_P, s_H)$, with which the farmers receive payments for both plantation and harvesting. Alternatively, a selective subsidy of $(s_P, s_H)$ can be implemented, with which the farmers choose to get subsidized on either plantation or harvesting, but not both. We use a superscript $P, H, C, S$ to denote, respectively, planting, harvesting, combined, and selective subsidy programs. We do not impose any constraint on the values of subsidies $s_P$ and $s_H$. The extreme cases of overly subsidizing (i.e., exceeding the associated cost) and negative subsidies (i.e., taxing) are allowed.

**The Sequence of Events.** The subsidy program is executed with the following sequence of events.

1. The government sets the target for the market output and allocates the budget for the subsidy program. The format of the program is announced.

2. Given the subsidy program, farmer $j$ determines a plantation quantity, $x_j$, at the beginning of the season, which leads to a harvesting quantity, $q_j = x_j/z_j$, at the end of the season.

3. Farmers bring their outputs to the market for sale, and the market price is determined. Subsidy payments are made to the farmers based on the format of the program.
3.1 A Benchmark Model: Without Subsidy

In this subsection, we briefly discuss the case when no subsidy is offered to the farmers. We will build on these results to analyze the effect of subsidies in the next sections. Without any subsidy, farmer \( j \)'s profit is

\[
\pi_j(q) = \left( \alpha - z_j c_P - c_H - \beta \sum_{i \in \bar{N}} q_i \right) q_j, \quad j \in \bar{N}.
\]

**Lemma 1 (Condition for Production: Without Subsidy)** The set of farmers producing positive amounts satisfies

\[ N = \left\{ j \in \bar{N} : g(j) < \frac{\alpha - c_H}{c_P} \right\}, \]

where \( g(j) = (j + 1)z_j - \sum_{i=1}^{j} z_i \) is increasing in \( j \). The number of active farmers satisfies \( n = |N| = \max\{N\} \).

The ratio \( (\alpha - c_P)/c_H \) is an index for the profitability of the crop, as this ratio increases with the market potential \( \alpha \) and decreases with the farmers’ costs. The function \( g(j) \) defines farmer \( j \)'s efficiency level relative to those who are more productive than farmer \( j \) in the market. Lemma 1 suggests that when the farmer’s efficiency level exceeds the index of crop profitability, the farmer produces a positive amount. With this result, we can identify a threshold \( n \in \bar{N} \) so that farmers in the set \( \{1, 2, \ldots, n\} \) produce, while the rest do not, as suggested in the next proposition.

**Proposition 1 (Production Equilibrium: Without Subsidy)** In equilibrium, farmer \( j \)'s output quantity is

\[
q_j^* = \begin{cases} \frac{1}{\beta} \left( \frac{\alpha - c_H - \bar{z} c_P}{n+1} - (z_j - \bar{z}) c_P \right), & j \in \{1, 2, \ldots, n\}, \\ 0, & j \in \{n+1, n+2, \ldots, \bar{n}\}, \end{cases}
\]

and farmer \( j \)'s profit is

\[
\pi_j^* = \beta (q_j^*)^2, \quad j \in \bar{N}.
\]

The overall industry input is

\[
X = \frac{n}{\beta(n+1)} \left( \bar{z}(\alpha - c_H - \bar{z} c_P) - (n+1) v_z c_P \right),
\]

the overall market output is

\[
Q = \frac{n}{\beta(n+1)} (\alpha - c_H - \bar{z} c_P),
\]
and the overall farmers’ profit is

\[ \Pi = \frac{n}{\beta(n+1)^2} \left( (\alpha - c_H - \bar{z}c_P)^2 + (n+1)^2v_zc_P^2 \right). \]

Moreover, the following results hold.

i) \( q_j^* \) is decreasing in \( j \) and is increasing in \( \bar{z} \).

ii) When the average input-to-output ratio \( \bar{z} \) increases while the variability of the input-to-output ratio \( v_z \) is kept constant, the overall industry input increases \([decreases]\) for \( \bar{z} < > \left( \alpha - c_H \right) / (2c_P) \), the overall market output decreases, and the overall farmers’ profit decreases.

iii) When the variability of input-to-output ratio \( v_z \) increases while the average input-to-output ratio \( \bar{z} \) is kept constant, the overall industry input decreases, the overall market output does not change, and the overall farmers’ profit increases.

We observe from Proposition 1 that a farmer’s output level and profit depend on the productivity distribution among all farmers only through the average input-to-output ratio \( \bar{z} \). When \( \bar{z} \) increases with \( z_j \) unchanged, farmer \( j \)'s production quantity increases. The overall market output level depends on the productivity distribution only through its average. Thus, neither the individual output nor the market output is affected when the variability of the input-to-output ratio changes.

The overall industry input, however, exhibits a very different response to the productivity distribution. When it is profitable to produce at the average productivity (i.e., the marginal profit \( \alpha - 2\bar{z}c_P - c_H \geq 0 \)), the industry responds to a reduced average productivity (i.e., an increased \( \bar{z} \)) through an increased plantation. When producing at the average productivity level becomes not profitable (i.e., the marginal profit \( \alpha - 2\bar{z}c_P - c_H < 0 \)), the reduced average productivity further reduces the profitability of plantation, leading to a reduced overall industry input. An increased variability of the input-to-output ratio, increasing the differences between the efficient farmers and the inefficient ones, makes the inefficient farmers less competitive in the market. As a result, the inefficient farmers reduce their input quantities in view of the competition pressure, while the efficient farmers also reduce their input quantities because of the increased productivity.

From the policy maker’s standpoint, the distributions of individual farmers’ outputs and wealth are also important concerns. To further understand the implication of the competition equilibrium, we discuss the concept of majorization in the next subsection.
3.2 Preliminaries: The Majorization Order

Many agriculture markets are highly fragmented. The level of market fragmentation is reflected in two aspects, the number of active producers and the market share distribution among the producers. Both aspects depend on the distribution of the farmers’ productivity levels. If the farmers are similar in their productivity levels, then the farmers are likely to take similar market shares. If, however, there is a high disparity among the farmers in their productivity levels, then the market is likely to be dominated by a few highly productive farmers.

The notion of fairness or evenness in distribution has always been an important aspect in evaluating government policies (e.g., Ma et al. 2022). Specifically, in a government subsidy program, the incentive provided to the farmers and the resulting wealth allocation among the farmers need to be carefully examined to understand the strategic social implications of the program. We apply the concept of majorization, which has been widely used to study resource allocation problems (e.g., Yao 1987, Tong 1997, Feng and Shanthikumar 2018a).

**Definition 1 (Majorization)** A vector $u$ of size $n$ majorizes another vector $v$ of the same size, written as $u \geq_m v$ if

i) $\sum_{i=1}^{k} u[i] \geq \sum_{i=1}^{k} v[i], k = 1, \ldots, n$, and $\sum_{i=1}^{n} u[i] = \sum_{i=1}^{n} v[i]$, or, equivalently,

ii) $\sum_{i=1}^{k} u(i) \leq \sum_{i=1}^{k} v(i), k = 1, \ldots, n$, and $\sum_{i=1}^{n} u(i) = \sum_{i=1}^{n} v(i)$,

where $u(i)$ and $u[i]$ [v(i) and v[i]] are, respectively, the $i$-th smallest and largest elements in $u$ [v].

When the condition of equal total sums is removed in (i) [(ii)], we say $u$ weakly sub-majorizes [weakly sup-majorizes] $v$, written as $u \geq_{wm} v$ [u $\geq_{ws} v$].

The notion of majorization is a general measure of evenness or dispersiveness. For any Schur convex function $f : \mathbb{R}^n \to \mathbb{R}$, we have $f(z_A) \geq f(z_B)$ whenever $z_A \geq_m z_B$. It is trivial to show that the average sum of square deviations, i.e., $\frac{1}{n} \sum_{i=1}^{n} (z_i - \bar{z})^2$, and the average sum of absolute deviations, i.e., $\frac{1}{n} \sum_{i=1}^{n} |z_i - \bar{z}|$, are both Schur convex functions. Thus, $z_A \geq_m z_B$ implies that the farmers in market $A$ are more dispersed in their productivity levels than those in market $B$ no matter which of the two deviation measures is used.

**Proposition 2 (Distribution among Farmers: Without Subsidy)** Consider two otherwise identical markets indexed by $A$ and $B$ with the farmers in market $A$ exhibiting a more dispersed productivity distribution than those in market $B$ (measured by the majorization order), i.e., $z_A \geq_m z_B$. If all farmers produce positive quantities, then the following results hold.
i) \( q_A^* \geq^m q_B^* \).

ii) \( \pi_A^* \geq_{wm} \pi_B^* \).

By the definition of majorization order, the market output levels are the same in the two markets, as implied by Proposition 1. The distribution of the farmers’ outputs, however, becomes more balanced when their input-to-output ratios are more evenly distributed. The farmers’ profits also become more evenly distributed. In the next section, we examine whether the subsidy programs may increase or decrease the disparity among the farmers.

4 The Effect of Subsidies on the Farmers’ Incentives

In this section, we analyze the farmers’ responses to the subsidy programs. Section 4.1 focuses on the case when a combined subsidy is offered and Section 4.2 discusses the case when a selective subsidy is implemented. The planting or harvesting subsidy can be regarded as a special case of a combined subsidy by setting \( s_H = 0 \) or \( s_P = 0 \), respectively.

4.1 Combined Subsidy

Under a combined subsidy, the government announces \((s_P, s_H)\) with \( s_P \) paid for each unit of planting input and \( s_H \) for each unit of harvesting output. Effectively, the subsidies change a farmer’s production cost to \( c_P - s_P \) and harvesting cost to \( c_H - s_H \). Then, farmer \( j \)'s profit under an output vector \( q \) becomes

\[
\pi_j(q, s_P, s_H) = \left( a_j(s_P, s_H) - \beta \sum_{i \in \bar{N}} q_i \right) q_j, \quad j \in \bar{N},
\]

where \( a_j(s_P, s_H) = \alpha - z_j(c_P - s_P) - (c_H - s_H) \) is the market potential of farmer \( j \).

Given subsidies \((s_P, s_H)\), we can derive the set of active farmers, denoted by \( \bar{N}^c(s_P, s_H) \); see the detailed derivation of Lemma 2 in Online Appendix A. Intuitively, the subsidies reduce the farmers’ costs and induce more farmers to produce. This intuition is, however, only true when the planting subsidy is below the plantation cost (i.e., \( s_P \leq c_P \)). Overly subsidizing on planting (i.e., \( s_P > c_P \)) can induce aggressive competition from the low-yield farmers, driving the high-yield farmers out of the market.

Replacing a farmer’s planting cost to \( c_P - s_P \) and harvesting cost to \( c_H - s_H \) in Proposition 1, we can directly derive the equilibrium outcome when the combined subsidy is offered.
Proposition 3 (Production Equilibrium: Combined Subsidy) Suppose that the government implements a combined subsidy \((s_P, s_H)\). In equilibrium, farmer \(j\)’s output quantity is

\[
q^C_j(s_P, s_H) = \begin{cases} 
\frac{1}{\beta} \left( \bar{\alpha}(s_P, s_H) - (z_j - \bar{z})(c_P - s_P) \right), & j \in N^C(s_P, s_H), \\
0, & j \in \bar{N} \setminus N^C(s_P, s_H),
\end{cases}
\]

and farmer \(j\)’s profit is

\[
\pi^C_j(s_P, s_H) = \beta q^C_j(s_P, s_H)^2, \quad j \in \bar{N}.
\]

The overall industry input is

\[
X^C(s_P, s_H) = \frac{n}{\beta(n + 1)} \left( \bar{z} \bar{a}(s_P, s_H) - (n + 1)v_z(c_P - s_P) \right),
\]

the overall market output is

\[
Q^C(s_P, s_H) = \frac{n}{\beta(n + 1)} \bar{a}(s_P, s_H),
\]

and the overall farmers’ profit is

\[
\Pi^C(s_P, s_H) = \frac{n}{\beta(n + 1)^2} \left( \bar{a}^2(s_P, s_H) + (n + 1)^2 v_z(c_P - s_P)^2 \right),
\]

where \(\bar{a}(s_P, s_H) = \alpha - \bar{z}(c_P - s_P) - (c_H - s_H)\). Moreover, the following results hold.

i) \(q^C_j(s_P, s_H)\) is decreasing [increasing] in \(j\) for \(s_P \leq [>]c_P\).

ii) Each farmer’s input quantity, output quantity, and profit are increasing [decreasing] in \(s_P\) when \(z_j \geq \lfloor \frac{n \bar{z}}{n + 1} \rfloor\). Each farmer’s input quantity, output quantity, and profit are increasing in \(s_H\).

iii) When the average input-to-output ratio \(\bar{z}\) increases while the variability of input-to-output ratio \(v_z\) remains constant, the overall industry input decreases [increases] for \(s_P \leq c_P\) and \(\bar{z} \geq (\alpha - c_H + s_H)/(2(c_P - s_P))\) [otherwise], the overall market output decreases [increases] for \(s_P \leq [>]c_P\), and the overall farmers’ profit decreases [increases] for \(s_P \leq [>]c_P\).

iv) When the variability of input-to-output ratio \(v_z\) increases while the average input-to-output ratio \(\bar{z}\) is kept constant, the overall industry input decreases [increases] for \(s_P \leq [>]c_P\), the overall market output does not change, and the overall farmers’ profit increases.
We explain Proposition 3 with the help of Figure 1. When \( s_P \leq c_P \), a more productive farmer (with a small \( z_j \)) produces more and makes more money, and the major insights observed from the case without subsidy in Proposition 1 remain.

While a larger harvesting subsidy incentivizes all farmers to plant more, harvest more, and profit more, a larger planting subsidy can discourage high-yield farmers to produce, as suggested by Proposition 3(ii). This is because a low-yield farmer needs a larger amount of input than a high-yield one for the same amount of output, and thus obtains a larger payment on plantation. Moreover, when \( s_P > c_P \), a farmer with a lower productivity level plants even more and harvests more than one with a higher productivity level, according to Proposition 3(i). In this case, the overall industry input, the overall market output, and the overall farmers’ profit always increase when \( \bar{z} \) increases, as suggested in Proposition 3(iii).

![Figure 1: The farmers’ production equilibrium under the combined subsidy \((s_P, s_H)\).](image)

**Notes.** \( \alpha = 3 \), \( \beta = 1 \), \( c_P = 0.2 \), \( c_H = 0.3 \), and \( z = (1, 1.5, 2.1, 2.8, 3.6) \).

Figure 1: The farmers’ production equilibrium under the combined subsidy \((s_P, s_H)\).

Proposition 3 characterizes the first-order effect of the subsidy program (i.e., how the farmers’ outputs and profits change with the subsidies). To understand the second-order effect (i.e., how the subsidies impact the distributions of the increased outputs and profits among the farmers), we evaluate the responses of the individual farmer’s output and profit to the increase of planting subsidy and harvesting subsidy. Specifically, for \( \delta > 0 \), define the output changes as

\[
\Delta_{s_P} q_j^C \equiv q_j^C(s_P + \delta, s_H) - q_j^C(s_P, s_H) \quad \text{and} \quad \Delta_{s_H} q_j^C \equiv q_j^C(s_P, s_H + \delta) - q_j^C(s_P, s_H),
\]

and the profit changes as

\[
\Delta_{s_P} \pi_j^C \equiv \pi_j^C(s_P + \delta, s_H) - \pi_j^C(s_P, s_H) \quad \text{and} \quad \Delta_{s_H} \pi_j^C \equiv \pi_j^C(s_P, s_H + \delta) - \pi_j^C(s_P, s_H).
\]

**Corollary 1 (Distribution of the Increased Output and Profit: Combined Subsidy)**

*Suppose all farmers produce positive quantities in equilibrium.*
i) **Planting subsidy:** $\Delta_{sp} q_j^C$ is increasing in $j$. When $c_P \leq s_{P1} < s_{P2}$, $q_j^C(s_{P2}, s_H) \geq_{wm} q_j^C(s_{P1}, s_H)$ and $\pi_j^C(s_{P2}, s_H) \geq_{wm} \pi_j^C(s_{P1}, s_H)$, and when $s_{P1} < s_{P2} \leq c_P$, $q_j^C(s_{P1}, s_H) \geq_{wm} q_j^C(s_{P2}, s_H)$.

ii) **Harvesting subsidy:** $\Delta_{sh} q_j^C$ is constant in $j$ and $\Delta_{sh} \pi_j^C$ is decreasing [increasing] in $j$ for $s_P \leq [>] c_P$. $q_j^C(s_P, s_{H2}) \geq_{wm} q_j^C(s_P, s_{H1})$ and $\pi_j^C(s_P, s_{H2}) \geq_{wm} \pi_j^C(s_P, s_{H1})$ for $s_{H1} < s_{H2}$.

Corollary 1 suggests that the farmers with lower productivity levels increase their outputs more when the planting subsidy is larger. This suggests that when $s_P < c_P$ (i.e., the farmers’ output levels are increasing in their productivity levels), an increased planting subsidy leads to a more balanced output distribution among the farmers. When $s_P > c_P$, the planting subsidy can overly remix the distribution, making the high-yield farmers produce less than the low-yield farmers. In this case, a higher planting subsidy leads to a more dispersed output distribution. It is worth noting that, when $s_P = c_P$, the farmers’ outputs are not affected by their productivity levels and thus all farmers produce the same amounts.

An increase in the harvesting subsidy leads to the same amount of output increase for all farmers. However, the high-yield farmers increase their profits more than the low-yield farmers only if the plantation is not overly subsidized (i.e., $s_P \leq c_P$). Overall, an increased harvesting subsidy widens the gaps among the farmers in both their outputs and profits.

The next result, examining the impact of productivity distribution, extends our earlier observation in Lemma 2.

**Proposition 4 (Distribution among Farmers: Combined subsidy)** Consider two otherwise identical markets indexed by $A$ and $B$ with the farmers in market $A$ exhibiting a more dispersed productivity distribution than those in market $B$ (measured by the majorization order), i.e., $z_A \geq^{m} z_B$. Suppose a combined subsidy $(s_P, s_H)$ is given and all farmers produce positive quantities. The following results hold.

i) $q_A^C(s_P, s_H) \geq_{wm} q_B^C(s_P, s_H)$, $\Delta_{sp} q_A^C \geq_{wm} \Delta_{sp} q_B^C$, and $\Delta_{sh} q_A^C =_{wm} \Delta_{sh} q_B^C$.

ii) $\pi_A^C(s_P, s_H) \geq_{wm} \pi_B^C(s_P, s_H)$, and $\Delta_{sh} \pi_A^C \geq_{wm} \Delta_{sh} \pi_B^C$.

Proposition 4 suggests that when the farmers’ productivity is more evenly distributed, their output levels and profits are more evenly distributed under the subsidy. Moreover, the increased
outputs and profits induced by the increase in the subsidies are more evenly distributed among
the farmers. The difference between the responses to the planting subsidies and the harvesting
subsidies lies in the fact that the effect of the former on the individual farmer’s output depends on
the farmer’s productivity, but the effect of the latter does not.

4.2 Selective Subsidy

Under a selective subsidy, the farmers are offered \((s_P, s_H)\), and each may choose to get paid based on
plantation or harvesting but not both. The individual farmer’s choice depends on the comparison
between the plantation payment \(z_j s_P\) and the harvesting payment \(s_H\). Certainly, a low-yield
farmer would prefer the planting subsidy and a high-yield farmer would prefer the harvesting
subsidy. Let \(m = \max\{j \in \hat{N} : z_j s_P \leq s_H\}\) (when \(z_1 > s_H/s_P\), we set \(m = 0\)) be the index
of the least productive farmer among those who prefer the harvesting subsidy. Let \(N^{S_p}(s_P, s_H)\)
and \(N^{S_h}(s_P, s_H)\) denote the set of active farmers who choose planting and harvesting subsidies,
respectively, and let \(N^S(s_P, s_H) = N^{S_p}(s_P, s_H) \cup N^{S_h}(s_P, s_H)\); see the detailed derivation of
Lemma 3 in Online Appendix A. As demonstrated in Figure 2, the farmers in set \(N^{S_p}(s_P, s_H)\) are
less productive than those in set \(N^{S_h}(s_P, s_H)\). When planting is overly subsidized (i.e., \(s_P > c_P\)),
it is possible that the farmers with intermediate levels of productivity choose not to produce.

![Segmentation of the farmers with the selective subsidy.](image)

Notes. \(\beta = 1, c_P = c_H = 0.3\) and \(\alpha = (1, 1.5, 2.1, 2.8, 3.6)\). In the left panel, \(\alpha = 2, s_P = 0.15,\n\sum s_H = 0.375\) and \(m = 3\). In the right panel, \(\alpha = 0.5, s_P = 0.35, s_H = 0.6\) and \(m = 2\).

Figure 2: Segmentation of the farmers with the selective subsidy.

**Proposition 5 (Production Equilibrium: Selective Subsidy)** Suppose that the government
implement a selective subsidy \((s_P, s_H)\). In equilibrium, farmer \(j\)'s output quantity is

\[
q^S_j(s_P, s_H) = \begin{cases} 
q^C_j(s_P, s_H) - \frac{1}{\beta} \left( z_j s_P - \sum_{i=1}^{n^{S,h}} z_i s_H + \frac{n^{S,h} s_H}{n+1} \right) & \text{for } j \in N^{S,h}(s_P, s_H), \\
0 & \text{for } j \in \bar{N} \setminus N^S(s_P, s_H), \\
q^C_j(s_P, s_H) - \frac{1}{\beta} \left( s_H - \sum_{i=1}^{n^{S,h}} z_i s_P + \frac{n^{S,h} s_P}{n+1} \right) & \text{for } j \in N^{S,P}(s_P, s_H),
\end{cases}
\]

and farmer \(j\)'s profit is

\[
\pi^S_j(s_P, s_H) = \beta \left( q^S_j(s_P, s_H) \right)^2, \quad j \in \bar{N}.
\]

The overall industry input is

\[
X^S(s_P, s_H) = X^C(s_P, s_H) - \frac{1}{\beta} \left( \sum_{i=1}^{n^{S,h}} z_i^2 s_P + \sum_{i \in N^{S,H}(s_P, s_H)} z_i s_H - \frac{n^z}{n+1} \left( \sum_{i=1}^{n^{S,h}} z_i s_P + n^{S,P} s_H \right) \right),
\]

and the overall market output is

\[
Q^S(s_P, s_H) = Q^C(s_P, s_H) - \frac{1}{\beta(n+1)} \left( \sum_{i=1}^{n^{S,h}} z_i s_P + n^{S,P} s_H \right),
\]

where \(n^{S,P} = |N^{S,P}(s_P, s_H)|\) and \(n^{S,h} = |N^{S,h}(s_P, s_H)|\). Moreover, the following results hold.

i) When \(s_P \leq c_P\), \(q^S_j(s_P, s_H)\) is decreasing in \(j\). When \(s_P > c_P\), \(q^S_j(s_P, s_H)\) is decreasing [increasing] in \(j\) for \(j < [>]m\) and \(q^S_{m+1}(s_P, s_H) \leq [\geq] q^S_m(s_P, s_H)\) for \((s_{m+1} s_P - s_H) + (s_m - z_{m+1})c_P \geq [\leq] 0\).

ii) Each farmer’s input quantity, output quantity, and profit are increasing [decreasing] in \(s_P\) when \(z_j \geq \frac{\sum_{i \in N^{S,P}(s_P,s_H)} z_i}{n+1}\) and \(j > m\) [otherwise]. Each farmer’s input quantity, output quantity, and profit are increasing [decreasing] in \(s_H\) when \(j \leq m\) [\(j > m\)].

The result in Proposition 5 is demonstrated in Figure 3. Unlike that under the combined subsidy, the market equilibrium under the selective subsidy depends on the entire vector \(z\) and can no longer be characterized by \(\bar{z}\) and \(v_z\) alone. When \(s_P \leq c_P\), the farmers’ outputs are increasing in their productivity levels. When \(s_P > c_P\), however, this is only true for high-yield farmers, who choose the harvesting subsidy. For the low-yield farmers who choose the planting subsidy, their outputs are decreasing in their productivity levels.

It is intuitive that an increased harvesting subsidy \(s_H\) benefits the farmers who choose the harvesting subsidy, but hurts those who choose the planting subsidy. An increased planting subsidy \(s_P\) certainly hurts the farmers who choose the harvesting subsidy, as the increase makes those
highly unproductive farmers more efficient in competition. Interestingly, not all farmers choosing
the planting subsidy benefit from an increased $s_P$. Rather, only the extremely unproductive ones
do.

Corollary 2 (Distribution of the Increased Output and Profit: Selective Subsidy) Suppose
all farmers produce positive quantities in equilibrium and $s_P \leq c_P$.

i) **Planting subsidy:** $\Delta s_P \Delta q_j^S$ is increasing in $j$. For $s_{P1} < s_{P2}$, $q^S(s_{P1}, s_H) \geq_{wm} q^S(s_{P2}, s_H)$.

ii) **Harvesting subsidy:** $\Delta s_H \Delta q_j^S$ is decreasing in $j$. For $s_{H1} < s_{H2}$, $q^S(s_P, s_{H2}) \geq_{wm} q^S(s_P, s_{H1})$, and
$\pi^S(s_P, s_{H2}) \geq_{wm} \pi^S(s_P, s_{H1})$.

Corollary 2 characterizes how the increased market output and farmers’ profit led by the in-
creased subsidies are distributed among the farmers. With an increased planting subsidy, the
low-yield farmers’ output levels increase and those of the high-yield farmers decrease. This is again
because only the low-yield farmers receive the planting subsidy, and they become more competitive
in the market as the planting subsidy increases. For a similar reason, when the harvesting subsidy
increases, the increased amount of market output is shared among the high-yield farmers.

Overall, an increased planting subsidy leads to more evenly distributed outputs among the
farmers, while an increased harvesting subsidy induces an increased dispersion among the farm-
ers’ outputs, provided that the plantation is not overly subsidized. This is consistent with our
observation under the combined subsidy from Corollary 1.

Now we examine the effect of productivity distribution when the selective subsidy is offered.

**Proposition 6 (Distribution among Farmers: Selective subsidy)** Consider two otherwise iden-
tical markets indexed by $A$ and $B$ with the farmers in market $A$ exhibiting a more disperse produc-
tivity distribution than those in market B (measured by the majorization order), i.e., $z_A \succeq^m z_B$.

Suppose a selective subsidy $(s_P, s_H)$ is given and all farmers produce positive quantities. Let $m_k = \max\{j \in \mathcal{N} : z_{kj} s_P \leq s_H\}$, $k = A, B$ (when $z_{k1} > s_H/s_P$, we set $m_k = 0$). We assume

a) $m_A \leq m_B$,

b) $\frac{1}{l} \sum_{j=1}^{l} (z_{Aj} - z_{Bj}) c_P \leq \frac{1}{m+1} \left( \sum_{i=m+1}^{n} (z_{Bi} - z_{Ai}) s_P - \sum_{i=m+1}^{m} (z_{Ai}s_P - s_H) \right)$ for $1 \leq l \leq m_A$.

Then the following results hold.

i) $q^S_A(s_P, s_H) \succeq_{wm} q^S_B(s_P, s_H)$ for $s_P \leq c_P$.

ii) $\pi^S_A(s_P, s_H) \succeq_{wm} \pi^S_B(s_P, s_H)$ for $s_P \leq c_P$.

Proposition 6 echoes the message from the combined subsidy in Proposition 4 that the overall market output and farmers’ profit are more evenly distributed among the farmers when their input-to-output ratios are more similar. The difference is that this conclusion is true for the selective subsidy with additional conditions. In particular, fewer farmers choose the harvesting subsidy over the planting subsidy when the productivity distribution becomes more dispersed (condition a). At the same time, the increased dispersion in productivity does not lead to a significant increase in plantation costs incurred by the high-yield farmers who choose the harvesting subsidy (condition b).

4.3 Comparisons

With the equilibrium outcomes derived in previous subsections, we compare different subsidy programs to understand the farmers’ incentives. To facilitate the comparison and keep the exposition clear, we would focus on the case that all farmers produce positive amounts in equilibrium.

**Proposition 7** Suppose that subsidy $(s_P, s_H)$ is given and can be administered in the form of either combined subsidy or selective subsidy. The following results hold.

i) There exists a $j^o \leq m$ such that $q^C_j(s_P, s_H) \leq >q^S_j(s_P, s_H)$ and $\pi^C_j(s_P, s_H) \leq >\pi^S_j(s_P, s_H)$ for $j \leq >j^o$.

ii) $q^S(s_P, s_H) \succeq_{wm} q^C(s_P, s_H)$ for $s_P \leq c_P$.

iii) There exists a $j_P$ such that $\Delta_{s_P} q^C_j \geq >\Delta_{s_P} q^S_j$ for $j \leq >j_P$, and $\Delta_{s_P} q^S \succeq_{wm} \Delta_{s_P} q^C$ if $(n + 1)z_1 \geq \sum_{i=1}^{m} z_i$. 

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iv) There exists a \( j_H \) such that \( \Delta_{s_H} q_j^C \leq [\Delta_{s_H} q_j^S \text{ for } j \leq [\Delta_{s_H} j_H , \text{ and } \Delta_{s_H} q^S \geq w_{s_H} \Delta_{s_H} q^C.} \)

According to Proposition 7, the highly productive farmers have a stronger incentive to increase their outputs when offered the choice over the subsidies than they do when offered the combination of the same subsidies. While these farmers get only one payment under the selective subsidy, they raise their production quantities because of the market competition—The selective subsidy induces lower outputs from low-yield farmers than the combined subsidy does.

Under the combined subsidy, not only the overall market output is more evenly distributed among the farmers, but also the increased amount of output induced by raising either planting subsidy or harvesting subsidy is more evenly distributed among the farmer, than under the selective subsidy. In other words, the combined subsidy is more effective in reducing the differences in the market presence and wealth among the farmers than the selective subsidy.

5  The Government’s Objectives and Subsidy Design

After characterizing the farmers’ equilibrium behaviors, we are now ready to analyze the government’s subsidy design. Suppose that the government plans a target market output level of \( \bar{Q} \), larger than the market output level without subsidizing any farmers. The government’s goal is to meet the target market output level through a carefully designed subsidy program, while minimizing the government spending on the program.

When a combined subsidy is offered, the government’s problem is

\[
\min_{s_P, s_H} \{ b^C(s_P, s_H) \equiv s_P X^C(s_P, s_H) + s_H Q^C(s_P, s_H) : Q^C(s_P, s_H) \geq \bar{Q} \},
\]

where \( X^C(s_P, s_H) \) and \( Q^C(s_P, s_H) \) are, respectively, defined in (4) and (5). When only the planting subsidy or only the harvesting subsidy is offered, we set \( s_H = 0 \) or \( s_P = 0 \), respectively, in the above formulation.

When a selective subsidy is offered, the government’s problem becomes

\[
\min_{s_P, s_H} \left\{ b^S(s_P, s_H) \equiv s_H \sum_{i \in N^{s_H}(s_P, s_H)} q_i^S(s_P, s_H) + s_P \sum_{i \in N^{s_P}(s_P, s_H)} z_i q_i^S(s_P, s_H) : Q^S(s_P, s_H) \geq \bar{Q} \right\},
\]

where \( Q^S(s_P, s_H) \) is defined in (10).

5.1  The Optimal Subsidy Design

In this subsection, we characterize the government’s optimal subsidy design and discuss the resulting policy implications.
Proposition 8 (Combined Subsidy: Subsidy Design) The government’s optimal design of the combined subsidy is characterized as follows:

i) When only the planting subsidy is offered, \( s_P^P = \left( \frac{\beta(n^P + 1)}{n^P} \bar{Q} - \bar{a}(0,0) \right) / \bar{z} \), where \( n^P = |N^C(s_P^P, 0)| \).

ii) When only the harvesting subsidy is offered, \( s_H^H = \frac{\beta(n^H + 1)}{n^H} \bar{Q} - \bar{a}(0,0) \), where \( n^H = |N^C(0, s_H^H)| \).

iii) When the combined subsidy is offered, \( s_C^P = c_P / 2 \) and \( s_C^H = \frac{\beta(n^C + 1)}{n^C} \bar{Q} - \bar{a}(0,0) - \frac{c_P \bar{z}}{2} \), where \( n^C \) is defined in Lemma 2.

It is known in the context of subsidizing a single producer (e.g., Taylor and Xiao 2014, Berenguer et al. 2017) that if the ratio of harvesting subsidy to planting subsidy equals the producer’s input-to-output ratio, then implementing either subsidy leads to the same output level. Proposition 8 generalizes this result to multiple producers and to the combined subsidy. In particular, when the set of active farmers stays the same, \( s_C^P \bar{z} + s_C^H = s_H^H = s_P^P \bar{z} \). Observe that the dispersion of farmers’ productivity does not affect the per-unit subsidies needed to achieve the target output level.

We further note from Proposition 8 that the market competition has a major impact on the government’s subsidy design. When the market price is more sensitive to the overall output level (i.e., when \( \beta \) is large), the government needs to subsidize more on per-unit planting or harvesting quantity. When the number of active producers increases, however, the government would reduce the per-unit subsidy (as \( \frac{(n^k + 1)}{n^k} \) is decreasing in \( n^k, k \in \{P, H, C\} \)).

An important implication from Proposition 8, which we highlight in the corollary below, is the coordination of subsidizing and taxing in the policy design.

Corollary 3 (Combined Subsidy: Tax-Subsidy Regime) Under the combined subsidy program, the government should tax on harvesting, i.e., \( s_H^H < 0 \), when \( \bar{Q} < Q^C(c_P/2, 0) \). Moreover, there exists a \( Q^T = \frac{n}{2\beta(n+1)}(\bar{a}(0,0) + \sqrt{\bar{a}^2(0,0) + (n+1)v_z c_P^2}) \) such that a positive government income is generated through the subsidy program when \( \bar{Q} < Q^T \).

Corollary 3 suggests that when the industry capability is not too much below the market need, the government should subsidize on the input, while taxing on the output. This is demonstrated through an example in Figure 4. Under such a policy, the government helps the low-yield farmers through the planting subsidy, while financing the subsidy spending through the tax contribution from the high-yield farmers.
Combined tax-subsidy policies are often observed. For example, many governments including India\(^7\) and U.S.\(^8\) offer tax exemptions or credits to low-income farmers. Combined subsidy-tax interventions are also applied in other contexts including pollution reduction (e.g., Fullerton and Mohr 2002), food production and consumption (e.g., Abadie et al. 2016), and biofuel promotion (e.g., Cui et al. 2011). The observation from Corollary 3 complements this literature by highlighting that the subsidy-tax intervention allows the government to achieve the primary goal with efficient budget planning and effective redistribution in the society.

![Optimal Combined Subsidy](image1)

**Figure 4:** The optimal subsidies and budget with respect to the target \(\bar{Q}\): The combined subsidy.

![Optimal Selective Subsidy](image2)

**Figure 5:** The optimal subsidies and budget with respect to the target \(\bar{Q}\): The selective subsidy.

**Proposition 9 (Selective Subsidy: Subsidy Design)** The government’s optimal design of the selective subsidy satisfies the following:

i) When \(\bar{Q} < Q^S(c_P, 0)\), all farmers choose the planting subsidy, i.e., \(N^{S,h}(s^S_P, s^S_H) = \emptyset\). In this case, \(s^S_P = \left(\beta(n^S + 1)\bar{Q} - \bar{a}(0, 0)\right)/\bar{z}\) and \(s^S_H\) can be any value within \([0, z_1s^S_P]\), where 

\(^7\)See https://cleartax.in/s/agricultural-income (last accessed September 21, 2022)

\(^8\)See https://www.ers.usda.gov/topics/farm-economy/federal-tax-issues/ (last accessed September 21, 2022)
ii) When $\bar{Q} \geq Q^S(c_P/2,0)$, some farmers choose the harvesting subsidy, i.e., $N^S:h(s_P^S, s_H^S) \neq \emptyset$.

In this case, the subsidies are given in Lemma 4 in Online Appendix A.

The format of the subsidy offered depends on the target market output $\bar{Q}$ set by the government. When $\bar{Q} < Q^S(c_P/2,0)$, the government would make the harvesting subsidy unattractive enough, i.e., $s_H \in [0, z_1 s_P^S]$, so that all farmers choose to get compensated for plantation. In this case, the problem under the selective subsidy reduces to that under the planting subsidy in Proposition 8(i).

When the government imposes an aggressive target, i.e., $\bar{Q} \geq Q^S(c_P/2,0)$, the government’s objective function is not jointly concave in the subsidies $(s_P, s_H)$. There can be multiple local minima for the government’s problem, as suggested by Proposition 9 (ii).

To understand the government’s design of the selective subsidy, we refer to an example depicted in Figure 5. When the target is not too far from the market output without any subsidy, the policy focuses on incentivizing the low-yield farmers to produce more to raise the overall market output. In this case, the offer of the planting subsidy is more attractive than that of the harvesting subsidy. To meet a high target market output, however, the government needs to rely on the high-yield farmers to bring additional quantities to the market. In this case, a larger subsidy payment is offered for harvesting. We also note the ratio $s_H^S/s_P^S$ increases as the target output level increases. This suggests that the high-yield farmers gain an increasing advantage in competition and have an increasing incentive to produce compared with the low-yield farmers, as the former would choose the harvesting subsidy and the latter would choose the planting subsidy.

An observation from Propositions 8 and 9 is that the average subsidy per unit of output stays the same across the four subsidy programs (i.e., $s_P^C \bar{z} + s_H^C = s_H^S = s_P^H = (n^S:s_H^S + \sum_{i \in N^S:p(s_P^S, s_H^S)} z_i s_P^S/n^S)$ with the same set of active farmers. However, it is worth highlighting that the actual subsidy payment made by the government can be significantly different because the farmers’ input-to-output ratios can be higher or lower than $\bar{z}$. The government’s spending in each subsidy program is analyzed in detail in the next section.

Our earlier discussion in Section 4 suggests that some choice of subsidies may attract more farmers to enter the market or drive some farmers out of the market. The next corollary confirms that these situations do arise when the government implements the most budget-efficient subsidies.

**Corollary 4 (Farmers’ Participation)** Under the government’s optimal subsidy design to meet a target output $Q$, the set of active farmers is characterized as follows:
i) Under the planting subsidy, \( N^C(s^p_P, 0) = \{1, \ldots, n^P\} \) for \( \bar{Q} \leq Q^C(c_P, 0) \) (with \( n^P = \bar{n} \) when \( \bar{Q} = Q^C(c_P, 0) \)) and \( N^C(s^p_P, 0) = \{\bar{n} - n^P + 1, \ldots, \bar{n}\} \) for \( \bar{Q} > Q^C(c_P, 0) \).

ii) Under the harvesting subsidy, \( N^C(0, s^H_H) = \{1, \ldots, n^H\} \).

iii) Under the combined subsidy, \( N^C(s^C_P, s^C_H) = \{1, \ldots, n^C\} \).

Moreover, \( n^P \) is increasing [decreasing] in \( \bar{Q} \) for \( \bar{Q} \leq [>]Q^C(c_P, 0) \), and \( n^H \) and \( n^C \) are increasing in \( \bar{Q} \).

![Diagram of planting subsidy, harvesting subsidy, combined subsidy, and selective subsidy](image)

Notes. \( \times \) indicates farmer \( j \) receives the plantation subsidy, \( \circ \) indicates farmer \( j \) receives the harvesting subsidy.

Figure 6: An illustration of active farmers for different values of target \( \bar{Q} \).

The result in Corollary 4 is demonstrated in Figure 6. Intuitively, subsidies would reduce the farmers’ costs and induce production. This intuition, however, is not true when only the planting subsidy is offered or when the selective subsidy is offered.

### 5.2 Key Performance Indicators of the Subsidy Programs

Though raising output is a primary goal, the government may evaluate the effectiveness of the subsidy design along other dimensions including budget spending, resource usage, wealth distribution, and social welfare. In this subsection, we examine these performance indicators.

**Proposition 10 (Budget Comparison)** Suppose that the government sets a target output \( \bar{Q} \) and all farmers produce.

\[
i) b^C(s^C_P, s^C_H) \leq b^S(s^S_P, s^S_H) \leq b^C(s^P_P, 0) \leq b^C(0, s^H_H) \text{ for } \bar{Q} \leq Q^C(c_P, 0). 
\]

\[
ii) b^C(s^C_P, s^C_H) \leq b^S(s^S_P, s^S_H) \leq b^C(0, s^H_H) \leq b^C(s^P_P, 0) \text{ for } \bar{Q} > Q^C(c_P, 0). 
\]

Moreover, \( b^C(s^P_P, 0) = b^C(0, s^H_H) = b^C(s^C_P, s^C_H) = b^S(s^S_P, s^S_H) \) for \( v_z = 0 \).

In the special case where all farmers produce with the same yields, i.e., \( z_j = \bar{z} \) or \( v_z = 0 \), the budgets required by all of the four subsidy programs are equivalent. This is in line with our
observation from Propositions 8 and 9 that the average subsidy per unit of output stays the same across all programs.

When the farmers vary in their productivity, they would receive different payments per unit of output. Consequently, the government spending on subsidy programs depends on the format of the subsidy. Proposition 10 suggests that the government gains the most flexibility in policy implementation, leading to the least budget required. The two-subsidy format, either in combination or in selection, requires less government spending than the single-subsidy format, either in planting or in harvesting. This is intuitive as the government can better incentivize the farmers with a two-parameter policy than with a one-parameter policy.

When a single-subsidy format is administered, a lower budget is needed for the planting subsidy when the target market output is low, while a lower budget is required for the harvesting subsidy when the target market output is high. This observation echoes our earlier discussion in Corollary 4 that the planting subsidy provides a strong incentive for low-yield farmers to produce, while the harvesting subsidy stimulates the high-yield farmers. A low target can be met with small spending on planting to raise the production level by the low-yield farmers, while a high target must be achieved with significant spending on harvesting to incentivize the high-yield farmers to bring additional quantities to the market.

Proposition 11 (Resource Requirement) Suppose that the government sets a target output $\bar{Q}$ and all farmers produce.

i) The industry average input-to-output ratio under any subsidy program is higher than that without subsidy. That is, $X^C(0, s^H_H)/\bar{Q}$, $X^C(s^P_P, 0)/\bar{Q}$, $X^C(s^C_C, s^C_H)/\bar{Q}$ and $X^S(s^S_P, s^S_H)/\bar{Q}$ are higher than $X^*/Q^*$.

ii) When $\bar{Q} \leq Q^C(c_p/2, 0)$, $X^C(0, s^H_H) \leq X^S(s^S_P, s^S_H) = X^C(s^P_P, 0) \leq X^C(s^C_C, s^C_H)$. When $\bar{Q} > Q^C(c_p/2, 0)$, $X^C(0, s^H_H) \leq [X^C(s^C_C, s^C_C), X^S(s^S_P, s^S_H)] \leq X^C(s^P_P, 0)$.

The amount of input used for achieving a target output is a key indicator for efficient resource utilization. There are many reported instances in which the government subsidies induce overly used resources, which not only results in waste but also leads to irreversible side effects (see, e.g., Zhu et al. 2016, Chen et al. 2017). Proposition 11(i) suggests that the subsidy programs always lead to increased average resource usage per unit of output. This is understandable because, without any subsidy, the high-yield farmers compete more effectively and produce more than the low-yield
ones, lowering the overall industry input-to-output ratio. The introduction of subsidies reduces the farmers’ costs, increasing the average amount of resources needed for each unit of output.

To understand Proposition 11(ii), we refer to the example depicted in Figure 7. The implementation of the harvesting subsidy always requires the least input consumption. This is because the harvesting subsidy enhances the competitiveness of the high-yield farmers, as opposed to that of the low-yield farmers. Consequently, the high-yield farmers bring more outputs to the market, reducing the industry-wide input-to-output ratio.

When the target output level is low, i.e., $\bar{Q} \leq Q^C(c_P/2, 0)$, the implementation of the combined subsidy leads to the highest overall industry input. This is because the combination of the planting subsidy and the harvesting tax (recall Corollary 3) discourages the high-yield farmers to generate large outputs. As a result, a big portion of the market output is contributed by low-yield farmers, who consume a large amount of input.

When the target output level is high, i.e., $\bar{Q} > Q^C(c_P/2, 0)$, in contrast, the harvesting subsidy, if offered, is always positive. The high-yield farmers, when receiving payments for harvesting, have a strong incentive to raise their production levels, reducing the industry-wide input-to-output ratio. The low-yield farmers, when receiving compensations from plantation, tend to increase their production levels, increasing the industry-wide input-to-output ratio. Therefore, the input consumption is the highest under the planting subsidy and lowest under the harvesting subsidy. The input level under the combined subsidy or the selective subsidy is somewhere in between. We also note that when the target output level becomes significantly high, the industry-wide input-to-output ratio under the planting subsidy can even exceed the average input-to-output ratio $\bar{z}$. This, again, underscores our earlier observation from Corollary 4 that the planting subsidy can induce significant waste on resource usage and hurt the overall productivity.

![Figure 7: Overall industry input comparison.](https://ssrn.com/abstract=4378693)
Proposition 12 (Distribution among Farmers) Suppose that the government sets a target output $\bar{Q}$ and all farmers produce without any subsidy.

i) For $\bar{Q} \leq Q^C(c_p/2, 0)$, $q^C(0, s_H^H) \geq m q^C(s_p^C, 0) = m q^S(s_p^S, s_H^S) \geq m q^C(s_p^C, s_H^C)$. For $Q^C(c_p/2, 0) < \bar{Q} \leq Q^C(c_p, 0)$, $q^C(0, s_H^H) \geq m [q^C(s_p^C, s_H^C), q^S(s_p^S, s_H^S)] \geq m q^C(s_p^C, 0)$. For $\bar{Q} > Q^C(c_p, 0)$, $q^C(0, s_H^H) \geq m q^C(s_p^C, s_H^C).

ii) For $\bar{Q} \leq Q^C(c_p/2, 0)$, $\pi^C(0, s_H^H) \geq \pi^C(s_p^P, 0) = m \pi^S(s_p^S, s_H^S) \geq m \pi^C(s_p^C, s_H^C)$. For $Q^C(c_p/2, 0) < \bar{Q} \leq Q^C(c_p, 0)$, $\pi^C(0, s_H^H) \geq \pi^C(s_p^C, s_H^C), \pi^S(s_p^S, s_H^S)] \geq \pi^C(s_p^C, 0)$. For $\bar{Q} > Q^C(c_p, 0)$, $\pi^C(0, s_H^H) \geq \pi^C(s_p^C, s_H^C).

The example in Figure 8 demonstrates the results described in Proposition 12. It is immediate to observe that the harvesting subsidy can induce a large disparity among the farmers in their outputs and profits. If poverty reduction is one of the purposes of the subsidy program, funding a harvesting subsidy can have adversarial effects.

When the output target is set low, the combined subsidy allows for the most evenly distributed outputs and profits among the farmers. This is achieved through taxing the high-yield farmers to subsidize the low-yield farmers. This observation makes an interesting contrast to the conclusion obtained by Tang et al. (2019). In their analysis of a two-farmer model, they find that it is never optimal for the government to offer the combined subsidy when the government concerns the profit difference between the farmers.

When the target output level is in an intermediate range, the planting subsidy leads to the most balanced output and profit distributions among the farmers. As the target output level increases, the government offers an increased planting subsidy. When the target output level becomes too high, the planting subsidy exceeds the planting cost, making the high-yield farmers completely lose their advantages in competition. In this case, either the combined subsidy or the selective subsidy can be more effective than the planting subsidy in reducing the discrepancies in outputs and profits among the farmers.

At an aggregate level, it is important for the government to understand the implication of a subsidy program on the overall well-being of the society. A commonly used measure for that is the social welfare. This requires the evaluation on the overall farmers’ profit, the consumer surplus, and the government budget. Specifically, given subsidies $(s_p, s_H)$, the social welfare is defined as

$$W_X(s_p, s_H) = \Pi_X(s_p, s_H) + \frac{\beta}{2} (Q_X(s_p, s_H))^2,$$

and the net social welfare is defined as $NW_X(s_p, s_H) = W_X(s_p, s_H) - b_X(s_p, s_H)$ with $X \in \{C, S\}$. 

Electronic copy available at: https://ssrn.com/abstract=4378693
Notes. $\alpha = 4$, $\beta = 1$, $c_P = c_H = 0.3$, $z = (1, 1.5, 2, 8, 3.6)$.

Figure 8: The farmers’ output distribution with respect to the target output level.

**Proposition 13 (Social Welfare)** Suppose that the government sets a target output $\bar{Q}$ and all farmers produce.

i) For $\bar{Q} \leq Q^C(\frac{c_P}{2}, 0) \text{ or } Q^C(\frac{3c_P}{2}, 0) \leq \bar{Q} \leq Q^C(2c_P, 0)$, $W_C(s_P^C, s_H^C) \leq W_C(s_P^P, 0) \leq W_C(0, s_H^H)$. For $Q^C(\frac{c_P}{2}, 0) \leq \bar{Q} \leq Q^C(\frac{3c_P}{2}, 0)$, $W_C(s_P^P, 0) \leq W_C(s_P^C, s_H^C) \leq W_C(0, s_H^H)$. For $\bar{Q} \geq Q^C(2c_P, 0)$, $W_C(s_P^C, s_H^C) \leq W_C(0, s_H^H) \leq W_C(s_P^P, 0)$.

ii) For $\bar{Q} \leq Q^C(\frac{c_P}{2}, 0)$, $NW_C(s_P^C, s_H^C) \leq NW_C(s_P^P, 0) \leq NW_C(0, s_H^H)$. For $\bar{Q} \geq Q^C(\frac{c_P}{2}, 0)$, $NW_C(s_P^P, 0) \leq NW_C(s_P^C, s_H^C) \leq NW_C(0, s_H^H)$.

We explain Proposition 13 with the reference to Figure 9. To make a clear comparison, we use the combined subsidy as a benchmark (which corresponds to the constant line crossing 1).

We observe that the harvesting subsidy is generally efficient, measured by both the social welfare and the net social welfare. This is in line with the observation that the harvesting subsidy leads to the highest industry-wide productivity with the lowest input resource consumption (recall Proposition 11). The exception occurs when the target output level becomes very high. In this case, the planting subsidy generates the highest social welfare because of the significantly increased income earned by the low-yield farmers. However, the high social welfare is at a cost of large government spending, resulting in a low net social welfare. By the same token, the planting subsidy always leads to the lowest net social welfare unless the output target is set low.

The comparison between the combined subsidy and the selective subsidy is consistent for the social welfare and for the net welfare. The selective subsidy tends to perform better than the combined subsidy except that the target output level is within a range of intermediate values.
6 Discussions and Concluding Remarks

We study the government-led subsidy programs offered to farmers to raise the market output for an agricultural product. Given that the farmers’ market is fragmented and the farmers differ from one another in their productivity levels, the farmers’ production incentive depends not only on the profitability of the product, but also on their input-to-output ratios in relation to the industry distribution. With or without subsidy, a more balanced distribution of input-to-output ratios always leads to more evenly distributed outputs and profits among the farmers. All farmers always obtain more profits when the government subsidizes more on harvesting. Interestingly, depending on the extent to which the government subsidizes on plantation, the farmers may or may not make more profit when being subsidized more on plantation.

Though we analyze output-oriented budget planning, the government’s subsidy design often needs to weigh against other considerations including resource usage, disparity among the farmers and net social welfare; see Table 1. Our analysis suggests that the planting subsidy shows the advantages of minimizing the discrepancies among the farmers in their outputs and profits when an intermediate target for the market output is set. However, the planting subsidy can lead to the highest government spending, the largest waste on input resources, and the lowest net social welfare when an aggressive output target is set. In contrast, the harvesting subsidy performs the best on resource usage and welfare generation, but creates the highest disparity among the farmers. The combined subsidy wins over other subsidy formats in minimizing the budget and balancing the profit distribution, but loses on efficient resource usage and welfare generation under a low output target. It is worth noting that the selective subsidy does not stand out with any of these performance measures, nor is it the bottom performer. These observations, shedding light on the farmers’ responses to the policies, provide practical guidance to the subsidy design.
Our analysis is based on fixed farmer yields. Consideration of yield uncertainty would certainly enrich the insights. The challenge of analyzing uncertain yields lies in the non-uniqueness of the dependent structure (e.g., Feng et al. 2019) and the stochastic majorization orders (e.g., Feng and Shanthikumar 2018a), and there are only limited developments for centralized resource planning in the literature. In Online Appendix B, we show that the main results derived from our analysis remain when the random yields are positively dependent and satisfy majorization order with respect to the increasing concave order. Significant technical development is needed in this direction to understand the effect of yield uncertainty and correlation. Another natural consideration under yield uncertainty is the risk preference of individual farmers. The existing work (e.g., Peng and Pang 2019, Ye et al. 2020) in this context heavily concentrates on the single-producer settings. Analysis of general risk measures based on the heterogeneity among the producers’ risk preferences can generate new understandings of subsidy design for distributed markets.

It is possible to consider alternative cost structures or alternative means of encouraging production to understand the producers’ incentives. In Online Appendix C, we show that under a convex production cost, capturing diminishing returns of input resources, the farmers with intermediate productivity levels can have a stronger production incentive than those with low or high productivity levels. One could also examine alternative subsidy schemes like Price Loss Coverage, Agriculture Risk Coverage, and supported prices (Alizamir et al. 2019, Gupta et al. 2017, Guda et al. 2021).

**Acknowledgments**

This work was partially supported by the National Natural Science Foundation of China [Grants 72032001, 71972071] and the Shanghai Sailing Program [Grant 22YF1451000].
References


A Proofs of Formal Results

Proof of Lemma 1. According to Proposition 1, $q^*_j$ is decreasing in $j$. For farmer $j \in \bar{N}$ to participate production, his output quantity should be positive even when farmers $j+1, j+2, \ldots, \bar{n}$ quit the market. By Proposition 1, farmer $j$’s output quantity should be

$$q^*_j = \frac{1}{\beta} \left( \frac{\alpha - c_H - (\frac{1}{j} \sum_{i=1}^{j} z_i) c_P}{j + 1} - \left( z_j - \frac{1}{j} \sum_{i=1}^{j} z_i \right) c_P \right).$$

$q^*_j > 0$ gives $g(j) < (\alpha - c_H)/c_P$, where $g(j) = (j + 1)z_j - \sum_{i=1}^{j} z_i$. For $j < \bar{n}$, $g(j + 1) - g(j) = (j + 1)(z_{j+1} - z_j) \geq 0$. Thus, $g(j)$ is increasing in $j$. □

Proof of Proposition 1. The results follow from Proposition 3 by setting $s_P = s_H = 0$. □

Proof of Proposition 2. We shall remark that the majorization order is preserved under the affine transformation, and the sub-majorization order is preserved under the increasing convex transformation; see Theorem A.1.f. and Theorem A.2.(i) in Marshall et al. (1979). Thus, $q_A^* \succeq_m q_B^*$ follows from the fact that $q_j^*$ is affine in $z_j$ for all $j \in \bar{N}$, given $\bar{z}$ is fixed. $\pi_A^* \succeq_m \pi_B^*$ is then obvious because $\pi_j^* = \beta(q_j^*)^2$ for all $j \in \bar{N}$. □

Lemma 2 (Condition for Production: Combined Subsidy) Farmer $j$ produces a positive amount under a combined subsidy $(s_P, s_H)$ if and only if

$$j \in N_C(s_P, s_H) = \left\{j \in \bar{N} : I_{\{s_P \leq c_P\}}g(j) + I_{\{s_P > c_P\}}\tilde{g}(j) < \frac{\alpha - c_H + s_H}{c_P - s_P}\right\},$$

where $g(j)$ is defined in Lemma 1 and $\tilde{g}(j) = \sum_{i=j}^{\bar{n}} z_i - (\bar{n} - j + 2)z_j$ is decreasing in $j$. The number of active farmers satisfies

$$n_C = |N_C(s_P, s_H)| = \begin{cases} \max\{N_C(s_P, s_H)\} & \text{if } s_P \leq c_P, \\ \bar{n} + 1 - \min\{N_C(s_P, s_H)\} & \text{if } s_P > c_P. \end{cases}$$
Proof of Lemma 2. According to Proposition 3, \( q_j^* \) is decreasing in \( j \) for \( s_P \leq c_P \). Following the lines of the proof of Lemma 1, we have \( g(j) < (\alpha - c_H + s_H)/(c_P - s_P) \). For \( s_P > c_P \), \( q_j^* \) is increasing in \( j \). For farmer \( j \in \bar{N} \) to produce, his output quantity should be positive even when farmers \( 1, 2, \ldots, j-1 \) quit the market. By Proposition 3, farmer \( j \)'s output quantity should be

\[
q_j^C = \frac{1}{\beta} \left( \alpha - (c_H - s_H) - \frac{1}{\bar{n} - j + 2} \sum_{i=j}^{\bar{n}} \zeta_i (c_P - s_P) \right) - \left( z_j - \frac{1}{\bar{n} - j + 1} \sum_{i=j}^{\bar{n}} \zeta_i (c_P - s_P) \right).
\]

\( q_j^C > 0 \) gives \( \bar{g}(j) < (\alpha - c_H + s_H)/(c_P - s_P) \), where \( \bar{g}(j) = \sum_{i=j}^{\bar{n}} \zeta_i - (\bar{n} - j + 2)z_j \). For \( j < \bar{n} \), \( \bar{g}(j + 1) - \bar{g}(j) = (\bar{n} - j + 1)(z_j - z_{j+1}) \leq 0 \). Thus, \( \bar{g}(j) \) is decreasing in \( j \).

Proof of Proposition 3. For farmer \( j \in \bar{N}\setminus N^C(s_P, s_H) \), it is clear that \( q_j^C(s_P, s_H) = 0 \) and \( \pi_j^C(s_P, s_H) = 0 \). For farmer \( j \in N^C(s_P, s_H) \), \( \pi_j \) is concave in \( q_j \) and the first-order condition of \( q_j \) gives

\[
q_j(s_P, s_H) = \frac{1}{2\beta} \left( a_j(s_P, s_H) + \beta q_j(s_P, s_H) - \beta \sum_{i \in N^C(s_P, s_H)} q_i(s_P, s_H) \right).
\]

The equilibrium is thus obtained by solving the above system of linear equations:

\[
q_j^C(s_P, s_H) = \frac{1}{\beta} \left( a_j(s_P, s_H) - \frac{1}{n+1} \sum_{i \in N^C(s_P, s_H)} a_i(s_P, s_H) \right) = \frac{1}{\beta} \left( \frac{\bar{a}(s_P, s_H)}{n+1} - (z_j - \bar{z})(c_P - s_P) \right).
\]

It follows that the farmer \( j \)'s equilibrium profit can be computed as

\[
\pi_j^C(s_P, s_H) = \beta(q_j^C(s_P, s_H))^2 = \frac{1}{\beta} \left( \frac{\bar{a}(s_P, s_H)}{n+1} - (z_j - \bar{z})(c_P - s_P) \right)^2.
\]

The overall industry input is

\[
X^C(s_P, s_H) = \sum_{i \in N} \zeta_i q_i^C(s_P, s_H) = \frac{n}{\beta(n+1)} (\bar{z}\bar{a}(s_P, s_H) - (n+1)v_2(c_P - s_P)),
\]

and the overall market output is

\[
Q^C(s_P, s_H) = \sum_{i \in N} q_i^C(s_P, s_H) = \frac{1}{\beta(n+1)} \sum_{i \in N} a_i(s_P, s_H) = \frac{n}{\beta(n+1)} \bar{a}(s_P, s_H),
\]

and the farmers’ profit is

\[
\Pi^C(s_P, s_H) = \sum_{i \in N} \pi_i^C(s_P, s_H) = \frac{n}{\beta(n+1)^2} (\bar{a}^2(s_P, s_H) + (n+1)^2v_2(c_P - s_P)^2).
\]

To show part (i), we have, for any \( j, j + 1 \in N^C(s_P, s_H) \),

\[
q_{j+1}(s_P, s_H) - q_j^C(s_P, s_H) = \frac{1}{\beta} (c_P - s_P)(z_j - z_{j+1}) \begin{cases} 
\leq 0 & \text{for } s_P \leq c_P, \\
\geq 0 & \text{for } s_P > c_P.
\end{cases}
\]
Note that when \( s_P \leq c_P \), \( q_j^C(s_P, s_H) = 0 \) for \( j \in \tilde{N} \setminus \tilde{N}^C(s_P, s_H) = \{ n + 1, n + 2, \ldots, \tilde{n} \} \); when \( s_P > c_P \), \( q_j^C(s_P, s_H) = 0 \) for \( \tilde{N} \setminus \tilde{N}^C(s_P, s_H) = \{ 1, 2, \ldots, \tilde{n} - n \} \). Thus, \( q_j^C(s_P, s_H) \) is decreasing [increasing] in \( j \) for \( s_P \leq [>]c_P \). We conclude part (i).

To show part (ii), we differentiate (2) with respect to \( s_P \) and \( s_H \), respectively, to obtain

\[
\frac{\partial q_j^C(s_P, s_H)}{\partial s_P} = \frac{(n + 1)z_j - n\bar{\gamma}}{\beta(n + 1)} \quad \text{and} \quad \frac{\partial q_j^C(s_P, s_H)}{\partial s_H} = \frac{1}{\beta(n + 1)}.
\]

It is clear that farmer \( j \)'s output quantity is increasing [decreasing] in \( s_P \) for \( z_j \geq [\leq]n\bar{\gamma}/(n + 1) \) and is increasing in \( s_H \). Since the input quantity and the output quantity vary by a scale of \( z_j \), farmer \( j \)'s input quantity exhibits the same monotone property as \( q_j^C \) with respect to \( s_P \) and \( s_H \). Moreover, from (2) and (3), we have \( \pi_j^C(s_P, s_H) = \beta(q_j^C(s_P, s_H))^2 \). Thus, farmer \( j \)'s profit exhibits the same monotone property as \( q_j^C \) with respect to \( s_P \) and \( s_H \). Hence we obtain part (ii).

To see part (iii), we differentiate (4) with respect to \( \bar{\gamma} \) to obtain

\[
\frac{\partial X_j^C(s_P, s_H)}{\partial \bar{\gamma}} = \frac{n}{\beta(n + 1)} \left( 2(s_P - c_P)\bar{\gamma} + \alpha - c_H + s_H \right).
\]

Thus, \( X_j^C(s_P, s_H) \) is increasing in \( \bar{\gamma} \) when \( s_P > c_P \). For \( s_P \leq c_P \), \( X_j^C(s_P, s_H) \) is increasing [decreasing] in \( \bar{\gamma} \) when \( \bar{\gamma} \leq [\geq](\alpha - c_H + s_H)/(2(c_P - s_P)) \).

Now differentiating (5) with respect to \( \bar{\gamma} \) yields

\[
\frac{\partial Q_j^C(s_P, s_H)}{\partial \bar{\gamma}} = \frac{n(s_P - c_P)}{\beta(n + 1)}.
\]

It is clear that \( Q_j^C(s_P, s_H) \) is decreasing [increasing] in \( \bar{\gamma} \) for \( s_P \leq [>]c_P \).

Now differentiating (6) with respect to \( \bar{\gamma} \) yields

\[
\frac{\partial \Pi_j^C(s_P, s_H)}{\partial \bar{\gamma}} = \frac{n}{\beta(n + 1)^2} \left( 2\bar{\gamma}(s_P, s_H)(s_P - c_P) \right).
\]

It is clear that \( \Pi_j^C(s_P, s_H) \) is decreasing [increasing] in \( \bar{\gamma} \) for \( s_P \leq [>]c_P \). We conclude part (iii).

To see part (iv), we note from (5) that \( Q_j^C(s_P, s_H) \) is independent of \( v_\bar{\gamma} \). Differentiating (4) with respect to \( v_\bar{\gamma} \) yields

\[
\frac{\partial X_j^C(s_P, s_H)}{\partial v_\bar{\gamma}} = \frac{n}{\beta}(s_P - c_P).
\]

Thus, \( X_j^C(s_P, s_H) \) is decreasing [increasing] in \( v_\bar{\gamma} \) for \( s_P \leq [>]c_P \).

Differentiating (6) with respect to \( v_\bar{\gamma} \) yields

\[
\frac{\partial \Pi_j^C(s_P, s_H)}{\partial v_\bar{\gamma}} = \frac{n}{\beta}(s_P - c_P)^2.
\]
Thus, $\Pi^C(s_p, s_H)$ is increasing in $v_z$. We conclude part (iv).

Proof of Corollary 1. To see part (i), we have

$$\Delta_{s_p}q_j^C = \frac{\delta}{\beta}(z_j - \frac{n}{n + 1}z)$$

Thus, $\Delta_{s_p}q_j^C$ is increasing in $j$. Next, we note that

$$\sum_{j=1}^{n} \Delta_{s_p}q_j^C = \frac{\delta}{\beta} \left(\sum_{j=1}^{n} z_j - \frac{(n-1)n}{n + 1}z\right) > 0.$$ 

Because $q_j^C(s_p, s_H)$ is decreasing in $j$ for $s_p \leq c_p$, the above inequality implies $q_j^C(s_p, s_H)$ weakly sup-majorizes $q_j^C(s_p, s_H)$ when $s_{p1} < s_{p2} \leq c_p$. Because $q_j^C(s_p, s_H)$ is increasing in $j$ for $s_p > c_p$, $q_j^C(s_p, s_H)$ weakly sub-majorizes $q_j^C(s_p, s_H)$ when $c_p \leq s_{p1} < s_{p2}$. It then follows that $\pi^C(s_p, s_H)$ weakly sub-majorizes $\pi^C(s_p, s_H)$ as sub-majorization order is preserved under the increasing convex transformation. We conclude part (i).

To see part (ii), we have

$$\Delta_{s_h}q_j^C = \frac{\delta}{\beta(n + 1)} \quad \text{and} \quad \Delta_{s_h}\pi_j^C = \frac{2\delta}{(n + 1)} q_j^C(s_p, s_H + \delta/2).$$

It is clear that $\Delta_{s_h}q_j^C$ is constant in $j$ and $\Delta_{s_h}\pi_j^C$ exhibits the same monotone property as $q_j^C$ with respect to $j$. Because $\Delta_{s_h}q_j^C > 0$ and $\Delta_{s_h}\pi_j^C > 0$, we have $q_j^C(s_p, s_{H2}) \geq_{wm} q_j^C(s_p, s_{H1})$ and $\pi_j^C(s_p, s_{H2}) \geq_{wm} \pi_j^C(s_p, s_{H1})$ for $s_{H1} < s_{H2}$. We conclude part (ii).

Proof of Proposition 4. To see part (i), we note that $q_j^C(s_p, s_H)$ is affine in $z_j$ for all $j \in \bar{N}$ (given $\bar{z}$ is fixed). Because the majorization order is preserved under the affine transformation, we have $q_j^C(s_p, s_H) \geq_{wm} q_j^C(s_p, s_H)$. Because $\Delta_{s_p}q_j^C$ is affine in $z_j$ (see the proof of Corollary 1), we have $\Delta_{s_p}q_j^C \geq_{wm} \Delta_{s_p}q_j^C$. Because $\Delta_{s_h}q_j^C$ is independent of $z$, we have $\Delta_{s_h}q_j^C = \Delta_{s_h}q_j^C$.

To see part (ii), it is clear that $\pi_j^C(s_p, s_{H}) \geq_{wm} \pi_j^C(s_p, s_{H})$. Because $\Delta_{s_h}\pi_j^C$ is affine in $z_j$ for all $j \in \bar{N}$, we have $\Delta_{s_h}\pi_j^C \geq_{wm} \Delta_{s_h}\pi_j^C$. This concludes the proof.

Lemma 3 (Condition for Production: Selective Subsidy) Farmer $j \in \bar{N}$ produces a positive amount under a selective subsidy $(s_p, s_H)$ if and only if

$$j \in N^S(s_p, s_H) = N^{SH}(s_p, s_H) \cup N^{Sp}(s_p, s_H),$$
where

\[ N^{S,h}(s_P, s_H) = \begin{cases} 
  \{ j \leq m : g(j) < \frac{\alpha - c_{H} + s_H}{c_P} \} & \text{if } s_P \leq c_P, \\
  \{ j \leq m : g^a(j) < \frac{\alpha - (c_{H} - s_H) + (\bar{n} - i^a(j))s_H - \sum_{i=1}^{m} z_i s_P}{c_P} \} & \text{if } s_P > c_P,
\end{cases} \]

\[ N^{S,p}(s_P, s_H) = \begin{cases} 
  \{ j > m : g(j) < \frac{\alpha - c_{H} - (i^b(j) - 1)s_H + \sum_{i=1}^{m} z_i s_P}{s_P - c_P} \} & \text{if } s_P \leq c_P, \\
  \{ j > m : g^b(j) < \frac{\alpha - c_{H} - (i^b(j) - 1)s_H + \sum_{i=1}^{m} z_i s_P}{s_P - c_P} \} & \text{if } s_P > c_P,
\end{cases} \]

\( g(j) \) is defined in Lemma 1, \( g^a(j) = (\bar{n} - i^a(j) + j)z_j + \sum_{i=j}^{\bar{n}} z_i - \sum_{i=1}^{\bar{n}} z_i - \sum_{i=1}^{j} z_i - (\bar{n} - j + i^b(j))z_j, \quad i^a(j) = \max\{ i \in \{m + 1, \ldots, \bar{n}\} : z_i(s_P - c_P) < s_H - z_j c_P \} \)

(when \( z_{m+1}(s_P - c_P) \geq s_H - z_j c_P \), we set \( i^a(j) = m \); when \( m = \bar{n} \), we set \( i^a(j) = \bar{n} \)), and \( i^b(j) = \min\{ i \in \{1, 2, \ldots, m\} : z_i c_P > s_H - z_j(s_P - c_P) \} \) (when \( z_m c_P \leq s_H - z_j(s_P - c_P) \), we set \( i^b(j) = m + 1 \); when \( m = 0 \), we set \( i^b(j) = 1 \)). Moreover, \( i^a(j) \) and \( i^b(j) \) are decreasing in \( j \).

The number of active farmers is \( n^S = |N^{S}(s_P, s_H)| = n^{S,h} + n^{S,p} \) with \( n^{S,h} = |N^{S,h}(s_P, s_H)| \) and \( n^{S,p} = |N^{S,p}(s_P, s_H)| \).

**Proof of Lemma 3.** By Proposition 5, \( q^S_j(s_P, s_H) \) is decreasing in \( j \) for \( s_P \leq c_P \). For farmer \( j \in \bar{N} \) to participate production, his output quantity should be positive even when farmers \( j+1, j+2, \ldots, \bar{n} \) quit the market. We have two cases to consider.

**Case i):** If \( j \leq m \), all farmers choose the harvesting subsidy and farmer \( j \)'s output becomes

\[ q^S_j = \frac{1}{\beta} \left( \frac{\alpha - (j - 1) \sum_{i=1}^{j} z_i c_P - (c_{H} - s_H)}{j + 1} - (z_j - \frac{1}{j} \sum_{i=1}^{j} z_i c_P) \right). \]

\( q^S_j > 0 \) gives \( g(j) < (\alpha - c_{H} + s_H)/c_P \), where \( g(j) \) is defined in Lemma 1.

**Case ii):** If \( j > m \), farmer \( j \) chooses the planting subsidy. Thus, farmer \( j \)'s output becomes

\[ q^S_j = \frac{1}{\beta} \left( \frac{\alpha + \sum_{i=1}^{j} z_i c_P - s_P}{j + 1} - (z_j - \frac{1}{j} \sum_{i=1}^{j} z_i c_P) \right) - \frac{1}{\beta} \left( s_H - \sum_{i=1}^{m} z_i s_P + (j - m) s_H \right). \]

\( q^S_j > 0 \) gives \( g(j) < (\alpha - c_{H} - m s_H + \sum_{i=1}^{m} z_i s_P)/c_P \).

When \( s_P > c_P \), \( q^S_j(s_P, s_H) \) is decreasing [increasing] in \( j \) for \( j < [>] m \). We have two cases to consider.

**Case i):** If \( j \leq m \), farmer \( j \)'s output should be positive even when farmers \( j + 1, \ldots, i^a(j) \) quit the market, where \( i^a(j) = \max\{ i \in \{m + 1, \ldots, \bar{n}\} : z_i(s_P - c_P) < s_H - z_j c_P \} \) (when \( z_{m+1}(s_P - c_P) \geq s_H - z_j c_P \), we set \( i^a(j) = m \); when \( m = \bar{n} \), we set \( i^a(j) = \bar{n} \)). Thus, farmer \( j \)'s output should be

\[ q^S_j = \frac{1}{\beta} \left( \frac{\alpha + \sum_{i=1}^{\bar{n}} z_i - \sum_{i=1}^{i^a(j)+1} z_i}{\bar{n} - i^a(j) + j + 1} - (z_j c_P + \sum_{i=1}^{j} z_i s_P + (\bar{n} - i^a(j)) s_H) \right). \]
\( q_j^S > 0 \) gives \((\bar{n} - i^a(j) + j)z_j + \sum_{i=j}^{i^a(j)} z_i - \sum_{i \in N} z_i < \frac{\alpha - c_H + (\bar{n} - i^s(j) + 1)s_H - \sum_{i=1}^{i^a(j)+1} z_is_P}{c_p} \). 

Case ii): If \( j > m \), farmer \( j \)'s output quantity should be positive even when farmers \( i^b(j), i^b(j) + 1, \ldots, j-1 \) quit the market, where \( i^b(j) = \min\{i \in \{1, 2, \ldots, m\} : z_ic_P > s_H - z_j(s_p - c_P) \} \) (when \( z_mc_P \leq s_H - z_j(s_p - c_p) \), we set \( i^b(j) = m + 1 \); when \( m = 0 \), we set \( i^b(j) = 1 \) ). Thus, farmer \( j \)'s output should be 

\[
q_j^S = \frac{1}{\beta} \left( \frac{\alpha + \left( \sum_{i \in \bar{N}} z_i - \sum_{i = i^b(j)}^{j-1} z_i \right) (c_p - s_P) - c_H}{\bar{n} - j + i^b(j) + 1} - z_j(c_p - s_P) + \sum_{i=1}^{i^b(j)-1} z_is_P - (i^b(j) - 1)s_H \right). 
\]

\( q_j^S > 0 \) gives \( \sum_{i \in \bar{N}} z_i - \sum_{i = i^b(j)}^{j} z_i - (\bar{n} - j + i^b(j))z_j < \frac{\alpha - c_H - (i^b(j))s_H + \sum_{i=1}^{i^b(j)-1} z_is_P}{s_p - c_P} \). This concludes the proof.

\[ \square \]

**Proof of Proposition 5.** For farmer \( j \in \bar{N} \setminus NS(s_p, s_H) \), it is clear that \( q_j^S(s_p, s_H) = 0 \) and \( \pi_j^S(s_p, s_H) = 0 \). For farmer \( j \in NS(s_p, s_H) \), farmer \( j \)'s profit is

\[
\pi_j(q) = \begin{cases} 
(a_j(0, s_H) - \beta \sum_{i \in NS(s_p, s_H)} q_i)q_j, & \text{for } j \in NS^h(s_p, s_H), \\
(a_j(s_p, 0) - \beta \sum_{i \in NS(s_p, s_H)} q_i)q_j, & \text{for } j \in NS^p(s_p, s_H).
\end{cases}
\]

Define \( \hat{a}_j(s_p, s_H) = a_j(0, s_H)\mathbb{I}_{(j \leq m)} + a_j(s_p, 0)\mathbb{I}_{(j > m)} \). Then the analysis directly follows that for Proposition 3 with \( a_j(s_p, s_H) \) replaced by \( \hat{a}_j(s_p, s_H) \).

We deduce, from the proof of Proposition 3 and (2),

\[
q_j^S(s_p, s_H) = q_j^C(s_p, s_H) - \begin{cases} 
\frac{1}{\beta} \left( z_js_p - \sum_{i=1}^{n^S_h} z_i + n^S_p s_H \right), & \text{for } j \in NS^h(s_p, s_H), \\
\frac{1}{\beta} \left( z_j s_H - \sum_{i=1}^{n^S_h} z_i + n^S_p s_H \right), & \text{for } j \in NS^p(s_p, s_H).
\end{cases}
\]

We can then compute

\[
X^S(s_p, s_H) = X^C(s_p, s_H) - \frac{1}{\beta} \left( \sum_{i=1}^{n^S_h} z_i^2 s_p + \sum_{i \in NS^p(s_p, s_H)} z_is_H - \frac{n \bar{z}}{n+1} \left( \sum_{i=1}^{n^S_h} z_i + n^S_p s_H \right) \right)
\]

and

\[
Q^S(s_p, s_H) = Q^C(s_p, s_H) - \frac{1}{\beta(n+1)} \left( \sum_{i=1}^{n^S_h} z_i + n^S_p s_H \right).
\]

From (7), we have, for any \( j, j \in NS(s_p, s_H) \),

\[
q_{j+1}^S(s_p, s_H) - q_j^S(s_p, s_H) = \frac{1}{\beta} \times \begin{cases} 
c_p(z_j - z_{j+1}), & \text{for } j < m, \\
((c_p - s_P)z_m + (z_ms_P - s_H)), & \text{for } j = m, \\
(c_p - s_P)(z_j - z_{j+1}), & \text{for } j > m.
\end{cases}
\]

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Because $z_ms_p \leq s_H$, we have $q_{j+1}^S(s_p, s_H) - q_j^S(s_p, s_H) \leq 0$ when $s_p \leq c_p$, and $q_{j+1}^S(s_p, s_H) - q_j^S(s_p, s_H) \leq [\geq] 0$ for $j < [\geq] m$ when $s_p > c_p$. Note that when $s_p \leq c_p$, $q_j^S(s_p, s_H) = 0$ for $j \in \tilde{N} \backslash N^S(s_p, s_H) = \{n + 1, n + 2, \ldots, \bar{n}\}$ and thus $q_j^S(s_p, s_H)$ is decreasing in $j$; when $s_p > c_p$, $q_j^S(s_p, s_H) = 0$ for $j \in \tilde{N} \backslash N^S(s_p, s_H) = \{n^{S_H} + 1, \ldots, \bar{n} - n^{S_p}\}$ and thus $q_j^S(s_p, s_H)$ is decreasing [increasing] in $j$ for $j < [\geq] m$.

We differentiate (7) with respect to $s_p$ and $s_H$, respectively, to obtain

$$
\frac{\partial q_j^S}{\partial s_p} = \begin{cases} 
-\frac{\sum_{i \in N^S, P(s_p, s_H)} z_i}{(n+1)z_j - \sum_{i \in N^S, P(s_p, s_H)} z_i} & \text{for } j \leq m, \\
\frac{\partial q_j^S}{\partial s_H} = \begin{cases} 
\frac{q_j^S + 1}{\beta (n+1)} & \text{for } j \leq m, \\
-\frac{q_j^S + 1}{\beta (n+1)} & \text{for } j > m.
\end{cases}
\end{cases}
$$

It is clear that farmer $j$’s output quantity is increasing [decreasing] in $s_p$ for $z_j \geq \frac{\sum_{i \in N^S, P(s_p, s_H)} z_i}{n+1}$ and $j > m$ [otherwise], and is increasing [decreasing] in $s_H$ when $j \leq m$ $[j > m]$. Since the input quantity and the output quantity vary by a scale of $z_j$, farmer $j$’s input quantity exhibits the same monotone property as $q_j^S$ with respect to $s_p$ and $s_H$. Moreover, from (8), we have $\pi_j^S(s_p, s_H) = \beta (q_j^S(s_p, s_H))^2$. Thus, farmer $j$’s profit exhibits the same monotone property as $q_j^S$ with respect to $s_p$ and $s_H$. Hence we conclude the proof.

**Proof of Corollary 2.** Because all farmers produce positive amounts in equilibrium, we have $N = \tilde{N}$ and $n = \bar{n}$. To see part (i), we define $m_p = \max\{j \in N : z_j(s_p + \delta) \leq s_H\}$ (when $z_1 > s_H/(s_p + \delta)$, we set $m_p = 0$). Note that $m \geq m_p$. We also define $\Delta_p = \sum_{i=1}^{m_p} z_i \delta + \sum_{i=m_p+1}^{m} (s_H - z_is_p)$. It is clear that $0 \leq \Delta_p < \sum_{i=1}^{m} z_i \delta$. We have

$$
\Delta_p q_j^S = \begin{cases} 
\frac{\Delta_p - \delta n\bar{z}}{\beta (n+1)} & \text{for } j \leq m, \\
\frac{\Delta_p - \delta n\bar{z}}{\beta (n+1)} + \frac{(s_p + \delta)z_j - s_H}{\beta} & \text{for } m_p < j \leq m, \\
\frac{\Delta_p - \delta n\bar{z}}{\beta (n+1)} + \frac{\delta z_j}{\beta} & \text{for } m < j.
\end{cases}
$$

The first piece is constant in $j$, and the second and third pieces are increasing in $j$. Because $\Delta_p q_{m+1}^S - \Delta_p q_m^S = \delta z_{m+1}/\beta - ((s_p + \delta)z_m - s_H)/\beta > \delta(z_{m+1} - z_m)/\beta \geq 0$, $\Delta_p q_j^S$ is increasing in $j$. Because $\Delta_p q_n^S > 0$ and $\sum_{i \in N} \Delta_p q_i^S > 0$, we have $\sum_{j=1}^{m} \Delta_p q_j^S > 0$ for any $l \in N$. This implies that $q^S(s_{l1}, s_H)$ weakly sup-majorizes $q^S(s_{l2}, s_H)$ for $s_{l1} < s_{l2} \leq c_p$.

To see part (ii), we define $m_H = \max\{j \in N : z_js_p \leq (s_H + \delta)\}$ (when $z_1 > (s_H + \delta)/s_p$, we set $m_H = 0$). Note that $m \leq m_H$. We also define $\Delta_H = (n - m_H)\delta + \sum_{i=m+1}^{m_H} (z_is_p - s_H)$. 

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It is clear that $0 \leq \Delta_H \leq (n - m)\delta$. Note that

$$
\Delta_{sH} q_j^S = \begin{cases} 
\frac{\Delta_H + \delta}{\beta(n+1)} & \text{for } j \leq m, \\
\frac{\Delta_H + \delta}{\beta(n+1)} - \frac{z_i s_p - s_H}{\beta} & \text{for } m < j \leq m_H, \\
\frac{\Delta_H + \delta}{\beta(n+1)} - \frac{\delta}{\beta} & \text{for } m_H < j.
\end{cases}
$$

The first and third pieces are constant in $j$, and the second piece is decreasing in $j$. Because $\Delta_{sH} q_{m+1}^S - \Delta_{sH} q_{m}^S = (z_{mH} s_P - s_H)/\beta - \delta/\beta \leq 0$, $\Delta_{sH} q_j^S$ is decreasing in $j$. Because $\Delta_{sH} q_1^S > 0$ and $\sum_{i \in N} \Delta_{sH} q_i^S > 0$, we have $\sum_{j=1}^l \Delta_{sH} q_j^S > 0$ for any $l \in N$. That is, $q^S(s_P, s_{H2})$ sub-majorizes $q^S(s_P, s_{H1})$. It follows that $\pi^S(s_P, s_{H2})$ sub-majorizes $\pi^S(s_P, s_{H1})$. This concludes the proof. □

**Proof of Proposition 6.** We define

$$
\Delta \equiv \left( m_{A} s_{H} + \sum_{i=m_{A}+1}^{n} z_{Ais_{P}} \right) - \left( m_{B} s_{H} + \sum_{i=m_{B}+1}^{n} z_{Bis_{P}} \right)
$$

$$
= \sum_{i=m_{H}+1}^{n} \left( (z_{Ai} - z_{Bi}) s_{P} + \sum_{i=m_{A}+1}^{m_{B}} (z_{Ai} s_{P} - s_{H}) \right).
$$

We have $\Delta \geq 0$ because $z_{A} \geq m_{B}$ and $z_{Ais_{P}} > s_{H}$ for $i > m_{A}$. We then have

$$
\sum_{j=1}^{l} \left( q_{A}^S(s_{P}, s_{H}) - q_{B}^S(s_{P}, s_{H}) \right)
$$

$$
= \begin{cases} 
\frac{c_p}{\beta} \sum_{j=1}^{l} (z_{Bj} - z_{Aj}) - \frac{l \Delta}{\beta(n+1)} & \text{for } l \leq m_{A}, \\
\frac{(c_p - s_{P})}{\beta} \sum_{j=1}^{l} (z_{Bj} - z_{Aj}) + \frac{m_{B} \Delta}{\beta(n+1)} & \text{for } m_{A} < l < m_{B}, \\
\frac{(c_p - s_{P})}{\beta} \sum_{j=1}^{l} (z_{Bj} - z_{Aj}) + \frac{m_{B} \Delta}{\beta(n+1)} & \text{for } l \geq m_{B}.
\end{cases}
$$

It is easy to check that $\sum_{j=1}^{l} \left( q_{A}^S(s_{P}, s_{H}) - q_{B}^S(s_{P}, s_{H}) \right) \geq 0$ for any $l \in N$. That is, $q_{A}^S(s_{P}, s_{H})$ weakly sub-majorizes $q_{B}^S(s_{P}, s_{H})$, and thus $\pi_{A}^S(s_{P}, s_{H})$ weakly sub-majorizes $\pi_{B}^S(s_{P}, s_{H})$. □

**Proof of Proposition 7.** To see part (i), we have, from (2) and (7),

$$
q_{j}^C(s_{P}, s_{H}) - q_{j}^S(s_{P}, s_{H}) = \frac{1}{\beta(n+1)} \times \begin{cases} 
(n + 1) z_{j} s_{P} - \left( \sum_{i=1}^{m} z_{is_{P}} + (n - m)s_{H} \right) & \text{for } j \leq m, \\
(m + 1)s_{H} - \sum_{i=1}^{m} z_{is_{P}} & \text{for } j > m.
\end{cases}
$$

The first piece is increasing in $j$ and the second piece is constant in $j$. Because $(q_{m+1}^C(s_{P}, s_{H}) - q_{m+1}^S(s_{P}, s_{H})) - (q_{m}^C(s_{P}, s_{H}) - q_{m}^S(s_{P}, s_{H})) = (s_{H} - z_{m}s_{P})/\beta \geq 0$, $q_{j}^C(s_{P}, s_{H}) - q_{j}^S(s_{P}, s_{H})$ is increasing in $j$. Note that the second piece is above zero, there must exist some $j^0 \leq m$ such that $q_{j}^C(s_{P}, s_{H}) - q_{j}^S(s_{P}, s_{H}) \leq [>0] \text{ for } j \leq [>]j^0$. We conclude part (i).

To see part (ii), because $\sum_{i \in N} (q_{j}^C(s_{P}, s_{H}) - q_{j}^S(s_{P}, s_{H})) > 0$ and $q_{j}^C(s_{P}, s_{H}) - q_{j}^S(s_{P}, s_{H})$ is increasing in $j$, we have $\sum_{j=1}^{l} (q_{j}^C(s_{P}, s_{H}) - q_{j}^S(s_{P}, s_{H})) > 0$ for any $l \in N$. That is, $q^S(s_{P}, s_{H})$ weakly sup-majorizes $q^C(s_{P}, s_{H})$ for $s_{P} \leq c_{P}$. We conclude part (ii).
To see part (iii), we note that
\[
\phi_j = \Delta s_P q^C_j - \Delta s_H q^S_j = \begin{cases} 
\frac{\delta s_j}{\beta} - \frac{\Delta_P}{\beta(n+1)} & \text{for } j \leq m_\delta, \\
\frac{s_H - s_P z_j}{\beta} - \frac{\Delta_P}{\beta(n+1)} & \text{for } m_\delta < j \leq m, \\
-\frac{\Delta_P}{\beta(n+1)} & \text{for } m < j.
\end{cases}
\]

The first piece is increasing in \(j\), the second piece is decreasing in \(j\) and the third piece is constant in \(j\). Because \(\phi_m - \phi_{m+1} = s_H - z_m s_P \geq 0\), \(\phi_j\) is decreasing in \(j\) for \(j > m_\delta\). Note that \((n+1)z_1 \delta \geq \sum_{i=1}^m \delta \delta > \Delta_P\), we have \(\phi_1 > 0\). Thus, there exists a \(j_P\) such that \(\phi_j \geq [>0\text{ for } j \leq [>]j_P\). Note that \(\sum_{i \in N} \phi_i > 0\), we have \(\sum_{j=1}^l \phi_j > 0\) for any \(l \in N\). We conclude part (iii).

To see part (iv), we note that
\[
\psi_j = \Delta s_H q^C_j - \Delta s_H q^S_j = \begin{cases} 
-\frac{\Delta_H}{\beta(n+1)} & \text{for } j \leq m, \\
\frac{z s_H - s_H z_j}{\beta} - \frac{\Delta_H}{\beta(n+1)} & \text{for } m < j \leq m_\delta, \\
\delta - \frac{\Delta_H}{\beta(n+1)} & \text{for } m_\delta < j.
\end{cases}
\]

Note that \(\psi_j\) is increasing in \(j\), as \(\Delta s_H q^C_j\) is constant in \(j\), and \(\Delta s_H q^S_j\) is decreasing in \(j\). Note that \(\psi_1 < 0\) and \(\psi_n > 0\), there exists a \(j_H\) such that \(\psi_j \leq [>0\text{ for } j \leq [>]j_H\). Finally, note that \(\sum_{i \in N} \psi_i > 0\), we have \(\sum_{j=1}^l \psi_j > 0\) for any \(l \in N\). We conclude part (iv). \(\square\)

**Proof of Proposition 8.** Because \(Q^C(s_P, s_H) = \bar{Q}\), we have
\begin{equation}
\bar{z}s_P + s_H = \frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0).
\end{equation}

i) When only the planting subsidy is offered (i.e., \(s_H = 0\)), we must have
\[
s_P^P = \frac{1}{\bar{z}} \left( \frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0) \right) = \left( \frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0) \right)/\bar{z},
\]
and the optimal budget is
\begin{equation}
b^C(s_P^P, 0) = X^C(s_P^P, 0)s_P^P = \bar{Q} \left( \frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0) \right) - \frac{nv_s}{\beta} (c_P - s_P^P)s_P^P.
\end{equation}

ii) When only the harvesting subsidy is offered (i.e., \(s_P = 0\)), we must have
\[
s_H^H = \frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0),
\]
and the optimal budget is
\begin{equation}
b^C(0, s_H^H) = Q^C(0, s_H^H) s_H^H = \bar{Q} \left( \frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0) \right).
\end{equation}
iii) When the combined subsidy is given, (11) implies that \( ds_H/ds_P = -\tilde{z} \). By Proposition 3, we have

\[
\frac{\partial X^C(s_P, s_H)}{\partial s_P} = \frac{n}{\beta(n+1)}((n+1)v_c + \tilde{z}^2) \quad \text{and} \quad \frac{\partial Q^C(s_P, s_H)}{\partial s_P} = \frac{n}{\beta(n+1)}\tilde{z},
\]

\[
\frac{\partial X^C(s_P, s_H)}{\partial s_H} = \frac{n}{\beta(n+1)}\tilde{z} \quad \text{and} \quad \frac{\partial Q^C(s_P, s_H)}{\partial s_H} = \frac{n}{\beta(n+1)}.
\]

Now differentiating \( b^C(s_P, s_H) \) with respect to \( s_P \), we have

\[
\frac{db^C(s_P, s_H)}{ds_P} = \frac{\partial b^C(s_P, s_H)}{\partial s_P} + \frac{\partial b^C(s_P, s_H)}{\partial s_H} \frac{ds_H}{ds_P}
= \frac{n}{\beta(n+1)}((n+1)v_c + 2(n+1)v_c s_P) \]

\[
= \frac{nv_c}{\beta}(-cp + 2s_P).
\]

Because the right-hand side is increasing in \( s_P \), \( b^C \) is convex in \( s_P \) and is minimized at \( s_P^C = cp/2 \).

Substituting this into (11), we obtain

\[ s_P^C = \frac{\beta(n+1)}{n}Q - \bar{a}(0, 0) - \frac{cp\tilde{z}}{2} \]

and the optimal budget is

\[ b^C(s_P^C, s_H^C) = Q\left(\frac{\beta(n+1)}{n}Q - \bar{a}(0, 0)\right) - \frac{nv_c^2}{4\beta}. \]

Hence, we conclude the proof. \( \square \)

**Proof of Corollary 3.** Setting \( s_H^C < 0 \) gives \( \bar{Q} < \frac{n}{\beta(n+1)}(\bar{a}(0, 0) + cp\tilde{z}/2) = Q^C(cp/2, 0) \). Setting \( b^C(s_P^C, s_H^C) = 0 \) gives \( Q^T \) directly. This concludes the proof. \( \square \)

**Lemma 4** When the selective subsidy is offered and \( Q \geq Q^S(cp/2, 0), (s_P^S, s_H^S) = (0, s_H^S) \) or \((s_P^S, s_H^S)\) is an element in the set

\[
\left\{(s_P, s_H) \mid s_P = \max\{\phi^l(\bar{s}_P), \min\{\bar{s}_P, \phi^u(\bar{s}_P)\}\}, \quad s_H = (\beta(n^S + 1)\bar{Q} - n^S\bar{a}(0, 0) - \sum_{i \in N^P} z_i s_P)/n^h,\right\}
\]

where \( N^h = N^{S,h}(s_P, s_H), N^p = N^{S,p}(s_P, s_H), n^h = |N^h| > 0, n^p = |N^p| > 0, n^S = |N^S(s_P, s_H)|, \)

\[
\bar{s}_P = c(N^h, N^p)\left(\frac{\beta(n^S + 1)}{n^S}Q - \bar{a}(0, 0)\right)/\tilde{z} + (1 - c(N^h, N^p))(cp/2),
\]

\[
c(N^h, N^p) = \left(\frac{n^S\tilde{z}}{\sum_{i \in N^p} z_i} + 1 - \sum_{i \in N^p} z_i^2\right)^{\frac{1}{2}}
\]

\[
\phi^l(s_P) = \begin{cases} 
\frac{\beta\bar{Q} - (\alpha - c_h)}{\tilde{z}_{n^S}} + cp, & \text{if } s_P \leq cp, \\
\frac{\beta\bar{Q} - (\alpha - c_h)}{\tilde{z}(n^S + 1)} + cp, & \text{if } s_P > cp \text{ and } n^S = \bar{n}, \\
\max\left\{\frac{\beta\bar{Q} - (\alpha - c_h)}{\tilde{z}(n^S + 1)} + cp, \min\left\{\frac{\beta(n^S + 1)\bar{Q} - n^p(\alpha - c_h) + (n^S - h(\alpha - c_h))cp}{\sum_{i \in N^p} z_i}, \frac{\beta(n^S + 1)\bar{Q} - n^S(\alpha - c_h) + n^S\tilde{z} cp}{\sum_{i \in N^p} z_i}\right\}\right\}, & \text{if } s_P > cp \text{ and } n^S < \bar{n},
\end{cases}
\]

Electronic copy available at: https://ssrn.com/abstract=4378693
\[ \phi^s(s_p) = \begin{cases} \mathbb{I}_{n^S < \bar{n}} \{ \frac{\beta Q - (\alpha - \epsilon H)}{z(\alpha + 1)} + c_p \} + \mathbb{I}_{n^S = \bar{n}} \{ \infty \} & \text{for } s_P \leq c_P, \\ \frac{\beta(n^p + 1)Q - n^p(\alpha - c_H) + (n^S - n^H z_{n^H})c_P}{\sum_{i \in N^p} z_i} & \text{for } s_P > c_P \text{ and } n^S = \bar{n}, \\ \min \left\{ \frac{\beta Q - (\alpha - c_H)}{z(\alpha + 1)} + c_p, \frac{\beta(n^p + 1)Q - n^p(\alpha - c_H) + n^S z_{c_c}}{n^H z(\alpha + n^p) + \sum_{i \in N^p} z_i} \right\} & \text{for } s_P > c_P \text{ and } n^S < \bar{n}. \end{cases} \]

**Proof of Lemma 4.** Because \( Q^S(s_p, s_H) = \bar{Q} \), we have

\[
\left( n^h s_H + s_P \sum_{i \in N^p} z_i \right)/n = \frac{\beta(n + 1)}{n} \bar{Q} - \bar{a}(0, 0) \equiv C. \tag{16}
\]

This implies that \( ds_H/ds_P = -\sum_{i \in N^p} z_i/n^h \). For a fixed set of active farmers \( N \), we must have

\[
\begin{align*}
&\{ \bar{z} s_P = C \text{ and } s_H < (z_1/\bar{z})C \\
&\quad \left( n^h z_{n^h} + \sum_{i \in N^p} z_i \right) s_P \leq nC < \left( n^h z_{(\min(N))} \right) + \sum_{i \in N^p} z_i \} s_P \quad \text{for } n^h = 0, \\
&\quad (n^h z_{n^h} + \sum_{i \in N^p} z_i) s_P \leq nC < \left( n^h z_{(\min(N))} \right) s_P \quad \text{for } 1 \leq n^h < n, \\
&\quad z_{(\max(N))} s_P \leq C \text{ and } s_H = C \quad \text{for } n^h = n. \tag{17}
\end{align*}
\]

We now differentiate \( b^S(s_p, s_H) \) with respect to \( s_P \) and obtain

\[
\frac{db^S(s_p, s_H)}{ds_P} = \frac{\partial b^S(s_p, s_H)}{\partial s_P} + \frac{\partial b^S(s_p, s_H)}{\partial s_H} \frac{ds_H}{ds_P} = 2 \beta \left( \sum_{i \in N^p} z_i^2 s_P - \sum_{i \in N^p} z_i s_H \right) + \frac{c_P}{n^h \beta} \left( \sum_{i \in N^p} z_i \right) \left( \sum_{i \in N^p} z_i \right) - \sum_{i \in N^p} z_i^2 n^h = \frac{1}{n^h \beta} \left( \sum_{i \in N^p} z_i^2 n^h + \sum_{i \in N^p} z_i \right) \left( 2s_P - c_P \right) + \sum_{i \in N^p} z_i n(\bar{z} c - 2C). \]

Because the right-hand side is increasing in \( s_P \) for every pair of \( (N^h, N^p) \), \( b^S(s_p, s_H) \) is convex in \( s_P \) and is (locally) minimized at \( s_P(N^h, N^p) = c(N^h, N^p)/C/\bar{z} + (1 - c(N^h, N^p))/(c_P/2) \), where

\[
c(N^h, N^p) = \frac{n^h}{\sum_{i \in N^p} \sum_{i \in N^p} z_i^2 n^h + (\sum_{i \in N^p} z_i)^2}.
\]

From (17), we must have \( s_P \leq C/\bar{z} \). Thus, when \( C < \bar{z} c_P/2 \), the optimal subsidy format is \( s_P^H = C/\bar{z} \) and \( s_H^S < (z_1/\bar{z})C \) (i.e., \( n^h = 0 \)). When \( C \geq \bar{z} c_P/2 \), we further deduce that any discontinuity point of \( db^S(s_p, s_H)/ds_P \), i.e., \( s_P^H = nC/(n^h z_{n^h} + \sum_{i \in N^p} z_i) \) for \( 1 \leq n^h \leq n \), is not the minimum point of \( b^S(s_p, s_H) \). This follows from the observation that

\[
\begin{align*}
\frac{db^S(s_p, s_H)}{ds_P} \bigg|_{s_p^-} & - \frac{db^S(s_p, s_H)}{ds_P} \bigg|_{s_p^+} = \begin{cases} \frac{2}{\beta} \sum_{i \in N^p} z_i (z_i - z_1) (C/\bar{z} - c_P/2), & n^h = 1, \\
\frac{c_P}{n^h (n^h - 1)} \left( n^h z_{n^h} + \sum_{i \in N^p} z_i \right) \left( n^h z_{n^h} - \sum_{i \in N^h} z_i \right), & n^h > 1. \end{cases}
\end{align*}
\]

That is, the left limit of \( db^S(s_p, s_H)/ds_P \) is greater than the right limit of \( db^S(s_p, s_H)/ds_P \) at \( s_P^H \).
Substituting $s_p(N^h, N^p)$ into (16) yields
\[ s_H(N^h, N^p) = \frac{nC - \sum_{i \in N^p} z_i s_p(N^h, N^p)}{n^h}. \]

Therefore, the budget is
\[ b^S(s_p(N^h, N^p), s_H(N^h, N^p)) = \frac{n}{4\beta} \left( \frac{\sum_{i \in N^p} z_i^2 n^p - \left( \frac{\sum_{i \in N^p} z_i}{n^h} \right)^2 (2C - \bar{z} c_p)^2 + \left( \frac{\sum_{i \in N^p} z_i}{n^h} \right)^2 - n^h \bar{z}^2}{n^h} \right) c_p^2. \]

Note that the above analysis is based on the condition that the set of active farmers is fixed. Now, we establish the condition of $(s_p, s_H)$ such that $Q^S(s_p, s_H) = \bar{Q}$ and the set of active farmers is $N = N^p \cup N^h$ with $|N| = n$, $|N^h| = n^h$ and $|N^p| = n^p$. It suffices to consider $n^h > 0$ and $n^p > 0$.

We discuss two cases.

**Case i):** When $s_p \leq c_p$, the set of active farmers should be $\{1, 2, \ldots, n\}$, in which the first $n^h$ farmers choose the harvesting payment and the rest choose plantation payment. Based on the condition characterized in Lemma 3, we have
\[ g(n) < \frac{\alpha - c_H - n^h s_H + \sum_{i \in N^p} z_i s_p}{c_p - s_p} \quad \text{and} \quad g(n + 1) \geq \frac{\alpha - c_H - n^h s_H + \sum_{i \in N^p} z_i s_p}{c_p - s_p}. \]

The first inequality suggests that farmer $n$ produces and the second one implies that farmer $n + 1$ does not produce. Together with $Q^S(s_p, s_H) = \bar{Q}$, we derive $\frac{\beta \bar{Q} - (\alpha - c_H)}{\bar{z}_n} + c_p < s_p \leq \frac{\beta \bar{Q} - (\alpha - c_H)}{\bar{z}_{n+1}} + c_p$ (when $n = \bar{n}$, only the first inequality is needed).

**Case ii):** When $s_p > c_p$, the set of active farmers should be $\{1, \ldots, n^h\} \cup \{\bar{n} - n^p + 1, \ldots, \bar{n}\}$.

According to Lemma 3, we have four conditions:

(i) Farmer $n^h$ (who chooses the harvesting subsidy) would produce, i.e., with $i^a(n^h) = \bar{n} - n^p$, $g^a(n^h) < \frac{\alpha - c_H + (n^p + 1) s_H - \sum_{i \in N^p} z_i s_p}{c_p - s_p}$. This yields $s_p < \frac{\beta(n^p + 1)Q - \bar{Q} - (\bar{n}^h s_H + za - n^h z_{n+1}) c_p}{\sum_{i \in N^p} z_i}$.

(ii) Farmer $\bar{n} - n^p + 1$ (who chooses the plantation subsidy) would produce, i.e., with $i^b(\bar{n} - n^p + 1) = n^h + 1$, $g^b(\bar{n} - n^p + 1) < \frac{\alpha - c_H - n^h s_H + \sum_{i \in N^p} z_i s_p}{s_p - c_p}$. This gives $s_p > \frac{\beta \bar{Q} - (\alpha - c_H)}{\bar{z}_n} + c_p$.

(iii) Farmer $n^h + 1$ (who chooses the harvesting subsidy) would not produce, i.e., with $i^a(n^h + 1) = \bar{n} - n^p$, $g^a(n^h + 1) = \frac{\alpha - c_H + (n^p + 1) s_H - \sum_{i \in N^p} z_i s_p}{c_p}$. This yields $s_p \geq -\frac{\beta(n^p + 1)Q - \bar{Q} - (\bar{n}^h s_H + za - n^h z_{n+1}) c_p}{\sum_{i \in N^p} z_i}$.

(iv) Farmer $\bar{n} - n^p$ (who chooses the plantation subsidy) would not produce, i.e., with $i^b(\bar{n} - n^p) = n^h + 1$, $g^b(\bar{n} - n^p) = \frac{\alpha - c_H - n^h s_H + \sum_{i \in N^p} z_i s_p}{s_p - c_p}$. This gives $s_p \leq \frac{\beta \bar{Q} - (\alpha - c_H)}{\bar{z}_n} + c_p$.

When $\bar{n} = n^h + n^p$ (i.e., all farmers produce), only (i) and (ii) are needed. When $n^h + n^p < \bar{n}$ and the farmer $n^h + 1$ chooses the plantation subsidy (i.e., $s_p > \frac{\beta(n^p + 1)Q - \bar{Q} - n^p (\alpha - c_H) + za c_p}{n^h z_{n+1} + \sum_{i \in N^p} z_i}$), only (i), (ii), and (iv) are needed. When $n^h + n^p < \bar{n}$ and the farmer $\bar{n} - n^p$ chooses the harvesting subsidy
(i.e., \( s_P \leq \frac{\beta(n+1)Q-n(\alpha-c_H)+n\bar{z}c_F}{n^2z(n-n^P) + \sum_{i \in N^P} z_i} \)), respectively, we obtain

\[ 0 < \frac{n^h + n^P}{n^h + 1} \]

only (i), (ii), and (iii) are needed. When \( n^h + n^P < \bar{n} \), the farmer

\[ n^h + 1 \]

chooses the harvesting subsidy, and the farmer \( \bar{n} - n^P \) chooses the plantation subsidy, (i), (ii), (iii), and (iv) are needed.

Combining the above cases gives the expressions of \( \phi'(s_P) \) and \( \phi''(s_P) \), respectively. We conclude the proof. □

**Proof of Proposition 9.** The results follow from Lemma 4 directly. □

**Proof of Corollary 4.** To see part i), it is clear that \( s_P^H \) is increasing in \( \bar{Q} \). Note that \((\alpha - c_H)/|c_P - s_P^P|\) is increasing [decreasing] in \( s_P^P \) for \( s_P^P \leq |c_P| \). By Lemma 2, \( n^P \) is increasing [decreasing] in \( \bar{Q} \) when \( \bar{Q} \leq |Q^C(c_P, 0) \). Also, all farmers produce when \( \bar{Q} = Q^C(c_P, 0) \).

To see part ii), it is clear that \( s_P^H \) is increasing in \( \bar{Q} \). Note that \((\alpha-c_H+s_P^H)/c_P \) is increasing in \( s_P^H \). By Lemma 2, \( n^H \) is increasing in \( \bar{Q} \).

To see part iii), it is clear that \( s_P^H \) is increasing in \( \bar{Q} \). Note that \((\alpha-c_H+s_P^H)/c_P \) is increasing in \( s_P^H \). Thus, \( n^C \) is increasing in \( \bar{Q} \). □

**Proof of Proposition 10.** It is clear that \( b^C(0, s_P^H) = b^C(s_P^P, 0) = b^C(s_P^C, s_P^H) = b^S(s_P^S, s_P^H) \) for \( v_z = 0 \). It is clear that either combined subsidy or selective subsidy leads to a lower budget than only the planting/harvesting subsidy. Note that

\[ b^C(0, s_P^H) - b^C(s_P^P, 0) = (nv_z/\beta)(c_P - s_P^P)s_P^P. \]

It is clear that \( b^C(0, s_P^H) - b^C(s_P^P, 0) \geq |0| \) for \( s_P^P \leq |c_P| \). Also note that

\[ b^S(s_P^S, s_P^H) - b^C(s_P^C, s_P^H) = \frac{n}{4\beta} \left( \sum_{i \in N^P} z_i^2 n^P - \left( \sum_{i \in N^P} z_i \right)^2 \right) (2C - \bar{z}c_P)^2 + \left( \sum_{i = 1}^{n^h} z_i^2 - \frac{1}{n^h} \left( \sum_{i = 1}^{n^h} z_i \right)^2 \right) \frac{c_P^2}{4\beta} \geq 0. \]

The inequality follows because \( \sum_{i \in N^P} z_i^2 n^P \geq \left( \sum_{i \in N^P} z_i \right)^2 \) and \( \sum_{i = 1}^{n^h} z_i^2 n^h \geq \left( \sum_{i = 1}^{n^h} z_i \right)^2 \), by the Cauchy-Schwartz inequality. This concludes the proof. □

**Proof of Proposition 11.** Substituting \((s_P^P, 0), (0, s_P^H)\) and \((s_P^C, s_P^H)\) into (4) and \((s_P^S, s_P^H)\) into (9), respectively, we obtain

\[ X^C(s_P^P, 0) = \bar{z}Q - \frac{nv_z}{\beta} (c_P - s_P^P), \quad X^C(0, s_P^H) = \bar{z}Q - \frac{nv_z}{\beta} c_P, \quad X^C(s_P^C, s_P^H) = \bar{z}Q - \frac{nv_z}{\beta} (c_P/2), \]

\[ X^S(s_P^S, s_P^H) = \bar{z}Q - \frac{nv_z}{\beta} (c_P - s_P^S) + \frac{1}{\beta} \left( \sum_{i \in N^h} z_i (\bar{z} - z_i) s_P^S + \sum_{i \in N^P} (\bar{z} - z_i) s_P^S \right). \]
To see part ii), we first note that $X^C(s_p, 0) - X^C(0, s_H^C) > 0$ and $X^C(s_p^C, s_H^C) - X^C(0, s_H^C) > 0$. Also, we note that $X^C(s_p^C, s_H^C) - X^C(s_p, 0) = n v_z (c_p/2 - s_p^2)/\beta \geq |\cdot|0$ for $s_p^2 \leq |\cdot|c_p/2$. Finally, we note that for $\bar{Q} > Q^C(c_p/2, 0)$,
\[
\frac{dX^S}{ds_p^S} = \frac{\partial X^S}{\partial s_p^S} ds_p^S = \frac{1}{n h \beta} \left( \sum_{i \in N_p} z_i^2 n_i - \sum_{i \in N^p} z_i \right) \geq 0.
\]
This implies that $X^C(0, s_H^C) < X^S(s_p^C, s_H^C) < X^C(s_p, 0)$. This concludes the proof.

To see part i), we note that $X^C(0, s_H^C)/\bar{Q} = \bar{z} - (n v_z c_p)/(|\beta|\bar{Q})$, being increasing in $\bar{Q}$. Thus, $X^C(0, s_H^C)/\bar{Q} \geq X^*/Q^*$. It follows that $X^C(s_p, 0)/\bar{Q} \geq X^*/Q^*$, $X^C(s_p, s_H^C)/\bar{Q} \geq X^*/Q^*$, and $X^S(s_p^C, s_H^C)/\bar{Q} \geq X^*/Q^*$.

We note that $X^C(s_p, 0)/\bar{Q} \leq |\cdot|\bar{z}$ for $s_p^2 \leq |\cdot|c_p$. Also, $X^C(0, s_H^C)/\bar{Q} \leq \bar{z}$, and $X^C(s_p^C, s_H^C)/\bar{Q} \leq \bar{z}$. This concludes the proof.

**Proof of Proposition 12.** By Lemmas 8 and 9, we have
\[
\sum_{j=1}^l q_j^C(s_p, 0) = \frac{lQ}{n} + \frac{c_p}{\beta} \left( l\bar{z} - \sum_{j=1}^l z_j \right) - \frac{C}{\beta \bar{z}} \left( l\bar{z} - \sum_{j=1}^l z_j \right),
\]
\[
\sum_{j=1}^l q_j^C(0, s_H^C) = \frac{lQ}{n} + \frac{c_p}{\beta} \left( l\bar{z} - \sum_{j=1}^l z_j \right),
\]
\[
\sum_{j=1}^l q_j^C(s_p^C, s_H^C) = \frac{lQ}{n} + \frac{c_p}{2\beta} \left( l\bar{z} - \sum_{j=1}^l z_j \right),
\]
\[
\sum_{j=1}^l q_j^C(s_p^S, s_H^S) = \frac{lQ}{n} + \frac{c_p}{\beta} \left( l\bar{z} - \sum_{j=1}^l z_j \right) + \frac{1}{\beta} \left( (n^h \wedge l) s_H^S + \sum_{j=1}^l z_j s_H^S - lC \right).
\]

It is easy to check that $q^C(0, s_H^C)$ majorizes $q^C(s_p^C, s_H^C)$ for any $\bar{Q}$, and $q^C(0, s_H^C)$ majorizes $q^C(s_p^C, 0)$ for $\bar{Q} = Q^C(c_p, 0)$. Note that $\sum_{j=1}^l q_j^C(s_p^C, 0) - \sum_{j=1}^l q_j^C(s_p^C, s_H^C) = (\bar{z}c_p/2 - C)(l\bar{z} - \sum_{j=1}^l z_j)/(\beta \bar{z}) \geq |\cdot|0$ for $C \leq |\cdot|\bar{z}c_p/2$. Thus, $q^C(s_p^C, 0)$ majorizes $q^C(s_p^C, s_H^C)$ for $\bar{Q} \geq Q^C(c_p/2, 0)$ and $q^C(s_p^C, s_H^C)$ majorizes $q^C(s_p, 0)$ for $Q^C(c_p/2, 0) < \bar{Q} \leq Q^C(c_p, 0)$.

Now we establish $q^C(0, s_H^C) \geq m$ $q^S(s_p^S, s_H^S) \geq m$ $q^C(s_p^C, 0)$ for $\bar{Q} \leq Q^C(c_p, 0)$. Note that $s_H^S < C$ and thus $\sum_{j=1}^l q_j^C(0, s_H^C) - \sum_{j=1}^l q_j^C(s_p^C, s_H^S) \geq 0$ for any $l \in N$. Also, note that $s_H^S > z_1 C/\bar{z}$ and $s_p^S < C/\bar{z}$. Thus, the sign of $\sum_{j \leq m} s_H^S + \sum_{j > m} z_j s_H^S - z_1 C/\bar{z}$ changes once and the change is from positive to negative. This suggests that $\sum_{j=1}^l q_j^C(s_p^S, s_H^S) - \sum_{j=1}^l q_j^C(s_p^C, 0) \geq 0$ for any $l \in N$.

Part ii) follows directly because the sub-majorization order is persevered under the increasing convex transformation. This concludes the proof.

**Proof of Proposition 13.** Because $Q^C(s_p, s_H) = \bar{Q}$, we substitute (5) and (6) into $W^C(s_p, s_H)$
and $NW^C(s_P, s_H)$, respectively, to obtain

$$W^C(s_P, s_H) = \frac{1}{\beta} \left( \frac{\beta^2 (n^2 + 2n)}{2n^2} Q^2 + n v z (c_P - s_P)^2 \right),$$

$$NW^C(s_P, s_H) = \frac{1}{\beta} \left( \frac{\beta^2 (n^2 + 2n)}{2n^2} Q^2 - \beta \frac{\beta(n + 1)}{n} \bar{Q} - \bar{a}(0, 0) \right) + n v z c_P (c_P - s_P).$$

The results then follow by inspecting the expressions of $W^C(s_P, s_H)$ and $NW^C(s_P, s_H)$. □

B The Effect of Output Uncertainty

In this section, we extend our model by incorporating the uncertainty in production outputs. The challenge of analyzing uncertain production outputs is two-fold. First, given the decentralized decision making by a large number of producers, there can be many ex-post scenarios in which different sets of producers are active. Second, when yields are uncertain, the modeling of general dependence among the yields can be challenging, and there are very limited developments in the existing literature on dependent yields even in the centralized systems (e.g., Chen and Gao 2019, Feng et al. 2019). Moreover, the stochastic notion of majorization order (e.g., Feng and Shanthikumar 2018a) needs to be extended to analyze allocations in decentralized systems. In view of all this, we present an analysis for each subsidy program under proportional random yields (i.e., an almost surely linear input-to-output relationship). We would focus on the case that all farmers produce ex-post in equilibrium (i.e., $\bar{N} = N$). This allows us to present a clean model without presenting tedious algebraic derivations for many cases, which would not offer many insights.

Let $Z = (Z_1, Z_2, \ldots, Z_n)$ denote the random vector of input-to-output ratios, with means $\mu_i = E[1/Z_i], i \in N$. Also, we denote $\mu_Z = (\mu_1, \mu_2, \ldots, \mu_n)$. Under random input-to-output ratios, farmer $j$’s objective function under the combined subsidy becomes

$$\pi_j(x, s_P, s_H) = E \left[ \left( \alpha - (c_H - s_H) - \beta \sum_i x_i/Z_j \right) x_j/Z_j - (c_P - s_P)x_j \right]$$

$$= (\alpha - c_H + s_H) x_j E[1/Z_j] - (c_P - s_P)x_j - \beta \sum_i x_i x_j E[1/(Z_i Z_j)], \quad j \in N.$$

**Lemma 5 (Production Equilibrium: Combined Subsidy)** Suppose that the government implements a combined subsidy $(s_P, s_H)$ and all farmers produce positive quantities. In equilibrium, farmers’ input quantities are

$$x^C(s_P, s_H) = \frac{1}{\beta} M^{-1} a(s_P, s_H),$$
and the farmers’ output quantities are

\[ Q^C(s_P, s_H) = (x_1^C(s_P, s_H)/Z_1, x_2^C(s_P, s_H)/Z_2, \ldots, x_n^C(s_P, s_H)/Z_n), \]

where \( M = (m_{ji}) \) is the correlation matrix with 

\[ m_{ji} = \begin{cases} I_{\{j=i\}}(2E[1/Z_j^2]) + I_{\{j\neq i\}}(E[1/(Z_jZ_i)]) \end{cases}, \]

\( \mathbf{1} = (1, 1, \ldots, 1) \), and 

\[ a(s_P, s_H) = (\alpha - c_H + s_H)\mu_Z - (c_P - s_P)\mathbf{1} \]

is the vector of average gross margins. Moreover, \( M \) is positive definite.

Proof of Lemma 5. It is clear that \( \pi_j \) is concave in \( x_j \) and thus the first-order condition of \( x_j \) gives

\[ \sum_{i \in N \setminus \{j\}} x_i E[1/(Z_iZ_j)] + 2x_j E[1/Z_j^2] = \frac{1}{\beta} \left( (\alpha - c_H + s_H)E[1/Z_j] - (c_P - s_P) \right), \quad j \in N. \]

Or, equivalently,

\[ M \mathbf{x} = \frac{1}{\beta} \left( (\alpha - c_H + s_H)\mu_Z - (c_P - s_P)\mathbf{1} \right), \]

where \( M = (m_{ji}) \) is the correlation matrix with 

\[ m_{ji} = \begin{cases} I_{\{j=i\}}(2E[1/Z_j^2]) + I_{\{j\neq i\}}(E[1/(Z_jZ_i)]) \end{cases}. \]

It is clear that \( M \) is positive definite (and thus invertible) and hence the above system of linear equations has a unique solution. □

According to Lemma 5, the market equilibrium depends on the vector of mean yields and the correlation matrix. It is important to recognize that the ranking of the farmers’ output levels depends on the realization of the random vector \( Z \). In other words, events \( \{Q^C_j(s_P, s_H) > Q^C_i(s_P, s_H)\} \) and \( \{Q^C_j(s_P, s_H) < Q^C_i(s_P, s_H)\} \) can both occur with positive probabilities.

Consequently, when measuring the evenness of the equilibrium output distribution, we must evaluate the order statistics of \( Q^C \). Let \( Q_{[j]} \) be the \( j \)th order statistic of the equilibrium output vector \( Q^C \). It is clear that for different realizations of the input-to-output ratios \( Z \), \( Q_{[j]} \) may correspond to the output of different farmers. One immediate extension of the majorization order in Definition 1 is to replace the equality by the stochastic equality in the usual stochastic order, i.e., replacing \( \leq \) by \( \leq_{st} \), to obtain the majorization or weak majorization order in the usual stochastic order, i.e., \( \leq_{m:st} \) or \( \leq_{wm:st} \) (see, Feng and Shanthikumar 2018a). One may also define majorization orders using convex order \( \leq_{cx} \), increasing convex order \( \leq_{icx} \), or increasing concave order \( \leq_{icv} \), depending on the joint distributions the random variables involved. In Figure 10, we demonstrate how the output distributions depend on the level of production uncertainty and the dependence among the input-to-output ratios. Here we examine the two extreme dependence structures. To
model general multi-variate distributions, one may apply copulas to examine the effect of dependence (e.g., Clemen and Reilly 1999, Feng et al. 2019), which requires a much involved analysis.

There are two farmers, and the figure plots the first order statistics of the output vector, \( Q_1 \), and the total output, \( Q_1 + Q_2 \). Four scenarios are considered here: The upper panels are for independent \((Z_1, Z_2)\), and the lower panels are for perfectly positively correlated \((Z_1, Z_2)\); The left panels are with low variability in the marginal distributions of \( Z_1 \) and \( Z_2 \), and the right panels are with high variability. We compare market \( A \) against market \( B \) with the former involving a larger yield dispersion between the two farmers. We observe from all four cases that \( Q^A_1 \leq_{st} Q^B_1 \) and \( Q^A_1 + Q^A_2 \leq_{ice} Q^B_1 + Q^B_2 \). Because the usual stochastic order implies the increasing concave order, we have the order statistics of outputs from market \( A \) weakly sup-majorizes those from market \( B \). In other words, when the dispersion of the marginal distributions of \((Z_1, Z_2)\) increases, the output distribution increases in the variability and decreases in the mean.

**Figure 10:** The distributions of partial sums of farmers’ output quantities: The combined subsidy

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Lemma 6 (Production Equilibrium: Selective Subsidy) Suppose that the government implements a selective subsidy \((s_P, s_H)\) and all farmers produce positive quantities. In equilibrium, farmers’ input quantities are

\[
x^S(s_P, s_H) = x^C(s_P, s_H) - \frac{1}{\beta}M^{-1}\left(s_H \text{diag}(\mu_Z)(1 - 1_m) + s_P 1_m\right),
\]

and the farmers’ output quantities are

\[
Q^S(s_P, s_H) = \frac{x^S_1(s_P, s_H)}{Z_1}, \frac{x^S_2(s_P, s_H)}{Z_2}, \ldots, \frac{x^S_n(s_P, s_H)}{Z_n},
\]

where \(1_m\) is a vector with the first \(m\) elements being one and the rest being zero, \(\text{diag}(\mu_Z)\) is a diagonal matrix with diagonal entries being \(\mu_Z\). \(M\) and \(a(0, 0)\) are defined in Lemma 5.

Proof of Lemma 6. It is clear that \(\pi_j\) is concave in \(x_j\) and thus the first-order condition of \(x_j\) gives

\[
Mx = \frac{1}{\beta}\left(a(0, 0) + s_H \text{diag}(\mu_Z) 1_m + s_P (1 - 1_m)\right).
\]

It is clear that \(M\) is positive definite and thus the above system of linear equations has a unique solution. This concludes the proof. \(\square\)

We present an example in Figure 11 to understand how the level of production uncertainty and the dependence among the input-to-output ratios affect the output distributions. In all scenarios, farmer 1 gets subsidized for plantation and farmer 2 gets compensated for harvesting. Like the combined subsidy, we can observe that \(Q^A_1 \leq_{st} Q^B_1\) and \(Q^A_1 + Q^A_2 \leq_{icv} Q^B_1 + Q^B_2\), implying that the order statistics of outputs from market \(A\) weakly sup-majorizes those from market \(B\). The difference is that this conclusion is true for the selective subsidy with the condition that the marginal distributions of farmers’ productivity levels have high variability (i.e., the right panels). When the marginal distributions of \((Z_1, Z_2)\) have low variability (i.e., the left panels), interestingly, we find that \(Q^A_1 + Q^A_2 \geq_{icx} Q^B_1 + Q^B_2\). In other words, as the dispersion of the marginal distributions of \((Z_1, Z_2)\) increases, the distribution of overall market output increases in both the mean and the variance.

As the farmers’ yield rates are random, the government aims to increasing the expected market output to meet the target level \(\bar{Q}\). Specifically, when a combined subsidy is offered, the government’s problem becomes

\[
\min_{s_P, s_H} \{ b^C(s_P, s_H) \equiv s_P X^C(s_P, s_H) + s_H E[1^\top Q^C(s_P, s_H)] : E[1^\top Q^C(s_P, s_H)] \geq \bar{Q}\},
\]
Notes. \( \alpha = 10, \beta = 1, c_F = c_H = 0.3, s_P = 0.13, s_H = 0.2, m = 1, z_A = (1.2, 1.8) \) and \( z_B = (1.4, 1.6) \), and \( \tilde{Z}_{ij} \) follows uniform distribution over \([\tilde{z}_{ij} - \delta, \tilde{z}_{ij} + \delta]\), \( i = A, B, j = 1, 2 \). \( \delta = 0.1 \) in the left panels and \( \delta = 0.4 \) in the right panels.

Figure 11: The distributions of partial sums of farmers’ output quantities: The selective subsidy where \( X^C(s_P, s_H) = 1^T x^C(s_P, s_H) \) and \( E[1^T Q^C(s_P, s_H)] \) are the overall industry input and the expected market output, respectively.

When a selective subsidy is offered, the government’s problem becomes

\[
\min_{s_P, s_H} \left\{ b^S(s_P, s_H) \equiv s_H E[1_m Q^S(s_P, s_H)] + s_P (1 - 1_m)^T x^S(s_P, s_H) : E[1^T Q^S(s_P, s_H)] \geq \bar{Q} \right\}.
\]

We now derive the optimal government subsidy designs and evaluate the key performance indicators.

**Proposition 14 (Combined Subsidy: Subsidy Design)** The government’s optimal design of the combined subsidy (with all farmers producing) is characterized as follows:

i) When only the planting subsidy is offered, \( s_P^P = \frac{\beta Q - \mu_Z M^{-1} a(0, 0)}{\mu_Z M^{-1} \mu_Z} \).

ii) When only the harvesting subsidy is offered, \( s_H^H = \frac{\beta Q - \mu_Z M^{-1} a(0, 0)}{\mu_Z M^{-1} \mu_Z} \).

iii) When the combined subsidy is offered, \( s_P^C = c_P / 2 \) and \( s_H^C = \frac{\beta Q - \mu_Z M^{-1} a(0, 0) - (c_P / 2) 1^T M^{-1} \mu_Z}{\mu_Z M^{-1} \mu_Z} \).

**Proof of Proposition 14.** Note that \( E[1^T Q^C(s_P, s_H)] = \bar{Q} \), i.e., the constraint is binding. Thus,

\[
\mu_Z M^{-1} (\mu_Z s_H + 1 s_P) = \beta \bar{Q} - \mu_Z M^{-1} a(0, 0) \quad \text{and} \quad \frac{ds_H}{ds_P} = -\frac{1^T M^{-1} \mu_Z}{\mu_Z M^{-1} \mu_Z}.
\]
Because the second term is positive, the right-hand side is increasing in $s$ and the optimal budget is

$$b^C(s_P, 0) = Q\left(\frac{\beta \bar{Q} - \mu_Z^\top M^{-1} a(0, 0)}{\mu_Z^\top M^{-1} \mu_Z}\right) - \frac{(\mu_Z^\top M^{-1} \mu_Z)(1^\top M^{-1} 1) - (1^\top C^{-1} \mu_Z)^2}{\beta \mu_Z^\top M^{-1} \mu_Z}(c_P - s_P^P)s_P^P.$$

(i) When only the planting subsidy is offered, we must have

$$s_P^P = \frac{\beta \bar{Q} - \mu_Z^\top M^{-1} a(0, 0)}{1^\top M^{-1} \mu_Z},$$

and the optimal budget is

$$b^C(s_P^P, 0) = Q\left(\frac{\beta \bar{Q} - \mu_Z^\top M^{-1} a(0, 0)}{\mu_Z^\top M^{-1} \mu_Z}\right).$$

(ii) When only the harvesting subsidy is offered, we must have

$$s_H^H = \frac{\beta \bar{Q} - \mu_Z^\top M^{-1} a(0, 0)}{\mu_Z^\top M^{-1} \mu_Z},$$

and the optimal budget is

$$b^C(0, s_H^H) = Q\left(\frac{\beta \bar{Q} - \mu_Z^\top M^{-1} a(0, 0)}{\mu_Z^\top M^{-1} \mu_Z}\right).$$

(iii) When the combined subsidy is offered, we must have

$$\frac{db^C(s_P, s_H)}{ds_P} = \frac{\partial b^C(s_P, s_H)}{\partial s_P} \frac{ds_P}{ds_P} + \frac{\partial b^C(s_P, s_H)}{\partial s_H} \frac{ds_H}{ds_P} = \frac{1}{\beta}(2s_P - c_P)\left(1^\top M^{-1} 1 - \frac{(1^\top M^{-1} \mu_Z)^2}{\mu_Z^\top M^{-1} \mu_Z}\right).$$

Because the second term is positive, the right-hand side is increasing in $s_P$. It follows that $b^C$ is convex in $s_P$ and is minimized at $s_P^C = c_P/2$. We then obtain $s_H^C = \frac{\beta \bar{Q} - \mu_Z^\top M^{-1} a(0, 0) - (c_P/2)1^\top M^{-1} \mu_Z}{\mu_Z^\top M^{-1} \mu_Z}$.

The optimal budget is

$$b^C(s_P^C, s_H^C) = Q\left(\frac{\beta \bar{Q} - \mu_Z^\top M^{-1} a(0, 0)}{\mu_Z^\top M^{-1} \mu_Z}\right) - \frac{(1^\top M^{-1} 1)(\mu_Z^\top M^{-1} \mu_Z) - (1^\top M^{-1} \mu_Z)^2 c_P^2}{4\beta},$$

This concludes the proof. \qed

Recall Proposition 8 that the optimal planting subsidy and harvesting subsidy is related through the industrial average input-to-output ratio. When the production outputs become uncertain, the two subsidy formats are related through the vector of mean yields and the correlation matrix. We also note that, under the combined subsidy, $s_H^C$ can be negative depending on the target output level, the mean yields, and the correlation among the yields. In this case, the half of plantation cost is paid by the government, while a tax is collected on harvesting.

**Proposition 15 (Selective Subsidy: Subsidy Design)** The government’s optimal design of the selective subsidy (with all farmers producing) satisfies $(s_P^S, s_H^S) = (s_P^P, 0)$, $(s_P^S, s_H^S) = (0, s_H^H)$, or $(s_P^S, s_H^S)$ is an element in the set

$$\left\{(s_P, s_H) \mid s_P = \max\{\phi(m + 1), \min\{c_1(m)s_H^P + c_2(m)(c_P/2), \phi(m)\}\}, \quad s_H = \frac{\beta \bar{Q} - \mu_Z^\top M^{-1} a(0, 0) + (1 - 1_m)s_P}{\mu_Z^\top M^{-1} \text{diag}(\mu_Z)^1_m}, \quad \text{for } m \in \mathbb{N}\setminus\{n\}\right\},$$

where $\phi(m) = (\beta \bar{Q} - \mu_Z^\top M^{-1} a(0, 0)) / (\mu_Z^\top M^{-1} \text{diag}(\mu_Z)^1_m(1 + 1_m) + (1 - 1_m))$, and $c_1(m)$ and $c_2(m)$ are functions of $M$, $\mu_Z$ and $1_m$.  

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Proof of Proposition 15. Note that \( E[1^T Q^S(s_p, s_H)] = \bar{Q} \). Thus,

\[ \mu Z^TM^{-1}(s_H\text{diag}(\mu Z)1_m + s_p(1 - 1_m)) = \beta \bar{Q} - \mu Z^TM^{-1}a(0, 0) \] and \( \frac{ds_H}{ds_p} = -\frac{\mu Z^TM^{-1}(1 - 1_m)}{\mu Z^TM^{-1}\text{diag}(\mu Z)1_m} \).

Now we differentiate \( b^S(s_p, s_H) \) with respect to \( s_p \), and obtain

\[
\frac{db^S(s_p, s_H)}{ds_p} = \frac{2}{\beta}(1 - 1_m)^T\left(\text{diag}(1) - \frac{M^{-1}\mu Z1_m^T\text{diag}(\mu Z)}{\mu Z^TM^{-1}\text{diag}(\mu Z)1_m}\right)M^{-1}\left(s_H\text{diag}(\mu Z)1_m + s_p(1 - 1_m) - \frac{c_p}{2}1\right)
\]

\[
= \frac{2}{\beta}(1 - 1_m)^T\left(\text{diag}(1) - \frac{M^{-1}\mu Z1_m^T\text{diag}(\mu Z)}{\mu Z^TM^{-1}\text{diag}(\mu Z)1_m}\right)M^{-1}\left(\text{diag}(\mu Z)1_m\beta \bar{Q} - \frac{\mu Z^TM^{-1}a(0, 0)}{\mu Z^TM^{-1}\text{diag}(\mu Z)1_m}\right)
\]

\[
+ s_p\left(\text{diag}(1) - \frac{\text{diag}(\mu Z)1_m\mu Z^M^{-1}}{\mu Z^M^{-1}\text{diag}(\mu Z)1_m}(1 - 1_m) - \frac{c_p}{2}1\right).
\]

Because the coefficient of \( s_p \) is positive, the right-hand side is increasing in \( s_p \). It follows that \( b^S \)

is convex in \( s_p \) for each \( m \). Setting \( db^S(s_p, s_H)/ds_p = 0 \) gives

\[
s_{P,m} = \frac{(1 - 1_m)^T\left(\text{diag}(1) - \frac{M^{-1}\mu Z1_m^T\text{diag}(\mu Z)}{\mu Z^M^{-1}\text{diag}(\mu Z)1_m}\right)M^{-1}}{(1 - 1_m)^T\left(\text{diag}(1) - \frac{M^{-1}\mu Z1_m^T\text{diag}(\mu Z)}{\mu Z^M^{-1}\text{diag}(\mu Z)1_m}\right)M^{-1}(1 - 1_m) - \frac{c_p}{2}} \cdot s_p^p.
\]

It follows that

\[
s_{H,m} = \frac{\beta \bar{Q} - \mu Z^M^{-1}(a(0, 0) + (1 - 1_m)s_{P,m})}{\mu Z^M^{-1}\text{diag}(\mu Z)1_m}.
\]

By the definition of \( m \), we must have \( s_{H,m}\mu Z_{m+1} < s_{P,m} \leq s_{H,m}\mu Z_m \). This yields \( \phi(m + 1) < s_{P,m} \leq \phi(m) \) with \( \phi(m) \) defined as

\[
\phi(m) = \frac{\beta \bar{Q} - \mu Z^M^{-1}a(0, 0)}{\mu Z^M^{-1}(\text{diag}(\mu Z)1_m(1/\mu Z_m) + (1 - 1_m))}.
\]

Finally, we note that only the planting subsidy and only the harvesting subsidy can be candidate solutions. This concludes the proof.

Similar to our observations in Proposition 9, the format of subsidy offered depends on the target output level set by the government when the farmers’ outputs become uncertain. However, in the presence of uncertainty, plantation and harvesting payments depend not only on the farmers’ choices of subsidies but also the correlation among the yields.

**Proposition 16 (Key Performance Indicators)** Suppose that the government sets a target output \( \bar{Q} \) and all farmers produces.
Proof of Proposition 16. The proof can be obtained by adapting the proofs of Propositions 10 and 11. We only provide the expression of overall industry input in each subsidy program, i.e.,

\[
X^C(s_p, 0) = \frac{1^\top M^{-1} \mu Z}{\mu Z M^{-1} \mu Z} \tilde{Q} - \frac{(\mu^\top Z M^{-1} \mu Z)(1^\top M^{-1} 1) - (1^\top C^{-1} \mu Z)^2}{\beta \mu Z M^{-1} \mu Z} (c_p - s_p),
\]

\[
X^C(0, s_H^*) = \frac{1^\top M^{-1} \mu Z}{\mu Z M^{-1} \mu Z} \tilde{Q} - \frac{(\mu^\top Z M^{-1} \mu Z)(1^\top M^{-1} 1) - (1^\top C^{-1} \mu Z)^2}{\beta \mu Z M^{-1} \mu Z} c_p,
\]

\[
X^C(s_p^*, s_H^*) = \frac{1^\top M^{-1} \mu Z}{\mu Z M^{-1} \mu Z} \tilde{Q} - \frac{(\mu^\top Z M^{-1} \mu Z)(1^\top M^{-1} 1) - (1^\top C^{-1} \mu Z)^2}{\beta \mu Z M^{-1} \mu Z} \frac{c_p}{2}.
\]

This concludes the proof. □

Compared with Propositions 10 and 11, Proposition 16 suggests the main insights obtained from the base model remain robust when production uncertainty is introduced. We refer to an example depicted in Figure 12. The planting subsidy requires the highest budget spending and input resources, but generates the lowest net social welfare, when the government sets an aggressive target output level. In contrast, the harvesting subsidy is the most efficient in the consumption of resources and achieves the highest net social welfare. The combined subsidy leads to the lowest government spending. The performance of a selective subsidy, in general, stays in the middle among the programs regarding government spending, resource requirement, and social welfare.

C Alternative Plantation Cost Structure

In this section, we analyze the model by replacing the linear planting cost \(c_p x_j\) by a quadratic cost \(c_p x_j + (\beta/2)c_p^2 x_j^2\). This cost structure captures the effect of diminishing returns of input resources. For example, farmer education or fertilizer use can improve productivity. However, the amount of investment needed for the marginal output increase gets higher as the investment becomes larger.

Compared with the linear planting cost, the effect of convex cost essentially makes the farmer’s marginal gain reduce rapidly with the decision \(x_j\). As Akkaya et al. (2016) point out, this effect, to a certain extent, reflects the risk aversion of the farmer’s production decision, inducing the farmers to be conservative and reduce the production. This is evident from the market equilibrium characterized in the next lemma.
Lemma 7 (Production Equilibrium: Combined Subsidy) Suppose that the government implements a combined subsidy \((s_P, s_H)\) and all farmers produce positive quantities. In equilibrium, farmers’ output quantities are

\[ q^C(s_P, s_H) = \frac{1}{\beta} C^{-1} a(s_P, s_H), \]

where \(C = (c_{ji})\) with \(c_{ji} = I_{[j=i]}(2 + c_P z_j^2) + I_{[j\neq i]}(1)\), and \(a(s_P, s_H) = (\alpha - c_H + s_H)1 - (c_P - s_P)z\). Moreover, \(C\) is positive definite.

Proof of Lemma 7. It is clear that \(\pi_j\) is concave in \(q_j\) and thus the first-order condition of \(q_j\) gives

\[ Cq = \frac{1}{\beta}\left( (\alpha - c_H + s_H)1 - (c_P - s_P)z \right), \]

where \(C = (c_{ji})\) with \(c_{ji} = I_{[j=i]}(2 + c_P z_j^2) + I_{[j\neq i]}(1)\). It is clear that \(C\) is invertible and hence the above system of linear equations has a unique solution. \(\square\)

The examples in Figure 13 present the effect of subsidies on the farmers’ market. When the subsidies are at a moderate level, increased subsidies lead to an increase in every farmer’s output.
The output increase due to harvesting subsidy is more significant for more productive farmers. These observations are in line with their counterparts in the case of the linear planting cost. The difference lies in the case when the government overly subsidizes on the plantation. In this case, we observe that the farmers with intermediate productivity levels can generate higher output than those with low or high productivity levels.

![Figure 13: The farmers’ production equilibrium with a quadratic cost structure under combined subsidy (s_P, s_H).](image)

Notes. α = 3, β = 1, c_P = c_P2 = 0.2, c_H = 0.3 and z = (1, 1.5, 2.1, 2.8, 3.6).

**Proposition 17 (Combined Subsidy: Subsidy Design)** The government’s optimal design of the combined subsidy (with all farmers producing) is characterized as follows:

i) When only the planting subsidy is offered, 
\[ s_P^* = \frac{\beta \tilde{Q} - 1^T C^{-1} a(0,0)}{1^T C^{-1} z} \cdot \]

ii) When only the harvesting subsidy is offered, 
\[ s_H^* = \frac{\beta \tilde{Q} - 1^T C^{-1} a(0,0)}{1^T C^{-1} z} \cdot \]

iii) When the combined subsidy is offered, 
\[ s_P^C = \frac{c_P}{2} \text{ and } s_H^C = \frac{\beta \tilde{Q} - 1^T C^{-1} a(0,0) - (c_P/2) 1^T C^{-1} z}{1^T C^{-1} z} . \]

**Proof of Proposition 17.** Note that \( Q^C(s_P, s_H) = \tilde{Q} \). Thus, 
\[ 1^T C^{-1} (1s_H + zs_P) = \beta \tilde{Q} - 1^T C^{-1} a(0,0) \] and 
\[ \frac{ds_H}{ds_P} = -\frac{1^T C^{-1} z}{1^T C^{-1} 1} . \]

(i) When only the planting subsidy is offered, we must have 
\[ s_P^* = \frac{\beta \tilde{Q} - 1^T C^{-1} a(0,0)}{1^T C^{-1} z} , \]

and the optimal budget is 
\[ b^C(s_P^*, 0) = \tilde{Q} \left( \frac{\beta \tilde{Q} - 1^T C^{-1} a(0,0)}{1^T C^{-1} 1} \right) - \frac{(z^T C^{-1} z)(1^T C^{-1} 1) - (1^T C^{-1} z)^2}{\beta 1^T C^{-1} z} (c_P - s_P^*) s_P^* . \]
(ii) When only the harvesting subsidy is offered, we must have
\[ s^H_H = \frac{\beta \bar{Q} - 1^\top C^{-1} a(0,0)}{1^\top C^{-1} 1}, \]
and the optimal budget is
\[ b^C(0, s^H_H) = \bar{Q} \left( \frac{\beta \bar{Q} - 1^\top C^{-1} a(0,0)}{1^\top C^{-1} 1} \right). \]

(iii) When the combined subsidy is offered, we must have
\[ \frac{\partial b^C(s_P, s^C)}{\partial s_P} = \frac{\partial b^C(s_P, s^H)}{\partial s_P} + \frac{\partial b^C(s_P, s^H)}{\partial s^H} \frac{ds^H}{ds_P} = \frac{1}{\beta} (2s_P - c_P) \left( z^\top C^{-1} z - \frac{(1^\top C^{-1} z)^2}{1^\top C^{-1} 1} \right). \]
Because the second term is positive, the right-hand side is increasing in \( s_P \). It follows that \( b^C \) is convex in \( s_P \) and is minimized at \( s^C_P = c_P/2 \). We then obtain \( s^C_H = \frac{\beta \bar{Q} - 1^\top C^{-1} a(0,0)}{1^\top C^{-1} 1} \frac{(c_P/2)^{1^\top C^{-1} 1}}{1^\top C^{-1} 1} \). The optimal budget is
\[ b^C(s^C_P, s^C_H) = \bar{Q} \left( \frac{\beta \bar{Q} - 1^\top C^{-1} a(0,0)}{1^\top C^{-1} 1} \right) - \frac{(z^\top C^{-1} z)(1^\top C^{-1} 1) - (1^\top C^{-1} z)^2}{4\beta} c^2_P. \]
This concludes the proof.

**Proposition 18 (Key Performance Indicators)** Suppose that the government sets a target output \( \bar{Q} \) and all farmers produces.

i) **Budget Comparison:** For \( \bar{Q} \leq Q^C(c_P,0) \), \( b^C(s^C_P, s^C_H) \leq b^C(s^P_P,0) \leq b^C(0, s^H_H) \). For \( \bar{Q} > Q^C(c_P,0) \), \( b^C(s^C_P, s^C_H) \leq b^C(0, s^H_H) \leq b^C(s^P_P,0) \).

ii) **Recourse Requirement:** For \( \bar{Q} \leq Q^C(c_P/2,0) \), \( X^C(0, s^H_H) \leq X^C(s^P_P,0) \leq X^C(s^C_P, s^C_H) \). For \( \bar{Q} > Q^C(c_P/2,0) \), \( X^C(0, s^H_H) \leq X^C(s^C_P, s^C_H) \leq X^C(s^P_P,0) \).

**Proof of Proposition 18.** The proof can be obtained by adapting the proof of Proposition 16. We only provide the expression of overall industry input in each subsidy program, i.e.,
\[
X^C(s^P_P,0) = 1^\top C^{-1} z \bar{Q} - \frac{(z^\top C^{-1} z)(1^\top C^{-1} 1) - (1^\top C^{-1} z)^2}{\beta 1^\top C^{-1} 1}(c_P - s^P_P),
\]
\[
X^C(0, s^H_H) = 1^\top C^{-1} z \bar{Q} - \frac{(z^\top C^{-1} z)(1^\top C^{-1} 1) - (1^\top C^{-1} z)^2}{\beta 1^\top C^{-1} 1} c_P,
\]
\[
X^C(s^C_P, s^C_H) = 1^\top C^{-1} z \bar{Q} - \frac{(z^\top C^{-1} z)(1^\top C^{-1} 1) - (1^\top C^{-1} z)^2}{\beta 1^\top C^{-1} 1} c_P/2.
\]
This concludes the proof.

The observations from Propositions 17 and 18 are similar to their counterparts from the linear plantation cost discussed in Section 5. The main difference lies in the fact that, with a convex cost structure, the planting subsidy and the harvesting subsidy are related through the entire vector of input-to-output ratios \( z \), rather than the average ratio \( \bar{z} \).
D Summary of Notation

Table 2: The summary of notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{N}$</td>
<td>the set of farmers</td>
</tr>
<tr>
<td>$\bar{n}$</td>
<td>the number of farmers</td>
</tr>
<tr>
<td>$c_P/c_H$</td>
<td>the cost per unit of input/output</td>
</tr>
<tr>
<td>$x_j/q_j$</td>
<td>farmer $j$’s input/output units</td>
</tr>
<tr>
<td>$z_j$</td>
<td>farmer $j$’s input-to-output ratio</td>
</tr>
<tr>
<td>$N$</td>
<td>the set of active farmers</td>
</tr>
<tr>
<td>$n$</td>
<td>the number of active farmers</td>
</tr>
<tr>
<td>$\bar{z}/v_z$</td>
<td>the average/variability of the input-to-output ratio of active farmers</td>
</tr>
<tr>
<td>$s_P/s_H$</td>
<td>the payment per unit planted/harvested</td>
</tr>
<tr>
<td>$\pi_j(q)$</td>
<td>farmer $j$’s profit function given an output vector $q$</td>
</tr>
<tr>
<td>$q_j^<em>/\pi_j^</em>$</td>
<td>farmer $j$’s equilibrium output quantity/profit without subsidy</td>
</tr>
<tr>
<td>$X/Q/\Pi$</td>
<td>the overall industry input/market output/farmers’ profit without subsidy</td>
</tr>
<tr>
<td>$N^X(s_P,s_H)$</td>
<td>the set of active farmers given subsidies $(s_P,s_H)$</td>
</tr>
<tr>
<td>$n^X$</td>
<td>the number of active farmers</td>
</tr>
<tr>
<td>$q_j^X(s_P,s_H)/\pi_j^X(s_P,s_H)$</td>
<td>farmer $j$’s equilibrium output quantity/profit given subsidies $(s_P,s_H)$</td>
</tr>
<tr>
<td>$X^X(s_P,s_H)$</td>
<td>the overall industry input given subsidies $(s_P,s_H)$</td>
</tr>
<tr>
<td>$Q^X(s_P,s_H)$</td>
<td>the market output given subsidies $(s_P,s_H)$</td>
</tr>
<tr>
<td>$\Pi^X(s_P,s_H)$</td>
<td>the farmers’ profit given subsidies $(s_P,s_H)$</td>
</tr>
<tr>
<td>$\Delta_{s_P}q_j^X/\Delta_{s_H}q_j^X$</td>
<td>farmer $j$’s output change to an increase in $s_P/s_H$</td>
</tr>
<tr>
<td>$\Delta_{s_P}\pi_j^X/\Delta_{s_H}\pi_j^X$</td>
<td>farmer $j$’s profit change to an increase in $s_P/s_H$</td>
</tr>
<tr>
<td>$b^X(s_P,s_H)$</td>
<td>the government’s budget spending</td>
</tr>
<tr>
<td>$s_P^P/s_H^P/(s_P^C,s_H^C)/(s_P^S,s_H^S)$</td>
<td>the optimal planting/harvesting/combined/selective subsidy</td>
</tr>
<tr>
<td>$W^X(s_P,s_H)/NW^X(s_P,s_H)$</td>
<td>social welfare/net social welfare</td>
</tr>
<tr>
<td>$Z_j$</td>
<td>farmer $j$’s random input-to-output ratio with mean $\mu_j = E[1/Z_j]$</td>
</tr>
</tbody>
</table>

Notes. $X \in \{P,H,C,S\}$. We use superscripts $P,H,C,S$ to denote the quantities under the planting, harvesting, combined, and selective subsidy programs.